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Insights from models that specify elementary particles, dark matter, and properties of objects and fields

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Abstract

This paper suggests insights that people might glean from so-called new modeling models that specify elementary particles, dark matter, and properties of objects and fields. New modeling has roots in successful popular physics modeling and, when combined with popular modeling, suggests explanations for data that popular modeling seems not to explain. The data pertain to elementary particles, dark-matter phenomena, and dark-energy phenomena. Possibly, new modeling points to a framework that provides a basis for successful popular modeling. Insights that this paper suggests pertain to elementary-particle physics, cosmology and astrophysics, general physics, and physics modeling.

Keywords: elementary particles, dark matter, dark energy, beyond the Standard Model, galaxy evolution, vacuum energy, quantum gravity

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1. Introduction

The following topics and questions highlight currently unexplained physics patterns.

1. Properties of objects: Might models catalog properties (such as charge, magnetic moment, energy, momentum, and intrinsic angular momentum) of objects?
2. Elementary particles: Might models catalog all known elementary particles and predict new elementary particles?
3. Dark matter: Might models explain data about ratios of not-ordinary-matter (as possibly in dark-matter) effects to ordinary-matter effects?
4. Dark energy: Might models explain data about gravity and so-called dark-energy phenomena?
5. Conservation laws: Might models catalog conservation laws?

People might consider the following two approaches to making progress regarding (at least the five above-mentioned) physics topics and unexplained patterns.

1. Focus - perhaps exclusively - on aspects that link directly to so-called (in this paper) popular modeling. Popular modeling tends to link to mathematical equations that feature continuous (space-time) coordinates and to the principle of stationary action.
2. Look for patterns and for so-called (in this paper) new modeling that does not violate successful (regarding explaining data) popular modeling and that does not necessarily link directly to either mathematics that features continuous coordinates or the principle of stationary action.

Today, much effort links to the first approach. (Section 2.3.2 of Ref. [1] discusses some endeavors that try to address some of the above-mentioned topics and that link to the first approach. The following notions link acronyms in Ref. [1] to terminology in the present paper. POST is popular modeling. SUPP is new modeling.)

Data might point to desirability to deploy the second approach. For example, data suggest the possibility that nature includes twice as much cosmic optical background radiation as popular modeling predicts. (Refs. [2, 3, 4, 5] provide data and discussion regarding the amount of cosmic optical background.) Also, data suggest the possibility that nature exhibits twice as much of a specific type of depletion of CMB (as in cosmic microwave background radiation) as popular modeling predicts. (Refs. [6, 7, 8] provide data and discussion regarding relevant absorption of CMB.) Popular modeling links to atomic phenomena the creation of half of the cosmic optical background radiation. Popular modeling links to atomic phenomena half of the specific depletion of cosmic microwave background radiation. Popular modeling links most known elementary particles with a notion of left-handedness. Perhaps, data point toward a concept - such as atoms that would link to right-handed elementary particles - that might prove useful. (Photons link to popular modeling notions of a left-circular polarization mode and a right-circular polarization mode, but not to popular modeling notions of elementary-particle handedness.) Exploring that concept might link to the second approach.

This paper suggests that the second approach leads to progress regarding the five above-mentioned unexplained patterns.

Discussion in this paper links to the following steps regarding progress in physics.

- People notice data that might suggest unexplained patterns.
- People propose patterns that the data might exhibit.
- People propose models that seem to output the patterns.
- People propose principles that might underlie the models.
- People propose reuses for the principles and models.

Within uses of the second approach, this paper discusses new modeling that stems from and does not disturb successful popular modeling and principles, seemingly somewhat decouples from directly linking to successful popular modeling, and usefully combines with successful popular modeling and principles.

Regarding classical physics, successful popular modeling and principles tend to link to mathematical equations that feature continuous variables. New modeling tends to link to mathematical equations that feature integer arithmetic.

2. Unexplained patterns that data seem to exhibit

This unit discusses data and patterns that popular modeling seems not to explain.

Section 3, section 4, and section 5 of Ref. [1] provide details (including dozens of citations) regarding data that underlie discussion in this unit. This unit does not repeat relevant citations.

2.1. Evolution of some physics modeling

This unit discusses patterns regarding the evolution of some physics modeling.

Eq. (1) points to notions for which patterns might pertain. (Ref. [9] discusses Eq. (1).)

$$F = ma \quad (1)$$

The symbol m (as in mass) links to notions of objects and to notions of properties of objects. The symbol a (as in acceleration) links to notions of motions of objects. The symbol F (as in force) links to notions of interactions between objects and to notions of fields.

Popular modeling has tended to treat as inputs (from inferences based on observations) the following - types of objects, catalogs of properties of objects, types of fields, and types of interactions. Such inputs link to notions of lists, with each list including a discrete set of items. In this context, the following three sentences pertain. Popular modeling tends to emphasize explaining motions of objects and changes to objects. Popular modeling classical physics modeling has roots in notions of continuous variables that link to notions of time, position, velocity, and acceleration. Popular modeling quantum physics modeling arose from classical physics modeling and notions that observations point to some properties for which notions of discrete values pertain.

A pattern of development of physics has tended to link to the first one of the two approaches that this paper discusses above.

Possibly, other patterns of development of physics might echo the second one of the two approaches that this paper discusses above.

Possibly, such other patterns of development of physics might not contradict successful popular modeling, might output lists that popular modeling inputs from inferences regarding experiments and observations, and might point to opportunities to advance popular modeling.

2.2. Properties of objects and fields

This unit discusses the notion that patterns might pertain regarding properties of objects and properties of fields.

Successful popular modeling includes electromagnetic properties such as charge, charge current, and magnetic moment. Successful popular modeling Newtonian dynamics includes two inertial properties - mass and momentum. Successful popular modeling includes gravitational properties such as energy and momentum.

In classical physics modeling, such properties link to continuous variables.

Successful popular modeling includes properties - such as atomic states and nuclear states - that link to systems that model as having more than one component. For example, atomic states link to two components - an electron cloud and an atomic nucleus. In quantum physics modeling, such properties can link to discrete variables. For example, for atomic physics, ionization charge and principal quantum number (for a shell or an orbital) are discrete variables that link to electron clouds.

Possibly, patterns pertain to properties of objects. Eq. (2) posits one such pattern. Here, the following notation pertains. q denotes charge. I denotes charge current. μ denotes magnetic moment. E denotes energy. P denotes momentum. s_{IAM} denotes spin (as in intrinsic angular momentum). Each one of q and E links to the word scalar. Each one of I, μ, E, and s_{IAM} links to the word vector. The symbol : links to the two-word phrase is to and pertains to the two symbols that bracket the symbol :. The symbol :: links to the word as and pertains to the two trios that bracket the symbol ::.

$$q : I : \mu :: E : P : s_{IAM} \tag{2}$$

Possibly, modeling can catalog properties of objects. Possibly, modeling can catalog properties of fields.

2.3. Elementary particles

This unit discusses elementary-particle data and patterns that popular modeling seems not to explain.

Successful popular modeling features 24 known elementary particles. Each known elementary particle links to exactly one of the word fermion and the word boson.

There are 12 elementary fermions. For each elementary fermion, the spin S (as in $S(S+1)\hbar^2$) is one-half. For three elementary fermions the magnitude $|q|$ of the charge is $|q_e|$, in which q_e denotes the charge of an electron. For three elementary fermions the magnitude $|q|$ of the charge is $(2/3)|q_e|$. For three elementary fermions the magnitude $|q|$ of the charge is $(1/3)|q_e|$. For three elementary fermions the magnitude $|q|$ of the charge is $0|q_e|$.

There are 12 elementary bosons. For the Higgs boson, $S = 0$. For each one of the Z boson, the W boson, the photon, and the eight gluons, $S = 1$.

Popular modeling suggests various bases for cataloging elementary particles. Examples of bases include (at least) fermion or boson, nonzero-mass or zero-mass, nonzero charge or zero-charge, baryon number, lepton number, and types of interactions in which specific particles partake. Popular modeling suggests various displays - each of which might provide a useful (for elementary particles) analog to the periodic table (for chemical elements) - for the set of known elementary particles.

Possibly, the seemingly large (for there being just 24 known elementary particles) set of bases and displays points to an opportunity to develop useful new techniques for cataloging elementary particles.

Popular modeling suggests that other (as in not known or not yet found) elementary particles exist.

Possibly, patterns pertain regarding all elementary particles. Possibly, patterns pertain regarding properties (including spin, charge, and mass) of elementary particles.

Possibly, modeling can catalog elementary particles and properties of elementary particles.

2.4. Elementary-particle properties, interactions, and physics constants

This unit discusses physics-constants data and patterns that popular modeling seems not to explain.

Section 4.2.1 of Ref. [1] suggests that Eq. (3) might link the masses (as in rest-energies) of the known nonzero-mass elementary bosons. m denotes mass. The symbols W, Z, and Higgs link to, respectively, the W boson, the Z boson, and the Higgs boson.

$$(m_{\mathrm{W}})^2 : (m_{\mathrm{Z}})^2 : (m_{\mathrm{Higgs}})^2 :: 7 : 9 : 17 \tag{3}$$

Section 4.3.1 of Ref. [1] suggests that Eqs. (4), (5), and (6) might link aspects of electromagnetic interactions with aspects of gravitational interactions. Eq. (4) defines β' as the ratio of the mass of the tau (a lepton elementary particle) to the mass of the electron (another lepton elementary particle). The right-hand side of Eq. (5) is - for two electrons - a ratio of electromagnetic repelling to gravitational attracting. Eq. (5) defines β. Eq. (6) might suggest a relationship between electromagnetism and gravity.

$$\beta' \equiv m_\tau / m_e \tag{4}$$

$$(4/3) \cdot (\beta^2)^6 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2) \tag{5}$$

$$\beta' = \beta \tag{6}$$

Popular modeling links the ratio $(m_{\mathrm{W}})^2 : (m_{\mathrm{Z}})^2$ to the three-word term weak mixing angle but not necessarily to the ratio $7 : 9$. Otherwise, popular modeling seems not to include Eqs. (3) and (6).

Possibly, Eqs. (3) and (6) link to patterns that point to useful modeling.

Table 1: Observed ratios of not-ordinary-matter effects to ordinary-matter effects. The notation NOM:OM abbreviates the seven-element phrase ratio of not-ordinary-matter effects to ordinary-matter effects.

(a) Ratios - that pertain to light that dates to about 380,000 years after a so-called Big Bang - of observed effects. The three-word phrase cosmic optical background links to radiation that - recently - measures as optical radiation or measures as close (with respect to wavelengths) to optical radiation. The acronym CMB links to radiation that - recently - measures as cosmic microwave background radiation.

