Thomas Precession Using the Selleri Transformations

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Abstract. In this paper, Thomas Precession is derived using the Selleri Transformations. These transformations belong to a set of "equivalent" transformations derived by Selleri which differ by a single parameter e_1 and include the Lorentz Transformations corresponding to a particular non-zero value of e_1 and the Selleri Transformations corresponding to $e_1 = 0$.

Key Words. Thomas precession, Selleri transformations, generalized transformations.

1. Introduction

Thomas Precession is named after Llewellyn Thomas who analyzed its action in 1926 [1]. It is a relativistic effect that occurs in the spin of an elementary particle or the movement of a gyroscope. Specifically, it refers to the rotation of a moving body which occurs when the body changes its velocity direction because of some external force. The action is predicted by the Lorentz transformations [2, 3] and occurs as a result of the application of two successive Lorentz transformations in two different directions. The rotation always occurs in the plane of the two velocities. According to Dragan [4], "Whenever a moving body changes the direction of velocity due to some external forces, it must rotate. This happens because the change of motion can be seen as a composition of two Lorentz transformations. Therefore any object moving along a curvilinear trajectory has to rotate, even if no torque is directly applied. This geometrical effect is called Thomas precession."

In this paper, we demonstrate precession and derive the associated angular velocity using the Selleri transformations for the first time. Selleri has studied these (inertial) transformations extensively [5-7] and has shown that they reproduce many of the predictions of the Lorentz transformations including length contraction [5], time dilation [5], Doppler effect [8], Aberration [8], the Sagnac effect [9] and clock synchronization [9] without any inconsistencies. He has also applied the transformations to dynamics involving particle collisions [10] and Buonaura [11] has applied the Selleri transformations to the area of electromagnetism. We here derive precession using the Selleri transformations and a very simple and elegant method developed by Dragan [4].

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2. Selleri Transformations

Consider an inertial system S with space and time coordinates x, y, z, t in which the speed of light is c, and another inertial system S' having space and time coordinates x', y', z', t' which is moving at speed v relative to S along the x axis. The two systems are coincident at $t = t' = 0$. Using two-way light speed constancy and experimentally confirmed dilated muon decay, Selleri [5-7] derived a set of transformations which differed by a single parameter e_1. This set can be written as

$$x' = \gamma(x - vt) \tag{1}$$

$$y' = y \tag{2}$$

$$z' = z \tag{3}$$

$$t' = \frac{t}{\gamma} + e_1(x - vt) \tag{4}$$

where $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = v/c$ and e_1 is now the only unknown parameter. Here the Lorentz transformations correspond to $e_1 = -\beta/c\sqrt{1-\beta^2}$ and the Selleri transformations correspond to $e_1 = 0$ which are

$$x' = \gamma(x - vt) \tag{5}$$

$$y' = y \tag{6}$$

$$z' = z \tag{7}$$

$$t' = \frac{t}{\gamma} \tag{8}$$

Selleri referred to the set contained in (1)-(4) as the "Equivalent" transformations since all members of the set (including the Lorentz transformations) are "equivalent" in the sense that they differ only by the clock synchronization parameter e_1 in the time component of the transformations and make the same predictions for many (but not all) phenomena. It is well known that the Lorentz transformations predict Thomas precession and its occurrence in orbiting electrons is considered experimental verification of the relativistic kinematics of the Lorentz transformations.

2.1 Generalized Selleri Transformations

In order to treat with frames moving in directions that are different from the direction of the x axis, we need generalized transformations. Consider a coordinate vector $\boldsymbol{r}(x, y, z)$ and a velocity $\boldsymbol{v}$ of the S' frame. Now the component $\boldsymbol{r}_{pd}$ of $\boldsymbol{r}$ that is perpendicular to the velocity vector is unaffected by the transformation while the component $\boldsymbol{r}_{pl}$ that is parallel to the velocity vector does change. Here $\boldsymbol{r} = \boldsymbol{r}_{pl} + \boldsymbol{r}_{pd}$ and $\boldsymbol{r}_{pl} = (\boldsymbol{r}.\frac{\boldsymbol{v}}{v})\frac{\boldsymbol{v}}{v}$ where $\frac{\boldsymbol{v}}{v}$ is the unit vector parallel to $\boldsymbol{v}$. Therefore, the space transformation becomes

$$\boldsymbol{r}'_{pl} = \gamma(\boldsymbol{r}_{pl} - \boldsymbol{v}t) = \gamma(\frac{\boldsymbol{r}.\boldsymbol{v}}{v^2} - t)\boldsymbol{v} \tag{9}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$ and

$$\boldsymbol{r}'_{pd} = \boldsymbol{r}_{pd} = \boldsymbol{r} - \boldsymbol{r}_{pl} = \boldsymbol{r} - (\frac{\boldsymbol{r}.\boldsymbol{v}}{v^2})\boldsymbol{v} \tag{10}$$