Aspect	NOM:OM
Amount of cosmic optical background	$1 : 1$
Some absorption of CMB	$1 : 1$

(b) Ratios - that pertain to some galaxies - of observed effects. Regarding galaxies, the notion of early links to observations that pertain to galaxies that link to high redshifts. High might link to $z > 7$ and possibly to smaller values of z. Here, z denotes redshift. The word later links to the notion that observations pertain to objects later in the history of the universe. Possibly, early NON:OM $1 : 0^+$ galaxies existed and people have yet to detect any such galaxies.

Objects	NOM:OM
Some early galaxies	$0^+ : 1$
Some later galaxies	$0^+ : 1$
Some later galaxies	$1 : 0^+$
Some later galaxies	$\sim 4 : 1$
Many later galaxies	$5^+ : 1$

(c) Ratios - that pertain to larger-than-galaxies-scale phenomena - of observed effects. For a galaxy cluster that has collided with another galaxy cluster, a ratio of other than $5^+ : 1$ might pertain.

Aspect	NOM:OM
Densities of the universe	$5^+ : 1$
Some galaxy clusters	$5^+ : 1$

2.5. Ratios of not-ordinary-matter effects to ordinary-matter effects

This unit discusses cosmology data and patterns that popular modeling seems not to explain.

Popular modeling suggests ratios of effects that seem not to link to ordinary matter to effects that seem to link to ordinary matter.

Table 1 lists observed ratios of NOM (as in not-ordinary-matter) effects to OM (as in ordinary-matter) effects. (This table reprises information in Table 12 in Ref. [1]. Section 4.11.1 of [1] cites sources of relevant data.)

Possibly, the notion that ratios that Table 1 lists are ratios of near-integers is a useful pattern.

2.6. Dark matter

This unit discusses data and patterns that might pertain regarding popular modeling notions about dark matter.

The two-word term dark matter arose in conjunction with data that link to the galaxy cluster row in Table 1c and data that link to the many later galaxies row in Table 1b. The term dark-matter galaxy links to the NON:OM $1 : 0^+$ row in Table 1b.

Popular modeling proposes and debates notions that link to the two-word term dark matter. Some popular modeling might suggest that gravitational phenomena might explain (without needing to consider matter other than ordinary matter) some of the ratios that Table 1 lists. (Ref. [10] provides an example.) Some popular modeling assumes that NOM links to dark matter being stuff (as in matter). Popular modeling proposes (various) general characteristics (beyond the characteristic of interacting with gravity) of dark matter. Popular modeling suggests ranges of properties of dark matter. Possibly, people consider that no data links directly to suggested general characteristics (beyond the characteristic of interacting with gravity) or specific properties.

Possibly, patterns link to the notion that ratios that Table 1 lists are ratios of near-integers.

Possibly, new modeling can suggest a new candidate combination of general characteristics and specific properties. Possibly people might consider that data to which Table 1a alludes links directly to the new candidate combination of dark-matter general characteristics and dark-matter specific properties. Possibly, data to which Table 1b and Table 1c allude link directly to the new candidate combination of dark-matter general characteristics and dark-matter specific properties.

2.7. Gravity and dark-energy phenomena

This unit discusses cosmology data and patterns that popular modeling seems not to explain.

Popular modeling suggests three eras regarding the so-called rate of expansion of the universe. (The notion of rate of expansion of the universe associates with notions of typical speeds of moving away from each other regarding neighboring large objects, such as galaxy clusters.) The first era (about which there may be no data) would feature a typical speed of moving away that rapidly increases. The second era (about which there is data) features a typical speed of moving away that - while remaining positive - decreases. The third (and also current) era (about which there is data) features a typical speed of moving away that increases. (Section 4.9.1 of Ref. [1] provides information about the eras and cites sources of data.)

Possibly, notions of such eras point to a useful pattern.

Popular modeling uses the two-word term dark energy to denote possible mechanisms that could lead to increases in the typical speed of moving away from each other of large clumps.

Popular modeling suggests that popular modeling may miscalculate the strength of some effects of gravity and dark energy. Popular modeling underestimates the third-era rate of expansion of the universe. Popular modeling overestimates so-called large-scale clumping of matter. (Section 4.9.2 and section 4.9.3 of Ref. [1] discuss these two possible miscalculations and cite sources of data.)

Possibly, notions of such miscalculations point to useful patterns.

2.8. Baryon asymmetry

This unit discusses baryon asymmetry, which popular modeling seems not to explain.

Popular modeling links the two-word term baryon asymmetry to inferences that - regarding known stuff - there are many more left-handed (or matter) fermion elementary particles than right-handed (or antimatter) fermion elementary particles. Popular modeling suggests that baryon asymmetry endures from early in the history of the universe. Popular modeling seems not to have settled on how baryon asymmetry came about.

Possibly, new patterns or modeling might link baryon asymmetry to a cause for baryon asymmetry.

2.9. N-body modeling, binding energy, and entanglement

This unit discusses aspects of multi-body modeling that popular modeling seems not to fully explain.

Popular modeling can model two-body (as in two-object) systems via notions of a center of mass, bulk motion (of the system), and reduced-mass modeling for the internal states of the system. Such modeling might, in effect, de-emphasize aspects that relate directly to the original two objects. Notions of binding energy and entanglement might gain (with respect to modeling that features the original two objects) emphasis.

Popular modeling can find intractability regarding modeling for N-body systems, for which N exceeds two.

Possibly, new patterns might pertain for two-body systems and might extend for at least some cases for N-body systems for which N exceeds two.

2.10. Uncertainty

This unit discusses a notion, that popular modeling seems not to include, of uncertainty.

Popular modeling suggests that notions of uncertainty (and notions of an uncertainty principle) can pertain for modeling that features popular modeling quantum mechanical waves or that features popular modeling classical mechanical waves. Popular modeling might suggest (at least by default) that notions of uncertainty do not link to popular modeling notions of classical mechanical objects.

Might new modeling point to the notion that popular modeling classical physics modeling for objects could include notions of uncertainty?

2.11. Vacuum energy and long-range-interaction-boson ground states

This unit discusses vacuum energy and a related discrepancy that popular modeling seems not to resolve.

Popular modeling uses Eq. (7) to describe the energy E of the ν-times excited state of a D-dimensional isotropic quantum harmonic oscillator. In popular modeling, ν is a nonnegative integer. In popular modeling, $\nu = 0$ links to the two-word term ground state. ω is a frequency. Popular modeling links a wavelength λ to the frequency via the equation $\lambda = c/\omega$. The symbol c denotes the speed of light.

$$E = 0.5(D + 2\nu)\hbar\omega \geq 0 \tag{7}$$

Popular modeling explores the notion of a so-called vacuum energy density that might link to the sum over all possible photon modes of the ground-state energies that Eq. (7) suggests. To keep the sum from being infinite, popular modeling explores notions of limiting the range of ω (or, equivalently, of limiting the range of λ).

Popular modeling also explores the notion of a so-called vacuum energy density that might link to the term Λ (as in a so-called cosmological constant) in the general relativity Einstein field equations. Popular modeling explores the notion that a positive value Λ might link to dark-energy phenomena.

Popular modeling indicates that, compared to values of Λ that might comport with data, vacuum energy densities that link to Eq. (7) are too large - perhaps by a factor of 10^{120}.

Possibly, new modeling might point to a way to resolve the discrepancy.

2.12. General relativity and quantum gravity

This unit discusses quantum gravity, a notion about which popular modeling seems unsettled.

People try so-called precision tests of general relativity. People assert that - so far - general relativity comports with data that links to all such precision tests. General relativity does not link to successful popular modeling for some physics regimes. For example, popular modeling does not use general relativity to address aspects for which Maxwell's equations prove useful, aspects of the strong interaction, or aspects of the weak interaction.

Possibly, popular modeling has yet to characterize adequately bounds on the use of general relativity.

Popular modeling has yet to develop seemingly compelling modeling regarding so-called quantum gravity or regarding quantization related to the Einstein field equations.

Possibly, inabilities to develop quantum modeling related to general relativity link to lacks of understanding about limits on the realm in which popular modeling based on general relativity is appropriate.

2.13. Properties, magnitudes of properties, forces, and conservation laws

This unit points to the possibility that new patterns might link properties of objects, minimum magnitudes for nonzero values of some properties, interaction forces, and conservation laws.

Successful popular modeling suggests that charge, energy, momentum, and angular momentum are conserved quantities. Popular modeling suggests that $B - L$ (as in baryon number minus lepton number) might be a conserved quantity.

Popular modeling does not necessarily suggest - regarding conservation laws - a comprehensive framework or a cataloging technique. For example, some popular modeling links conservation of energy to invariance regarding temporal translations (as in adding a constant to a coordinate that links to time), links conservation of momentum to invariance regarding spatial translations (as in adding a constant 3-vector to a 3-vector coordinate that links to position), and does not necessarily link conservation of charge to a translational invariance.

Possibly, new patterns might link to a set of conserved quantities and to the conservation laws that link to those conserved quantities.

Possibly, new patterns might link properties of objects, minimum magnitudes for nonzero values of some properties, interaction forces, and conservation laws.

3. Models that seem to output unexplained patterns

This unit discusses modeling that seems to explain data and patterns that popular modeling seems not to explain.

The following assumptions underlie new models that output patterns to which this paper alludes above.

1. Nature includes six isomers of most elementary particles. Stuff that has bases in five isomers underlies dark-matter effects (as in not-ordinary-matter effects).
2. New applications of notions that have roots in successful popular modeling that uses multipole mathematics can be useful regarding cataloging properties of objects, cataloging known and predicting new elementary particles, and modeling effects of gravity (including dark-energy effects).

Aspects key to developing the new models include the following.

1. Start from successful popular modeling.
2. Find concepts that seem to be invariant regarding choices of popular modeling models or to be invariant within popular modeling models.
3. Develop new modeling that stems from the invariant concepts, that has bases in integer-arithmetic equations, and that tends to output notions that popular modeling otherwise seems to treat as inputs based on experiments and observations.

Table 2: Aspects of popular modeling and aspects of new modeling. The symbol † denotes the two-word phrase list of. The symbol ‡ denotes the four-word phrase list of types of. The rightmost two columns position the aspects with respect to the types of modeling. For example, the word input links to the notion that popular modeling inputs (as in assumes) the aspect. The word output links to the notion that new modeling, in effect, outputs the aspect. One can consider combined (as in new plus popular) modeling in which popular modeling inputs such outputs. The word branch associates with notions that people might consider as branching from aspects of new modeling to underlie aspects of popular modeling. That branching can link to existing popular modeling uses of coordinate systems and might point to future other popular modeling uses of coordinate systems. Thereby, that branching can underlie popular modeling regarding motions of objects and changes to objects.

Aspects of modeling	Popular modeling	New modeling
† Properties of objects	Input from observations	Output from modeling
‡ Objects	Input from observations	Output from modeling
† Families of elementary particles	Input from observations	Output from modeling
† Fields	Input based on observations	Output from modeling
‡ Interactions between objects and fields	Input based on observations	Output from modeling
Popular modeling coordinate systems	Inherent in modeling	Branch from modeling
Motions of objects	Output from modeling	-
Changes to objects	Output from modeling	-

3.1. Evolution of some physics modeling

This unit discusses aspects of the evolution of physics modeling, provides some contrasts between popular modeling and new modeling, and anticipates some aspects (which this paper discusses below) of new modeling.

Popular modeling evolves.

Some (including early) popular modeling links to values, as inferred directly by people, of properties of specific objects. Some such modeling has bases in people-established coordinate systems.

Some popular modeling (including some Newtonian modeling) links to notions of one object and one field. Some such modeling has bases in people-established coordinate systems.

Some popular modeling (including modeling based on notions of relativity) links to values of properties of an object I, as would be inferred by an object O, with the values having bases in coordinate systems that link to object O.

Some relativity-centric popular modeling uses coordinate systems that are not centric to either object in a pair of an object O and an object I. New modeling suggests that some such popular modeling might link to notions of one evolutionary step too far. Use of such coordinate systems links, in effect, to a third object (as in an object that defines the coordinate system). In popular modeling, modeling regarding three objects can be more difficult than modeling regarding two objects.

Table 2 contrasts aspects of popular modeling and aspects of new modeling.

New modeling de-emphasizes direct use of continuous coordinates.

New modeling includes two uses of integer-centric mathematics. One use involves integer-arithmetic equations and uses integers to index catalogs to which the leftmost column in Table 2 alludes. Another use involves integer mathematics that links to three aspects - number of dimensions, excitation level, and energy - of popular modeling that has bases in mathematics that associates with the three-word term isotropic harmonic oscillator. New modeling use of harmonic-oscillator mathematics extends from popular modeling use of harmonic-oscillator mathematics.

New modeling can call attention to relationships between popular modeling notions of objects and popular modeling notions of scale.

For example, at a small scale (or for Newtonian modeling) a zero-spin object might be reasonably well-defined and notions of momentum and velocity might be more important (regarding modeling) than notions of angular momentum and angular velocity. On a larger scale, the notions of that object as a well-defined independent object might give way to notions of that object as a component of a multi-object system. Notions (regarding the original object) of angular momentum and angular velocity might gain in importance. Notions (regarding the original object) of momentum and velocity might decline in importance.

Discussion below provides details regarding new modeling and regarding branching from new modeling to popular modeling.

3.2. Properties of objects and fields

This unit discusses patterns regarding objects and fields, discusses mathematics that seems to express the patterns, and points to possibilities for extrapolated modeling that would have bases in the mathematics.

Useful new modeling might stem from the following notions.

- Successful popular modeling links the electromagnetic field to two modes - left-circular polarization and right-circular polarization - and to a spin of one (in units of $\hbar$).
- Successful popular modeling suggests that the electromagnetic field conveys to an object O (as in observer object) information about an object I (as in inferred object).
- Successful popular modeling includes the notion an object O can characterize contributions (that link to object I) to the electromagnetic field in terms of contributions (that link to object I) to an inferred (by object O) electric field and contributions (that link to object I) to an inferred (by object O) magnetic field.
- From the point of view of an object O and regarding an object I, the following notions pertain.
  - The inferred electric field that object O links to object I links to a property of inferred charge q (of object I), a property of inferred position (of object I), and an inferred time that links the charge and position. (For popular modeling in which object I models as point-like, the word position links to a specific position. For popular modeling in which object I models as point-like, the word time links to a specific time. For popular modeling in which object I does not model as point-like, position can link to a region and time can link to a time range.)
  - The inferred magnetic field that links to object I links to the following properties.
    * An inferred charge current I (of object I), an inferred position (of object I), and an inferred time that links the charge current and position. The inferred charge current links to the inferred charge (of object I) and an inferred velocity (of object I). (For popular modeling in which object I models as point-like, the word velocity links to change in point-like position. For popular modeling in which object I does not model as point-like, velocity can link to change in position of the region.)
    * An inferred magnetic moment μ (of object I), an inferred position (of object I), and an inferred time that links the magnetic moment and position. For some modeling (for example, regarding an object I that models as having a rigid distribution of charge), the inferred magnetic moment links to the inferred charge (of object I) and an inferred angular velocity (of object I).
- Successful popular modeling regarding a point-like object links the inferred electric field to the two-word phrase monopole potential.
- ND (as in Newtonian dynamics) aspects of successful popular modeling can link a total potential to a sum of a monopole (contribution to the total potential) potential, a dipole (contribution to the total potential) potential, a quadrupole (contribution to the total potential) potential, and so forth.
  - The monopole potential can link to a radial (as in distance r away from an object I) r^{-1} spatial dependence. The dipole potential can link to a radial (as in distance r away from an object I) r^{-2} spatial dependence. The quadrupole potential can link to a radial (as in distance r away from an object I) r^{-3} spatial dependence. And so forth. (For a dipole potential, a quadrupole potential, and so forth, angular dependence can also - along with radial dependence - pertain.)
  - Regarding a scalar property (such as charge, which links to electromagnetism), the following can pertain.
    * For a system (of objects) that models as having exactly one point-like object, a monopole potential can pertain. The point links to a popular modeling notion of zero dimensions.
    * For a system (of objects) that models as having exactly two identical point-like objects, a dipole potential can pertain. The line that links the two objects links to a popular modeling notion of one dimension.
    * For a system (of objects) that models as having exactly four identical objects arrayed as the corners of a square, a quadrupole potential can pertain. The square links to a popular modeling notion of two dimensions.

- * For a system (of objects) that models as having exactly eight identical objects arrayed as the corners of a cube, an octupole potential can pertain. The cube links to a popular modeling notion of three dimensions.

New modeling links to the following popular modeling notions regarding an object I.

- The electromagnetic field (that object O links to object I) links to two modes, each one of which links to a magnitude (as in $|s| = 1$) of spin s (in units of $\hbar$). New modeling (arbitrarily, but without unintended consequences) links $s = +1$ to the left-circular-polarization mode and links $s = -1$ to the right-circular-polarization mode.
- Regarding ND and popular modeling that considers object I to be point-like, the electric field (that object O links to object I) links to a position (that object O infers regarding object I), to a monopole (as in r^{-1}) potential and to one (as in q) property that is not linked directly to properties (such as position) that link directly to space-time-coordinates.

New modeling extends the above notions as follows.

- Notions of monopole potentials, dipole potentials, and so forth are not directly relevant within new modeling. (New modeling does not necessarily need to directly discuss the dynamics of objects. New modeling should be invariant with respect to popular modeling choices between, for example, ND and SR - as in special relativity. New modeling does not necessarily need to directly discuss the excitation of fields. New modeling should not disturb successful physics modeling regarding excitations of fields and regarding de-excitations of fields.)
- Fields with integer spins of $|s| \geq 2$ can pertain. For example, for gravity, new modeling links $s = +2$ to the left-circular-polarization mode and links $s = -2$ to the right-circular-polarization mode. (Refs. [11, 12, 13] discuss notions of gravitational circular polarization.)
- The symbol 1x> links to inferred properties - such as q (as in charge), μ (as in magnetic moment), E (as in energy) and s_{IAM} (as in intrinsic angular momentum) - that link (in the sense of discussion above) to inferred position but that do not necessarily link to inferred linear velocity. The symbol 2x> links to inferred properties - such as I (as in charge current) and P (as in momentum) - that link (in the sense of discussion above) to inferred position and to inferred (linear) velocity. Each one of 1x> and 2x> does not link to inferred (linear) acceleration. The symbol 3x> links to the hypothetical possibility of inferred properties that link to inferred position, inferred (linear) velocity, and inferred (linear) acceleration. (New modeling suggests that successful popular modeling suggests that the notion of an unchanging object and the notion of a nonzero 3x> property are not compatible with each other. For example, a linearly accelerating charged object I changes at least one of inferred E - as in inferred energy - and inferred P - as in inferred momentum. A related notion is that inferred acceleration links to the notion that an inferred object I [itself] would not link to at least one of conservation of energy and conservation of momentum. A related notion is that an inferred object I models as being part of a system [of inferred objects and/or inferred fields] that includes more than just object I and inferred fields that link to object I.) The symbol 1x> links to the successful popular modeling notion of zero derivatives (with respect to time) of position. The symbol 2x> links to the successful popular modeling notion of one derivative (with respect to time) of position. The symbol 3x> links to the successful popular modeling notion of two derivatives (with respect to time) of position.

The next paragraph discusses mathematics and not necessarily physics. (Notation from set theory that this paper uses includes the following. $\{a, b, \cdots\}$ denotes the set that has members a, b, and so forth. $a \in b$ denotes that a is a member of set b. $a \notin b$ denotes that a is not a member of set b. $a \subset b$ denotes that set a is a subset of set b. $a \cap b$ denotes the largest set for which each member is a member of each one of set a and set b. $\emptyset$ denotes the set that includes no - as in zero - members. Other mathematics notation that this paper uses includes the following. $a \Rightarrow b$ denotes that, if a pertains, b pertains. $a \Leftrightarrow b$ denotes that a pertains if and only if b pertains.)

The following rules pertain.

- Each term in a sum (of terms) has the form ks_k. k is a positive integer. One of $s_k = +1$ and $s_k = -1$ pertains. k appears no more than once in the sum.

- Eq. (8) depicts a sum. K denotes the subset (of the set of all positive integers k) for which one of $s_k = +1$ and $s_k = -1$ pertains.

$$s = \sum_{k \in K} k s_k \tag{8}$$

- Eqs. (9) and (10) define the symbols n_k and Σ.

$$n_k = \text{ the number of } k \text{ in the set } K \tag{9}$$

$$\Sigma = |s| \tag{10}$$

- New modeling links the word solution (as in solution to an integer-arithmetic equation) to a set of integers s, k, and s_k that satisfy Eq. (8).