Since $\boldsymbol{r}' = \boldsymbol{r}'_{pd} + \boldsymbol{r}'_{pl}$, the new space transformation for the Selleri Transformations is given by

$$\boldsymbol{r}' = \boldsymbol{r}'_{pd} + \boldsymbol{r}'_{pl} = \boldsymbol{r} - (\frac{\boldsymbol{r}.\boldsymbol{v}}{v^2})\boldsymbol{v} + \gamma(\frac{\boldsymbol{r}.\boldsymbol{v}}{v^2} - t)\boldsymbol{v} \tag{11}$$

The Selleri time transformation is from (8) given by $t' = t/\gamma$. Hence the generalized Selleri transformations are

$$\boldsymbol{r}' = \boldsymbol{r} - (\frac{\boldsymbol{r}.\boldsymbol{v}}{v^2})\boldsymbol{v} + \gamma(\frac{\boldsymbol{r}.\boldsymbol{v}}{v^2} - t)\boldsymbol{v} \tag{12}$$

$$t' = t/\gamma \tag{13}$$

A generalized velocity transformation formula can now be derived. Taking differentials in (12) and (13) we get

$$d\boldsymbol{r}' = d\boldsymbol{r} - (\frac{d\boldsymbol{r}.\boldsymbol{v}}{v^2})\boldsymbol{v} + \gamma(\frac{d\boldsymbol{r}.\boldsymbol{v}}{v^2} - dt)\boldsymbol{v} \tag{14}$$

and

$$dt' = dt/\gamma \tag{15}$$

Dividing (14) by (15) gives

$$\frac{d\boldsymbol{r}'}{dt'} = \frac{d\boldsymbol{r} - (\frac{d\boldsymbol{r}.\boldsymbol{v}}{v^2})\boldsymbol{v} + \gamma(\frac{d\boldsymbol{r}.\boldsymbol{v}}{v^2} - dt)\boldsymbol{v}}{dt\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{d\boldsymbol{r}}{dt} - (\frac{\frac{d\boldsymbol{r}}{dt}.\boldsymbol{v}}{v^2})\boldsymbol{v} + \gamma(\frac{\frac{d\boldsymbol{r}}{dt}.\boldsymbol{v}}{v^2} - 1)\boldsymbol{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{16}$$

Using $\boldsymbol{u} = \frac{d\boldsymbol{r}}{dt}$ and $\boldsymbol{u}' = \frac{d\boldsymbol{r}'}{dt'}$, (16) becomes

$$\boldsymbol{u}' = \frac{\sqrt{1-\frac{v^2}{c^2}}(\boldsymbol{u} - \frac{\boldsymbol{u}.\boldsymbol{v}}{v^2}\boldsymbol{v}) - (\boldsymbol{v} - \frac{\boldsymbol{u}.\boldsymbol{v}}{v^2}\boldsymbol{v})}{1-\frac{v^2}{c^2}} \tag{17}$$

which is the generalized velocity transformation formula under the Selleri transformations.

3. Thomas Precession

Consider a stationary observer A and an observer B moving at velocity $\boldsymbol{v}$ relative to A. If $B's$ velocity changes by an infinitesimal value $\boldsymbol{dv}'$ relative to his initial reference frame B, then observer A sees $B's$ velocity change from $\boldsymbol{v}$ to $\boldsymbol{v} + d\boldsymbol{v}$ and $B's$ new frame B' is rotated by an angle $d\boldsymbol{\Omega}$ relative to $A's$ frame. The angle $d\boldsymbol{\Omega}$ can be viewed as the angle between the velocity $\boldsymbol{v} + d\boldsymbol{v}$ of B' with respect to A and its approximation $\boldsymbol{v} + d\boldsymbol{v}'$. For a small angle $d\boldsymbol{\Omega}$, $\sin d\Omega \simeq d\Omega$. Noting that $\boldsymbol{v} \times \boldsymbol{v} = 0$, the value of $d\boldsymbol{\Omega}$ can be found by taking the vector product of $\boldsymbol{v} + d\boldsymbol{v}$ and $\boldsymbol{v} + d\boldsymbol{v}'$ giving

$$d\boldsymbol{\Omega} \simeq \frac{(\boldsymbol{v} + d\boldsymbol{v}') \times (\boldsymbol{v} + d\boldsymbol{v})}{v^2} \simeq \frac{1}{v^2}(\boldsymbol{v} \times d\boldsymbol{v} - \boldsymbol{v} \times d\boldsymbol{v}') \tag{18}$$