- For each solution, there is exactly one second solution for which s is the negative of the s for the first solution and each s_k is the negative of the respective s_k for the first solution. New modeling links the one-element term solution-pair to such a pair of a first solution and the second solution. For a solution-pair, one nonnegative value of Σ pertains. (Regarding showing arithmetic pertaining to a solution-pair for which - for each solution - $k_{max \in K}$ denotes the maximum k in K, this paper adopts a convention of showing the solution for which $s_{k_{max \in K}} = +1$ and not showing the solution for which $s_{k_{max \in K}} = -1$. For example, for $K = \{1\}$, this paper might show the expression $1 = \Sigma = |+1|$ and would not show the expression $1 = \Sigma = |-1|$.)

- New modeling links the word cascade (and the phrase one-step cascade) to forming a new (as in second) K by adding one (new) positive integer to an original (as in first) K. For a one-step cascade that starts with a K for which $n_k = l_1$ pertains, $n_k = l_1 + 1$ pertains for the second K. New modeling links the two-element phrase multi-step cascade to multiple uses of the notion of a one-step cascade. At each step n_k increases by one.

- This paper extends the notion of cascade to refer to solution-pairs. With respect to one solution-pair, the notions of one Σ and one K pertain. A one-step cascade regarding that K results in a new K for which - across the one-step cascade solution-pairs - more than one Σ pertains. (For example, the solution-pair that associates with $1 = \Sigma = |+1|$ cascades in one step to the two solution-pairs that associate with, respectively, $1 = \Sigma = |-1+2|$ and $3 = \Sigma = |+1+2|$.) Except where this paper makes a specific statement such as non-same-Σ, the notion of one-step cascade solution-pair links to the notions of the original Σ and the new K. (For the example, $3 = \Sigma = |+1+2|$ does not equal the original Σ - as in $\Sigma = 1$ - and - absent a reference to non-same-Σ - would not pertain.)

The following discussion links mathematics and new (physics) modeling. Discussion regarding properties and fields implies (but does not explicitly state the word) inferred.

The following examples - for which $1 \in K$ - pertain.

- $K = \{1\}$ links to $1 = \Sigma = |+1|$.
  - The electric field and the property q (as in charge) link to 1x> use of $1 = \Sigma = |+1|$.
- $K = \{1, 2\}$ links to $1 = \Sigma = |-1+2|$ and to $3 = \Sigma = |+1+2|$.
  - The magnetic field and the property I (as in charge current) link to 2x> use of $1 = \Sigma = |-1+2|$.
  - The magnetic field and the property μ (as in magnetic moment) link to 1x> use of $1 = \Sigma = |-1+2|$.
  - New modeling suggests that the magnetic field might link also to 1x> use of $3 = \Sigma = |+1+2|$.
    - Regarding popular modeling classical modeling, 1x> use of $3 = \Sigma = |+1+2|$ might link to non-spherically symmetric (as in cylindrically symmetric and oval or as in cylindrically symmetric and oblate) distribution of charge.
    - Regarding popular modeling quantum modeling, 1x> use of $3 = \Sigma = |+1+2|$ might link to anomalous magnetic moment.
- $K = \{1, 2, 4\}$ links to $1 = \Sigma = |-1-2+4|$, to $3 = \Sigma = |+1-2+4|$, to $5 = \Sigma = |-1+2+4|$, and to $7 = \Sigma = |+1+2+4|$.

- Based on the series position, velocity, and acceleration, 3x> use of $1 = \Sigma = |-1-2+4|$ might link to a property of accelerating charge. New modeling de-emphasizes this notion, based on discussion above about inferred objects.
- Based on the series position and velocity, 2x> use $1 = \Sigma = |-1-2+4|$ might link to a property of velocity of magnetic moment.
- Based on the series charge, magnetic moment, and so forth, 1x> use of $1 = \Sigma = |-1-2+4|$ might link to a property of precessing (axis of) magnetic moment. (This notion of precessing magnetic moment does not link to Larmor precession. The Earth exhibits this notion of precessing magnetic moment.) New modeling suggests that popular modeling would consider this property of precessing magnetic moment to link to a combination of electromagnetic properties and at least one of inertial properties and gravitational properties.
- Based on the above notions, new modeling suggests that 1x> use of a solution-pair for which $4 \in K$ implies that the solution-pair does not link to notions of solely electromagnetic effects and does link to Eq. (11).

$$4 \in K \Rightarrow \text{ 1x> use of the solution-pair does not link directly to } q \tag{11}$$

Some gravitational properties link to doubling the integers in various sets, each consisting of one Σ and a set K, that link to electromagnetism.

- E (as in energy) links to $K = \{2\}$ and to 1x> use of $2 = \Sigma = |+2|$. P (as in momentum) links to $K = \{2, 4\}$ and to 2x> use of $2 = \Sigma = |-2+4|$. s_{IAM} (as in intrinsic angular momentum) links to $K = \{2, 4\}$ and to 1x> use of $2 = \Sigma = |-2+4|$. Possibly, a property of anomalous angular momentum links to $K = \{2, 4\}$ and to 1x> use of $6 = \Sigma = |+2+4|$.

Some gravitational properties do not link to doubling the integers in sets, each consisting of one Σ and a set K, that link to electromagnetism.

- 1x> use of $2 = \Sigma = |+1-2+3|$ links to object-internal stress-energy and to two distinct (axes and associated) moments of inertia.
- 1x> use of $2 = \Sigma = |-1+2-3+4|$ links to rotation relative to one of the two distinct axes of moment of inertia. 1x> use of $2 = \Sigma = |-1-2-3+4|$ links to rotation relative to the other one of the two distinct axes of moment of inertia.

Discussion above links to both (of the above-mentioned) popular modeling uses of the terms, monopole, dipole, and so forth.

- For an object I that an object O models as point-like, the following statements pertain. $n_k = 1$ links to monopole potential. $n_k = 2$ links to dipole potential. $n_k = 3$ links to quadrupole potential. And so forth.
- $n_k = 1$ links to one solution-pair (and to one value of Σ). $n_k = 2$ links to two solution-pairs (for which the Σ for one solution-pair does not equal the Σ for the other solution-pair). $n_k = 3$ links to four solution-pairs (and to up to four values of Σ). $n_k = 4$ links to eight solution-pairs (and, at least for the case of $K = \{1, 2, 3, 4\}$, to six values of Σ). And so forth.

Successful popular modeling includes the notion that useful descriptions of excitations of the electromagnetic field can have bases in mathematics of quantum harmonic oscillators. Regarding notions that this paper discusses above, the following notions pertain.

- Mathematics of a one-dimensional harmonic oscillator links (in popular modeling) to aspects regarding the $s = +1$ (as in left-circular polarization) mode.
- Mathematics of a one-dimensional harmonic oscillator links (in popular modeling) to aspects regarding the $s = -1$ (as in right-circular polarization) mode.
- New modeling suggests that individual terms in a sum of the form $1 \leq \Sigma = |\cdots|$ do not link directly to popular modeling notions of excitations of modes. Below, this paper discusses notions that some individual terms (that are in or that are lacking from sums of the form $1 \leq \Sigma = |\cdots|$) link to popular modeling notions of ground-state symmetries.

Modeling that this paper discusses assumes that Eq. (12) pertains. In other words, a set K does not include any odd integers that exceed seven.

$$\{9, 11, 13, 15, \cdots\} \cap K = \emptyset \tag{12}$$

3.3. Elementary particles

This unit discusses a catalog of all elementary particles of which people know or that new modeling suggests.

For $n_k \geq 3$, some solution-pairs link to $\Sigma = 0$. Three examples link (respectively) to the expressions $0 = \Sigma = |-1-2+3|$, $0 = \Sigma = |-1-3+4|$, and $0 = \Sigma = |-1+2-3+4|$.

New modeling posits that Eq. (13) links to $0 = \Sigma = |\cdots|$ solution-pairs that link to popular physics notions of elementary particles.

$$\{1,3\} \subset K,\ \{5,7,9,11,\cdots\} \cap K = \emptyset \tag{13}$$

New modeling uses the following notions to catalog elementary particles. A symbol of the form SΦ links to a so-called family of elementary particles. Each elementary particle links to one family. Each family links to one of one, three, or eight elementary particles. For a family, the value S denotes the spin (in units of $\hbar$) for each elementary particle in the family. S links to the popular modeling expression $\mathrm{S}(\mathrm{S}+1)\hbar^2$ that links to angular momentum. Regarding popular modeling, known values of S include 0, 0.5, and 1. The symbol Φ links to a symbol of the form X_Q, in which X is a capital letter and Q is the magnitude of the charge (in units of $|q_e|$, in which q_e denotes the charge of an electron) for each particle in the family. For cases for which $Q = 0$, new modeling omits - from the symbols for families - the symbol Q. Regarding quarks, new modeling uses the symbol $\mathrm{Q}_{>0}$ to link to cases for which either one of $\mathrm{Q}_{1/3}$ or $\mathrm{Q}_{2/3}$ pertains.

For an integer l, new modeling uses the notation $+l\hat{}$ to denote the series to which Eq. (14) alludes. Each item in the series totals to l.

$$+l\hat{}\ \text{denotes the series}\ +l,\ -l+2l,\ -l-2l+4l,\ -l-2l-4l+8l,\ \cdots \tag{14}$$

New modeling uses the notation $+l_n\hat{}$ to denote the item in Eq. (14) that includes exactly n terms. For example, $+l_2\hat{}$ denotes , $-l+2l$.

New modeling links the symbols 1t0 and 2t0 to uses of solution-pairs for which $0 = \Sigma = |\cdots|$. (For each one of 1t0 and 2t0, the zero in each symbol links to the zero in $0 = \Sigma = |\cdots|$. In contrast, for each one of 1x>, 2x>, and 3x>, the symbol > links to the notion of greater than - as in $\Sigma > 0$. Technically, regarding symbols that this paper uses, the 0 in t0 is redundant and the > in x> is redundant.) For elementary particles, 1t0 links to notions of a time. For elementary particles, 2t0 links to notions that time changes. The symbol t in the symbols 1t0 and 2t0 links to successful popular modeling notions of temporal. The symbol x in the symbols 1x>, 2x>, and 3x> links to successful popular modeling notions of spatial.

- For elementary particles, 1t0 uses of solution-pairs link to specific elementary particles.
- For boson elementary particles, 2t0 uses of solution-pairs link to types of interactions in which the counterpart 1t0 elementary bosons partake. Here, a 2t0 solution-pair is a one-step cascade from a counterpart 1t0 solution-pair.
- For elementary particles, 1t0 uses of a 2t0 solution-pair (that is a one-step cascade from a counterpart elementary-particle 1t0 use of a solution-pair) can link to specific elementary particles.

Table 3 catalogs all known elementary particles and some elementary particles that new modeling suggests nature might include. A primary organizing principle for the rows is that lesser n_k precedes greater n_k. A secondary organizing principle for the rows is that nonzero charge precedes zero charge.

The following points associate with new modeling. New modeling regarding elementary particles ...

- Links the 1t0 uses of three solution-pairs ($0 = \Sigma = |-1-2+3|$, $0 = \Sigma = |-1-3+4|$, and $0 = \Sigma = |-1+2-3+4|$, respectively) to the W boson, the Z boson, and the Higgs boson.
- Uses Eqs. (15) and (16), which link to Eq. (11).

$$\text{1t0 use and } 4 \notin K \Rightarrow |q| = |q_e| \tag{15}$$

$$\text{1t0 use and } 4 \in K \Rightarrow q = 0 \tag{16}$$

- Uses Eqs. (17) and (18), which link to Eq. (11).

Table 3: All known elementary particles and some elementary particles that new modeling suggests nature might include. For each one of the 1t0 column and the 2t0 column, the table alludes to solution-pairs by using sums that echo Eq. (8) and Eq. (10). Each 2t0 solution-pair cascades - in one step - from the corresponding 1t0 solution-pair or solution-pairs. The leftmost column lists solution-pairs. The next two columns provide interpretations that new modeling links to the 1t0 solution-pairs. Q is a magnitude of charge (in units of $|q_e|$, in which q_e denotes the charge of an electron). m denotes mass. The next three columns pertain to elementary particles. The three charged leptons are the electron, the muon, and the tau. n_{EP} denotes the number of elementary particles in a family or, for quarks, in the two families. Regarding quarks and the notion of $0.5\text{Q}_{l_Q/3}$, l_Q can be either two or one. Regarding the $0.5\text{Q}_{2/3}$ family of three quarks and the notion of charge, new modeling posits that a notion of two-thirds times the $Q = 1$ 1t0 solution-pair plus one-third times the $Q = 0$ 1t0 solution-pair pertains. Regarding the $0.5\text{Q}_{1/3}$ family of three quarks and the notion of charge, new modeling posits that a notion of one-third times the $Q = 1$ 1t0 solution-pair plus two-thirds times the $Q = 0$ 1t0 solution-pair pertains. The symbol † denotes that the elementary particles are as-yet unfound. The word inflaton links to popular modeling notions of a possible inflaton elementary particle. 2L cascades from 1L, 3L cascades from 2L, and so forth. The acronym TBD abbreviates the three-word phrase to be determined. Eq. (14) and related remarks define notation of the form $+16_n\hat{}$. The table de-emphasizes (but new modeling does not necessarily rule out) the possibilities that - for each one of some S≥ 1 - 1t0 use of the solution-pair $|+1-2-3-4-8+16_\text{S}\hat{}|$ links to an elementary boson that has spin S+1. Such elementary bosons might link to notions of (S + 1)G families or (S + 1)J families. The table de-emphasizes (but new modeling does not necessarily rule out) the possibilities that - for each one of some S ≥ 1 - 1t0 uses of combinations of the solution-pair $|-1+3-4-6-8+16_\text{S}\hat{}|$ and the solution-pair $|-1-2-3-4-6+16_{(\text{S}-1)}\hat{}|$ link to elementary fermions that have spins of S + 0.5. Such elementary fermions might link to notions of (S + 0.5)$\text{Q}_{>0}$ families. The 2t0 notation $|\pm(-1-3+4)-2-6+8|$ links to 2t0 use of the two solution-pairs $|-1-2-3+4-6+8|$ and $|+1-2+3-4-6+8|$. (Here, $-1-3+4 = +1+3-4 = 0$.) In this table, each 1t0 solution-pair for which $n_k \geq 4$ cascades (in $n_k - 3$ steps) from (at least) one of solution-pair $|-1-3+4|$ and solution-pair $|-1-2+3|$.

1t0	Q	m	Names	Families	n_{EP}	2t0
$\|-1-2+3\|$	1	>0	W	1W_1	1	$\|+1-2-3+4\|$; $\|-1-2-3+6\|$.
$\|-1-3+4\|$	0	>0	Z	1Z	1	$\|+1-2-3+4\|$; $\|-1-3-4+8\|$; $\|+1-3-4+6\|$.
$\|-1-2-3+6\|$	1	>0	Charged leptons	0.5C_1	3	$\|-1+3-4-6+8\|$; $\|-1+2-3-4+6\|$.
$\|+1-3-4+6\|$	0	>0	Neutrinos	0.5N	3	$\|-1+3-4-6+8\|$; $\|-1+2-3-4+6\|$.
$\|+1-2-3+4\|$	0	>0	Higgs boson	0H	1	$\|+1-2-3-4+8\|$; $\|-1+2-3-4+6\|$.
$\|-1-3-4+8\|$	0	=0	Inflaton	0I	1 †	$\|+1-2-3-4+8\|$; $\|-1-3-4-8+16\|$; $\|-1+3-4-6+8\|$.
$\|-1+2-3-6+8\|$ and $\|-1+2-3-4+6\|$	1, 0	>0	Quarks	$0.5\text{Q}_{l_Q/3}$	6	$\|\pm(-1-3+4)-2-6+8\|$
$\|+1-2-3-4+8\|$	0	=0	Gluons	1G	8	$\|\pm(-1-3+4)-2-6+8\|$
$\|+1-2-3-4+8\|$	0	=0	Jay	1J	1 †	$\|\pm(-1-3+4)-2-6+8\|$
$\|-1-3-4-8+16_1\hat{}\|$	0	=0	Photon	1L	1	$\|-1-3-4-8+16_2\hat{}\|$, $\|+1-2-3-4-8+16_1\hat{}\|$, $\|-1+3-4-6-8+16_1\hat{}\|$.
$\|-1-3-4-8+16_S\hat{}\|$, with S being the S in SL	0, 0, . . .	=0, =0, . . .	Graviton, TBD, . . .	2L, 3L, . . .	1 †, 1 †, . . .	$\|-1-3-4-8+16_{(S+1)}\hat{}\|$, $\|+1-2-3-4-8+16_S\hat{}\|$, $\|-1+3-4-6-8+16_S\hat{}\|$

$$\text{1t0 use and } 6 \notin K \Rightarrow \text{one boson or eight bosons} \tag{17}$$

$$\text{1t0 use and } 6 \in K \Rightarrow \text{three flavours of fermions} \tag{18}$$

- Links Eqs. (19) and (20) to spins.

$$\text{If the object is a boson, } S = |n_k - 4| \tag{19}$$

$$\text{If the object is a fermion, } S = |n_k - 4.5| \tag{20}$$

- Links each LRI (as in long-range interaction) elementary particle that nature includes to Eq. (21), in which S links to the S in the notation SL.

$$\text{1t0 use of } |-1-3-4-8+16_S{}^\wedge| \text{ and } S \geq 1 \tag{21}$$

- Points to all known elementary particles.
  - The eight gluons link to 1t0 use of one solution-pair.
  - Each one of the other known boson elementary particles links to 1t0 use of a unique (to that boson elementary particle) solution-pair.
  - Each trio of three equally charged lepton fermion elementary particles links to 1t0 use of a unique (to those three fermion elementary particles) solution-pair.
  - Each trio of three equally charged quark fermion elementary particles links to 1t0 use of a unique (to those three fermion elementary particles) combination of a pair of solution-pairs.
    - From the standpoint of charge, the $Q = 2/3$ quarks link to two-thirds times a $Q = 1$ solution-pair plus one-third times a $Q = 0$ solution-pair.
    - From the standpoint of charge, the $Q = 1/3$ quarks link to one-third times the same $Q = 1$ solution-pair plus two-thirds times the same $Q = 0$ solution-pair.
- Suggests (at least) the following new elementary particles.
  - An inflaton: Popular modeling concordance cosmology suggests that nature might include an inflaton elementary particle.
  - A so-called jay boson: Popular modeling does not necessarily link the repulsive component of the strong interaction or the Pauli exclusion principle to a boson elementary particle.
  - A graviton: Popular modeling suggests that nature might include a graviton elementary particle.
- Suggests that, for boson elementary particles, 2t0 uses of solution-pairs link to types of interactions or decays that nature exhibits (or might exhibit). Discussion related to Table 6 in Ref. [1] provides further information. (The following notions link symbols in Ref. [1] to symbols in the present paper. Z_Γ in Ref. [1] is K in the present paper. n_Γ is n_k. 0d>, 1d>, and 2d> are, respectively, 1x>, 2x>, and 3x>. 0d0 and 1d0 are, respectively, 1t0 and 2t0.)

Table 3 provides a possible analog (for elementary particles) to the periodic table for chemical elements. The periodic table organizes chemical elements based on two notions - the atomic weight that links to an element and the types of interactions in which an element participates. In the sense of Table 3, one might link atomic weight to 1t0 and the notion of property. Types of chemical interactions might link to 2t0.

Regarding Table 3, the following notions pertain.

- For a family of elementary particles, each 1t0 item links to one n_k and to the various k that appear in the sum that leads (or, for quarks, the two sums that lead) to $0 = \Sigma = |\cdots|$. For a boson family for which $n_{EP} = 1$, no further consideration pertains. For a fermion family (for which $n_{EP} = 3$), one further aspect - flavour - pertains. For a boson family for which $n_{EP} = 8$, one further aspect - choice of specific particle - pertains.
- Each 2t0 item links to one n_k and to the various k that appear in the sums that lead to $0 = \Sigma = |\cdots|$. For some families of boson elementary particles, the relevant set of k links to types of interactions that nature exhibits.

3.4. Elementary-particle properties, interactions, and physics constants

This unit discusses interrelations regarding properties of elementary particles, discusses aspects of new modeling that seem to link to popular modeling notions of Gauge symmetries and the Higgs mechanism, suggests interaction patterns - for elementary bosons and elementary fermions - beyond Gauge and Higgs notions, discusses notions regarding the new-modeling-suggested jay boson, and discusses a possible limit regarding a series that starts with electromagnetism and includes - as a second item - gravity. This unit points to possible new links between physics constants and to possibilities for reducing in number popular modeling independent fundamental constants.