Using the generalized velocity transformation (17) with $\boldsymbol{u} = \boldsymbol{v} + d\boldsymbol{v}$ and $\boldsymbol{u}' = d\boldsymbol{v}'$ gives

$$d\boldsymbol{v}' = \frac{\sqrt{1-\frac{v^2}{c^2}}(\boldsymbol{v} + \mathrm{d}\boldsymbol{v} - \frac{(\boldsymbol{v} + \mathrm{d}\boldsymbol{v}).\boldsymbol{v}}{v^2}\boldsymbol{v}) - (\boldsymbol{v} - \frac{(\boldsymbol{v} + \mathrm{d}\boldsymbol{v}).\boldsymbol{v}}{v^2}\boldsymbol{v})}{1-\frac{v^2}{c^2}} \tag{19}$$

Again using $\boldsymbol{v} \times \boldsymbol{v} = 0$, we vector multiply equation (19) by $\boldsymbol{v}$ to get

$$\boldsymbol{v} \times d\boldsymbol{v}' = \frac{\sqrt{1-\frac{v^2}{c^2}}\boldsymbol{v} \times \mathrm{d}\boldsymbol{v}}{1-\frac{v^2}{c^2}} = \frac{\boldsymbol{v} \times \mathrm{d}\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{20}$$

Substituting (20) in (18) results in

$$d\boldsymbol{\Omega} \simeq \frac{1}{v^2}(\boldsymbol{v} \times d\boldsymbol{v} - \frac{\boldsymbol{v} \times \mathrm{d}\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}) = -\frac{1}{v^2}(\gamma - 1)\boldsymbol{v} \times \mathrm{d}\boldsymbol{v} \tag{21}$$

Dividing both sides by the infinitesimal time dt gives

$$\frac{d\boldsymbol{\Omega}}{dt} = -\frac{1}{v^2}(\gamma - 1)\boldsymbol{v} \times \frac{\mathrm{d}\boldsymbol{v}}{dt} \tag{22}$$

Setting $\boldsymbol{\omega}_T = \frac{d\boldsymbol{\Omega}}{dt}$ and $\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt}$, (22) becomes

$$\omega_T = -\frac{1}{v^2}(\gamma - 1)\boldsymbol{v}\times\boldsymbol{a} \tag{23}$$

This is the Thomas precession rate [3] which can be written as

$$\omega_T = -\frac{\gamma^2}{1+\gamma}\frac{\boldsymbol{v}\times\boldsymbol{a}}{c^2} \tag{24}$$

For $v << c$, (24) reduces to

$$\omega_T = -\frac{1}{2}\frac{\boldsymbol{v}\times\boldsymbol{a}}{c^2} \tag{25}$$

Fig.1 taken from Christodoulides [3] shows the precession of an orbiting electron. Here the electron orbits the nucleus with a velocity $\boldsymbol{v}$, centripetal acceleration $\boldsymbol{a}$, spin $\boldsymbol{s}$ and angular momentum $\boldsymbol{l}$. The spin vector precesses about the normal to the orbital plane (which is in the same direction as $\boldsymbol{l}$) at an angular velocity ω_T . Thus, the Selleri transformations are able to correctly predict Thomas precession.

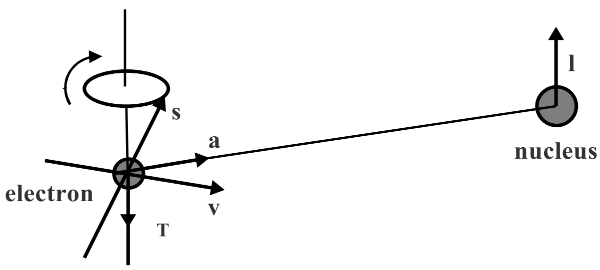

Fig.1 Thomas Precession occurring when an Electron orbits the Nucleus

4. Conclusion

Thomas precession using the Selleri transformations is demonstrated in this paper for the first time. This phenomenon, like the transverse Doppler effect and relativistic elastic collisions, is a strictly relativistic effect that has no counterpart in classical mechanics.

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