3.4.1. Interrelations regarding properties of elementary particles

Regarding boson elementary particles, new modeling defines $(N')^2$ via Eqs. (22) and (23). M' denotes $m/(m_{\mathrm{Z}}/3)$. S' denotes S (as in the spin, in units of $\hbar$), Q' denotes the magnitude of the charge, in units of the magnitude of the charge of the W boson. (Successful popular modeling equates the magnitude of the charge of the W boson to the magnitude of the charge of the electron.) μ' denotes the magnitude of the magnetic moment, in units of the magnitude of the magnetic moment of the W boson.

$$(N')^2 \equiv (M')^2 + (S')^2 + (Q')^2 + (\mu')^2 - (T')^2 \tag{22}$$

$$(T')^2 = 1 \ \Leftrightarrow \ M' > 0 \ ; \quad (T')^2 \ \Leftrightarrow \ M' = 0 \tag{23}$$

New modeling suggests that Eqs. (24) and (25) might pertain regarding all known boson elementary particles and all boson elementary particles that new modeling suggests. (New modeling does not suggest any elementary bosons for which $M' > 0$ and $N' \leq 2$.)

$$N' \in \{0, 1, 2, 3, 4\} \tag{24}$$

$$N' = 4 - S' \geq 3 \ \Leftrightarrow \ M' > 0 \ ; \quad N' = S' \ \Leftrightarrow \ M' = 0 \tag{25}$$

Eqs. (22), (23), (24), and (25) might point to the possibility that the notion of N' links to modeling that lies beyond both popular modeling and the scope of this paper.

Section 5.15 of Ref. [1] suggests the possibility that - for fermion elementary particles - an analog to Eq. (22) pertains and that the analogous equation has bases in logarithms (and not in squares) of ratios of magnitudes of particle properties to magnitudes of properties of specific fermion elementary particles.

3.4.2. Interaction Gauge symmetries and the Higgs mechanism

Popular modeling links the electromagnetic, electroweak, and strong interactions to the respective Gauge symmetries $U(1)$, $SU(2) \times U(1)$, and $SU(3)$.

Mathematics suggests, for any integer D for which $D \geq 2$, that the ground state of a D-dimensional isotropic quantum harmonic oscillator links to $SU(D)$ symmetry. (Section 5.10.2 of Ref. [1] notes that Ref. [14] provides this result.) Excitations of a $D = 1$ quantum harmonic oscillator link to $U(1)$ symmetry. (Section 5.10.3 of Ref. [1] discusses this notion.)

New modeling suggests, regarding Eq. (7), that (unlike in popular modeling) the range that Eq. (26) shows has physics-relevance. Per section 5.10.1 of Ref. [1], the following notions pertain. Wave functions have the form that Eq. (27) shows. Here, r_{HO} is a radial coordinate and has dimensions of length, η is a positive-number scaling factor with dimensions of length, and the wave functions do not necessarily vary based on angular coordinates. For $0 > \nu > -D/2$, the wave functions are defined almost everywhere (as in throughout the domain $0 < r_{HO} < \infty$, but not for $r_{HO} = 0$). The wave functions normalize. For $\nu = -D/2$, normalization pertains in the limit that η approaches zero.

$$0 > \nu \geq -D/2 \tag{26}$$

$$(r_{HO}/\eta)^{\nu} e^{-(r_{HO})^2/(2\eta^2)} \tag{27}$$

For boson elementary particles, popular modeling includes the following notions. For each $S = 1$ zero-mass elementary particle, there are two excitation modes (for example, left-circular polarization and right-circular polarization). For each $S = 1$ nonzero-mass elementary particle, there are three excitation modes (for example, relative to an axis, the spin state can measure as plus one, zero, or minus one). For each $S = 0$ nonzero-mass elementary particle, there is one excitation mode. Across all these cases, for each excitation mode, $D = 1$ harmonic oscillator mathematics links to notions about excitations.

For each of three interaction types - electromagnetic, weak, and strong - new modeling suggests excitation modeling that has bases in mathematics for a D^*-dimensional isotropic quantum harmonic oscillator. For each case, D^* is the product of the number of excitation modes and the number (in Table 3) of 2t0 uses of each K. The respective (regarding interaction types) products are two (as in two times one), three (as in three times one), and four (as in two times two). For each case, new modeling links the popular modeling notion of modeling excitations (via the mathematics of a one-dimensional harmonic oscillator) to the notion that a symmetry related to a $(D^* - 1)$-dimensional isotropic quantum harmonic oscillator pertains.

The respective symmetries are $U(1)$ for the electromagnetic interaction, $SU(2)$ for the weak interaction, and $SU(3)$ for the strong interaction.

Popular modeling includes the notion of a Higgs field and the notion that the ground state energy for Higgs bosons is less than the ground state energy of the Higgs field. Popular modeling links such to the two-word term Higgs mechanism.

New modeling suggests that the popular modeling for the Higgs field links to $D = 3$ (as in three spatial dimensions) and to the popular modeling notion that the ground state links to $\nu = 0$. Popular modeling links the ground state of the Higgs boson to a lower energy than the ground state energy of the Higgs field. New modeling suggests that popular modeling links the ground state of the Higgs boson to new modeling notions of $D = 3$ and $\nu = -1$. Relative to the ground state for the Higgs boson, popular modeling links modeling for excitations to a ground state that links to $D = 1$ and $\nu = 0$.

3.4.3. Interaction patterns - for bosons and fermions - beyond Gauge and Higgs notions

Eqs. (28) and (29) echo successful popular modeling regarding, respectively, a boson raising (as in excitation) operator and a boson lowering (as in de-excitation) operator. Here, a_b^+ denotes raising operator. $|l>$ denotes a state that links to the notion of l excitations. a_b^- denotes lowering operator.

$$a_b^+|n>= (1+n)^{1/2}|n+1>, \text{ for } 0 \leq n < \infty \tag{28}$$

$$a_b^-|n>= (n)^{1/2}|n-1>, \text{ for } 0 \leq n < \infty \tag{29}$$

New modeling suggests that Eqs. (30) and (31) might pertain regarding, respectively, a fermion raising (as in excitation) operator and a fermion lowering (as in de-excitation) operator.

$$a_f^+|n>= (1-n)^{1/2}|n+1>, \text{ for } 0 \leq n \leq 1 \tag{30}$$

$$a_f^-|n>= (n)^{1/2}|n-1>, \text{ for } 0 \leq n \leq 1 \tag{31}$$

New modeling suggests - regarding boson elementary particles - the following parallels to discussion above about Gauge and Higgs notions. Each LRI boson elementary particle links to a $U(1)$ symmetry. Regarding the inflaton boson elementary particle, a parallel - the inflaton field is to the inflaton elementary particle as the Higgs field is to the Higgs boson - to the Higgs boson pertains.

This paper does not discuss notions that - regarding aspects of the popular modeling elementary particle Standard Model - the Higgs mechanism might link to nonzero mass for one or both of nonzero-mass boson elementary particles and fermion elementary particles. This paper does not discuss notions that a would-be inflaton mechanism might link to nonzero properties for one or both of nonzero-property boson elementary particles and nonzero-property fermion elementary particles.

Regarding Eqs. (30) and (31), harmonic oscillator mathematics does not pertain. Regarding interactions that involve fermion elementary particles, new modeling suggests that Eqs. (32) and (33) can pertain. Here, n_l links to the excitation state of a left-handed fermion elementary particle. n_r links to the excitation state of a right-handed fermion elementary particle.

$$a_f^+|n_l, n_r>= (1-n_l)^{1/2}(1-n_r)^{1/2}|n_l+1, n_r+1>, \text{ for } 0 \leq n_l \leq 1 \text{ and } 0 \leq n_r \leq 1 \tag{32}$$

$$a_f^-|n_l, n_r>= (n_l)^{1/2}(n_r)^{1/2}|n_l-1, n_r-1>, \text{ for } 0 \leq n_l \leq 1 \text{ and } 0 \leq n_r \leq 1 \tag{33}$$

New modeling suggests that Eqs. (32) and (33) link to the notion that excitation of a left-handed fermion elementary particle can link to excitation of a right-handed fermion elementary particle.

New modeling suggests that Eq. (34) links to the notion that - for two fermion elementary particles that link to the same (either left or right) handedness, an interaction can de-excite one fermion (as in, in the equation, fermion one) and excite the other fermion (as in, in the equation, fermion two).

$$a_f^+|n_2> a_f^-|n_1>= (1-n_2)^{1/2}(n_1)^{1/2}|n_2+1>|n_1-1>, \text{ for } 0 \leq n_2 \leq 1 \text{ and } 0 \leq n_1 \leq 1 \quad (34)$$

3.4.4. Notions regarding the new-modeling-suggested jay boson

Regarding Table 3, except for the case of gluons and the jay boson, adding a six to the K set (for a boson 1t0 solution-pair) to produce a basis for a 2t0 solution-pair links to a notion of the number three, as in three fermion flavours for the 1t0 use of the new solution-pair. Similarly regarding 1x> use of a solution-pair, adding an integer to the K set for the 1x> solution-pair to produce a basis for a 2x> solution-pair links to a notion of a factor of three, as in three dimensions that link to velocity.

Regarding (gluons and the jay boson and) 1t0 use of the $0 = |+1-2-3-4+8|$ solution-pair, there are two 2t0 solution-pairs ($0 = |-1-2-3+4-6+8|$ and $0 = |+1-2+3-4-6+8|$) that share $K = \{1, 2, 3, 4, 6, 8\}$. New modeling suggests that a factor of three links to each one of the 2t0 solution-pairs - in the sense that $n_{EP} = 9$, as in three times three. Out of the nine elementary particles, eight elementary particles link to the successful popular modeling notion of eight gluons. New modeling suggests that the ninth elementary particle - the jay boson - links to repulsion between the two fermion objects (that might be fermion elementary particles but are not necessarily fermion elementary particles) in a pair of adequately similar fermion objects that (in the sense of popular modeling) are in adequately similar states. New modeling suggests considering that the jay boson interacts with the popular modeling notion of white (or clear) color charge.

Section 5.2 in Ref. [1] suggests popular modeling examples for which the notion of a jay boson might prove helpful.

Regarding groups and symmetries, new modeling suggests the following notions.

Popular modeling links a representation of the group $SU(3)$ to modeling regarding gluons. New modeling suggests that the notion of gluons plus the jay boson might link (in a sense of an extension to popular modeling) to the group $U(3)$. Here, one can consider that the jay boson links to the identity matrix that a representation of $U(3)$ includes but that a similar representation for $SU(3)$ lacks.

3.4.5. A possible limit regarding the types of long-range interactions

Regarding Eq. (5), new modeling suggests linking a factor of four to the solution-pair $1 = \Sigma = |+1|$ and linking a factor of three to the solution-pair $2 = \Sigma = |+2|$. New modeling suggests that the series four, three, and so forth might extrapolate to link a factor of zero to the solution-pair. $5 = \Sigma = |+5|$. New modeling suggests that, for an integer $l \geq 5$, the solution-pair $l = \Sigma = |+l|$ might not link to a physics property. New modeling suggests that such a limit regarding Σ links to the notion that, possibly, no boson elementary particles link - for example, in the sense of Eqs. (24) and (25) - to $S \geq 5$.

Possibly, no data pertains regarding whether nature includes LRI bosons for which $S \geq 3$.

3.5. Dark matter

This unit discusses a candidate specification for dark matter.

Useful modeling might link with the following notions.

- New modeling categorizes elementary particles into two categories.
  - LRI (as in long-range interaction) boson elementary particles include the photon, the (would-be) graviton, and any higher-spin zero-mass bosons that would extend the series that starts with the photon and the graviton.
  - Isomeric-set elementary particles include all elementary particles that are not LRI boson elementary particles.
- Nature includes six isomers (as in sets of isomeric-set elementary particles).
  - So-called isomer-zero links to one set of isomer-set elementary particles and to successful popular modeling notions of elementary particles, left-handedness, charge, charge current, and magnetic moment.
  - Each one of so-called isomer-one through so-called isomer-five links to its own instance of an isomer-set of elementary particles and to its own instance of each one of charge, charge current, and magnetic moment.

- Each one of the pair isomer-zero and isomer-three, the pair isomer-one and isomer-four, and the pair isomer-two and isomer-five links to the one-element term isomer-pair.
- Each one of isomer-two and isomer-four links to popular modeling notions of left-handedness. Each one of isomer-one, isomer-three, and isomer-five links to popular modeling notions of right-handedness.

- New modeling includes notions of n_i (as in number of instances) and $R_{/i}$ (as in reach per instance). Regarding isomers, $n_i = 6$. Each isomer includes its own instance of boson elementary particles (as in the W boson and the Z boson) that intermediate the weak interaction and its own instance of boson elementary particles (as in the eight gluons) that intermediate the strong interaction. For each of the weak interaction and the strong interaction, $n_i = 6$ and the reach (as in number of isomers) of an instance of the interaction is one isomer (as in $R_{/i} = 1$).
- New modeling includes Eq. (35).

$$n_i R_{/i} = 6 \tag{35}$$

- To a first approximation, each isomer does not detect (much in the way of) electromagnetic aspects of the other five isomers. (Data to which Table 1a alludes suggests that isomer-zero stuff detects some electromagnetic aspects of isomer-three stuff.) New modeling suggests that $n_i = 6$ and $R_{/i} = 1$ pertain regarding a 1x$> 1 = \Sigma = |+1|$ so-called component of electromagnetism (as in the electromagnetic interaction) and regarding the property of charge.
- To a first approximation, each isomer interacts gravitationally equally with itself and with each one of the other five isomers. New modeling suggests that $n_i = 1$ and $R_{/i} = 6$ pertain regarding the 1x$> 2 = \Sigma = |+2|$ component of gravity (as in the gravitational interaction) and regarding the property of energy.
- New modeling categorizes LRI-linked object properties (as in object properties that link to LRI) into three categories. (Section 5.11.1 of Ref. [1] suggests a means for computing - given a set K - the category to which - for $\Sigma \geq 1$ - a 1x> use of a solution-pair links. One considers the integers from 1 to the maximum k - in K - that is less than or equal to four. One counts the number of integers that are not in K but are in the range from one to the previously mentioned maximum k. If the count is zero or three, $R_{/i} = 1$. If the count is one, $R_{/i} = 2$. If the count is two, $R_{/i} = 6$. Based on the notion of an object, new modeling suggests that the reach that links to a 2x> use of a solution-pair equals the reach for a 1x> use of a solution-pair from which the 2x>-solution-pair cascades. The result regarding 2x>-use is independent of the choice of 1x>-used solution-pair. The following notions link symbols in Ref. [1] to symbols in the present paper. n_I in Ref. [1] is n_i in the present paper. R_I is $R_{/i}$.) The following notions pertain regarding properties.
  - Each so-called $R_{/i} = 1$ (as in reach-one or in reach of one per instance of property) LRI-linked property associates with the notion of six instances (as in one instance per isomer) of the property. Each one of charge (a 1x> property), charge current (a 2x> property for which the 1x> counterpart is charge), and magnetic moment (a 1x> property) is a reach-one property.
    - New modeling suggests that ordinary matter does not interact directly with dark-matter charge, dark-matter charge current, or with dark-matter magnetic moment.
  - Each so-called $R_{/i} = 2$ (as in reach-two or in reach of two per instance of property) LRI-linked property associates with the notion of three instances (as in one instance per isomer-pair) of the property.
  - Each so-called $R_{/i} = 6$ (as in reach-six or in reach of six per instance of property) LRI-linked property associates with the notion of one instance (as in one instance that pertains to all six isomers) of the property. Each one of energy (a 1x> property) and momentum (a 2x> property for which the 1x> counterpart is energy) is a reach-six property.
    - New modeling suggests that ordinary matter interacts directly with dark-matter energy and with dark-matter momentum.
- New modeling suggests that counterpart (across isomers) elementary particles are identical with respect to spin S and with respect to rest energy (as in mass). (New modeling suggests that the notion of counterpart links to mass but not necessarily to lepton flavour. Popular modeling suggests that - at least for neutrinos - mass eigenstates do not necessarily equal flavour eigenstates.)

- New modeling suggests that counterpart (across isomers) nonzero-mass elementary particles are identical with respect to ratios of magnitudes the respective charges to the respective masses.

- New modeling suggests that each isomer-pair links to a different pairing between charged lepton flavours and rest-energies. Table 9 in Ref. [1] provides more information. New modeling suggests that the pairings for isomer-one, isomer-two, isomer-four, and isomer-five lead to those isomers forming stable neutrons and to those isomers not forming significant numbers of counterparts to isomer-zero atoms. (Sections 3.21.1 and 4.8.1 of Ref. [1] provide more information.)

3.6. Gravity and dark-energy phenomena

This unit discusses phenomena that popular modeling associates with gravitational and dark-energy phenomena. This unit suggests insights regarding the evolution of the universe. This unit suggests insights regarding the formation and evolution of galaxies.

Useful modeling might link with the following notions.

- Regarding the gravity that an object O links to an object I, the following notions pertain.
  - Some gravitational properties (including energy) link to attracting and some gravitational properties (including momentum) link to repelling. An example that links with successful popular modeling regarding special relativity illustrates the attracting or repelling notion. For a non-rotating object I, a rest-energy mc^2 pertains. For any object O, inferences regarding the E (as in energy) of object I and the P (as in momentum) of object I link via Eq. (36). The greater the inferred value of $|P|$, the greater the inferred value of E and the greater the gap - between E and mc^2 - that object O infers. The inferred gravitational effect that links to momentum subtracts from the inferred gravitational effect that links to energy. (Section 3.6.2 of Ref. [1] points to this notion based on an analogy to the electromagnetic invariance of $(E')^2 - (B')^2c^2$, in which E' denotes the inferred magnitude of the electric field - that links to object I - and B' denotes the inferred magnitude of the magnetic field - that links to object I. E' scales with inferred charge. B' scales with inferred charge current plus inferred magnetic moment.)

$$E^2 - P^2c^2 = (mc^2)^2 \tag{36}$$

  - New modeling divides gravitational properties into the following two sets.
    - 1x> gravitational properties that link to $\Sigma = 2$ include the following properties. Here, the notion of an object I implies that each 1x> gravitational property links to one common inferred position.
      1. Energy, which links to $2 = \Sigma = |+2|$, links to $n_k = 1$. $R_{/i} = 6$ pertains.
      2. Angular momentum, which links to $2 = \Sigma = |-2+4|$, links to $n_k = 2$. $R_{/i} = 2$ pertains.
      3. Two moments of inertia, which link to $2 = \Sigma = |+1-2+3|$, link to $n_k = 3$. $R_{/i} = 1$ pertains.
      4. Two so-called rotations-that-link-to-moments-of-inertia which link to, respectively, $2 = \Sigma = |-1+2-3+4|$ and $2 = \Sigma = |-1-2-3+4|$, link to $n_k = 4$. $R_{/i} = 1$ pertains.
    - 2x> gravitational properties that link to $\Sigma = 2$ include the following properties. Here, the notion of an object I implies that each 2x> gravitational property links to the one common position and to one common inferred velocity. Here, the notion of an object I implies that the reaches match the reaches of the counterpart 1x> gravitational properties. For example, the $R_{/i}$ for momentum equals the $R_{/i}$ for energy.
      1. Momentum, which links to $2 = \Sigma = |-2+4|$, links to $n_k = 2$. $R_{/i} = 6$ pertains.
      2. So-called angular-momentum momentum, which links to $2 = \Sigma = |-2-4+8|$, links to $n_k = 3$. $R_{/i} = 2$ pertains.
      3. Two so-called moments-of-inertia momenta, which link to $2 = \Sigma = |-1+2-3+4|$ and $2 = \Sigma = |-1-2-3+4|$, link to $n_k = 4$. $R_{/i} = 1$ pertains.
      4. Two so-called rotations-that-link-to-moments-of-inertia momenta, which link to $2 = \Sigma = |-1+2-3-4+8|$ and $2 = \Sigma = |-1-2-3-4+8|$, link to $n_k = 5$. $R_{/i} = 1$ pertains.

  - $n_k = 1$, $n_k = 3$, and so forth $\Sigma = 2$ properties link to notions of attracting of object O toward object I.
    * The property energy links to $2 = \Sigma = |+2|$, $n_k = 1$, and attracting.
    * The properties moments of inertia link to $2 = \Sigma = |+1-2+3|$, $n_k = 3$, and attracting.
  - $n_k = 2$, $n_k = 4$, and so forth $\Sigma = 2$ properties link to notions of repelling of object O away from object I.
    * The property angular momentum links to $2 = \Sigma = |-2+4|$, $n_k = 2$, and repelling.

- New modeling suggests that, regarding two objects that move (generally radially) away from each other, the gravitational interaction transits some portion of the sequence dominance by $n_k = 4$-property-based repelling, dominance by $n_k = 3$-property-based attracting, dominance by $n_k = 2$-property-based repelling, and dominance by $n_k = 1$-property-based attracting. (Here, one might consider the notions that $n_k = 4$ links to Newtonian dynamics octupole potentials, $n_k = 3$ links to Newtonian dynamics quadrupole potentials, $n_k = 2$ links to Newtonian dynamics dipole potentials, and $n_k = 1$ links to Newtonian dynamics monopole potentials.)
- New modeling suggests that eras in the rate of expansion of the universe link to transitions regarding dominance.
  - Inflation might link to dominance by ($\Sigma = 2$) $n_k = 4$-property-based repelling.
  - The start of the first one of the two multi-billion-years eras links to dominance by ($\Sigma = 2$) $n_k = 3$-property-based attracting.
  - The start of the second (and current) multi-billion-years era links to dominance by ($\Sigma = 2$) $n_k = 2$-property-based repelling.
- New modeling suggests that popular modeling based on general relativity and an equation of state that suits a period early in the first multi-billion-years era would link to $R_{/i} = 1$ and extrapolate to underestimate later effects - which would link to $R_{/i} = 2$ - that link to the equation of state. Section 4.9.1 of Ref. [1] suggests (qualitatively) that the underestimate might account for a significant portion of the gap (regarding increases in the rate of expansion over the most recent some billions of years) between data and popular modeling.
- Section 4.9.2 of Ref. [1] discusses the notion that popular modeling overestimates large-scale clumping of stuff (as in stuff that has bases in at least one of DM - as in dark matter - and OM). New modeling suggests the following notions.
  - Popular modeling links to notions that each one of 1x$>$ use of $2 = \Sigma = |-2+4|$ repulsion and 2x$>$ use of $2 = \Sigma = |-2+4|$ repulsion links to $R_{/i} = 1$.
  - More appropriate application would link to $R_{/i} = 2$ regarding 1x$>$ use of $2 = \Sigma = |-2+4|$ repulsion and would link to $R_{/i} = 6$ regarding 2x$>$ use of $2 = \Sigma = |-2+4|$ repulsion
  - The popular modeling underestimates (based on $R_{/i} = 1$) link to (at least some part of) the popular modeling overestimates regarding clumping.
- Table 4 discusses six possible eras regarding a typical speed of moving away from each other of large clumps. (Discussion related to Table 10 in Ref. [1] cites sources of data and perspective.)

New modeling suggests that some new modeling notions regarding eras that follow the inflationary epoch might not necessarily depend significantly on new modeling notions regarding the inflationary epoch or on new modeling notions regarding eras that might precede the inflationary epoch.

This paper does not try to explore the possibility that (or to estimate a time at which) a transition - for the largest observable objects - from repelling based on $2 = |-2+4|$ to attracting based on $2 = |+2|$ might occur.

New modeling suggests that smaller astrophysical objects generally transit segments of the series that includes $n_k = 3$ attraction, $n_k = 2$ repulsion, and $n_k = 1$ attraction more rapidly than do larger astrophysical objects.

A proto-solar-system that features ordinary matter (as in isomer-zero stuff) forms based on $n_k = 3$, $R_{/i} = 1$ attraction. That proto-solar-system expels isomer-three stuff (and some ordinary-matter stuff) based on $n_k = 2$, $R_{/i} = 2$ repulsion. The proto-solar-system continues to evolve based on the presence of ordinary matter and on $n_k = 1$ attraction. (Section 4.8 in Ref. [1] suggests that isomer-three stuff

Table 4: Six possible eras regarding a typical speed of moving away from each other of large clumps. The rightmost three columns suggest eras. The leftmost three columns describe phenomena that new modeling suggests as noteworthy causes for the eras. Generally, a noteworthy cause links to dominant forces and to notions of accelerations. Generally, an era links to notions of speeds. The symbol $\rightarrow$ links to the notion that a noteworthy cause may gain prominence before an era starts. Subsequent rows associate with later eras. Popular modeling suggests notions of a Big Bang (or - at least - of a time that popular modeling links to the two-word term Big Bang). The symbol † denotes a possible association between the relevant era and some popular modeling notions of a Big Bang. New modeling points to the possibility for the first two eras that the table discusses. 1t0 use of the solution-pair $0 = |-1-2-3-4+8|$ links to the Pauli exclusion principle (and to the might-be jay boson). The 1x> solution-pairs to which the table alludes associate with gravitation. The notation $2 = |-1 \pm 2 - 3 + 4|$ links to the two solution-pairs $2 = |-1+2-3+4|$ and $2 = |-1-2-3+4|$. Pop-mod (as in popular modeling) uses the word inflation (or, the two-word term inflationary epoch) to name the era that links to the third row in the table. Popular modeling suggests that the inflationary epoch started about 10^{-36} seconds after the Big Bang. Popular modeling suggests that the inflationary epoch ended between 10^{-33} seconds after the Big Bang and 10^{-32} seconds after the Big Bang. Possibly, no direct evidence exists for the inflationary epoch. The following notions pertain regarding the column with the one-word label notes. The symbol 1 denotes the notion that popular modeling interpretations of data support the notions of each one of the two billions-of-years eras. The symbol 2 denotes the notion that popular modeling suggests the era. The symbol 3 denotes the notion that new modeling suggestions regarding resolving popular modeling tensions (between data and modeling) that associate with the fifth row do not necessarily depend on the existence of the era.

Force	1x> or 1t0 solution-pairs	$R_{/i}$	$\rightarrow$	Typical speed	Pop-mod duration	Notes
Attractive	$2 = \|+1-3-4-8+16\|$	6	$\rightarrow$	Is negative	-	3
Repulsive	$0 = \|-1-2-3-4+8\|$	1	$\rightarrow$	Turns positive †	-	3
Repulsive	$2 = \|-1 \pm 2-3+4\|$	1	$\rightarrow$	Increases rapidly	Less than a second	2, 3
Attractive	$2 = \|-1-2+3\|$	1	$\rightarrow$	Decreases	Billions of years	1
Repulsive	$2 = \|-2+4\|$	2	$\rightarrow$	Increases	Billions of years	1
Attractive	$2 = \|+2\|$	6	$\rightarrow$	Would decrease	-	3

Table 5: Possible stages and eras regarding galaxy formation and evolution. The rightmost three columns suggest stages and eras. The leftmost three columns describe phenomena that new modeling suggests as noteworthy causes for the stages and eras. Generally, a noteworthy cause links to dominant forces and to notions of accelerations. The symbol † denotes the notion the 2x> $2 = |-2+4|$ - with $R_{/i} = 6$ - also pertains. The symbol $\rightarrow$ links to the notion that a noteworthy cause may gain prominence before a stage starts. Subsequent rows associate with later stages. This table associates with a scenario in which a galaxy forms based on one original one-isomer clump and initially does not significantly collide with other galaxies. Currently, some galaxies associate with the first era, some galaxies associate with the second era, and some galaxies associate with the third era. The following notes pertain. (1) A one-isomer original clump forms. (2) The original clump repels (some) stuff that associates with the isomer that associates with the original clump and (most) stuff that associates with one other isomer. (3) The original clump attracts stuff that associates with the four not-repelled isomers and stuff that associates with the isomer that associates with the original clump. (4) Another galaxy subsumes the original clump and might subsequently merge with yet other galaxies.

Force	1x> solution-pair	$R_{/i}$	$\rightarrow$	Stage	Era	Note
Attractive	$2 = \|-1-2+3\|$	1	$\rightarrow$	1	First	(1)
Repulsive	$2 = \|-2+4\|$ †	2	$\rightarrow$	2	First	(2)
Attractive	$2 = \|+2\|$	6	$\rightarrow$	3	Second	(3)
Attractive	$2 = \|+2\|$	6	$\rightarrow$	4	Third	(4)

evolves similarly to isomer-zero stuff. Isomer-three solar systems would evolve similarly to isomer-zero solar systems. (Section 4.8 in Ref. [1] suggests that isomer-one, isomer-two, isomer-four, and isomer-five might not form protons or atoms. If so, stuff that links to those four isomers might not form large enough clumps for detection (by current means) - as clumps - in isomer-zero solar systems or in isomer-three solar systems.)

Table 5 discusses possible stages and eras regarding galaxy formation and evolution. (Discussion related to Table 10, Table 11, and - especially - Table 12b in Ref. [1] cites sources of data and perspective.)

New modeling suggests that galaxy clusters generally evolve based on somewhat equal presences of all isomers and on $n_k = 1$, $R_{/i} = 6$ attraction.

3.7. Ratios of dark-matter effects to ordinary-matter effects

This unit discusses the notion that the new modeling specifications for dark matter and dark energy suffice to explain a variety of inferred ratios of dark-matter effects to ordinary-matter effects.

New modeling suggests, based on the specification for dark matter and on discussion above regarding gravity and dark-energy phenomena, the following notions (which, in effect, extend Table 1).

- DM (as in dark-matter effects) replaces NOM (as in not-ordinary-matter effects) throughout Table 1.

- Successful popular modeling suggests that each one of cosmic optical background and CMB formed based on atomic transitions. New modeling suggests the following notions.
  - The amount of observed CMB links to a DM:OM ratio of 1 : 1.
  - New modeling suggests the following possibilities.
    - 1x$>$ uses of solution-pairs $1 = \Sigma = |-2-3+5-7+8|$ and $1 = \Sigma = |+2+3-5-7+8|$ associate with the charge of the electron cloud and with the charge of the atomic nucleus. The two solution-pairs associate with interactions via which the atom transits to a new overall charge state.
    - 1x$>$ use of solution-pair $1 = \Sigma = |-1-4+5-7+8|$ associates with electron-cloud principal energy levels and with electromagnetic interactions via which the atom transits to new principal energy-level states.
    - 1x$>$ uses of solution-pairs $1 = \Sigma = |-1-4-5-7+8|$ and $1 = \Sigma = |+1-4-5-7+8|$ associate with fine-structure energy levels and hyperfine energy levels and with electromagnetic interactions via which the atom transits to new fine-structure and hyperfine energy-level states.
    - For each one of the above five solution-pairs ($1 = \Sigma = |-2-3+5-7+8|$, $1 = \Sigma = |+2+3-5-7+8|$, $1 = \Sigma = |-1-4+5-7+8|$, $1 = \Sigma = |-1-4-5-7+8|$, and $1 = \Sigma = |+1-4-5-7+8|$), $R_{/i} = 6$ pertains.
  - Per discussion above in this paper, isomer-zero and isomer-three form atoms. Isomer-one, isomer-two, isomer-four, and isomer-five do not form atoms. New modeling suggests a notion of an effective reach $(R_{/i})_{\mathrm{ef}}$. New modeling suggests Eq. (37).

    $$\text{Regarding electromagnetism: for atomic states and atomic transitions, } (R_{/i})_{\mathrm{ef}} = 2 \qquad (37)$$
- Table 5 suggests galaxy formation and evolution scenarios that might link to the ratios that Table 1b lists.
  - Some early galaxies link to a DM:OM ratio of $1 : 0^{+}$, but possibly observations do not (yet) point directly to such DM galaxies.
- Section 4.11.2 and section 4.11.3 of Ref. [1] suggest that $R_{/i} = 6$ links with the fives in the $5^{+} : 1$ ratios that Table 1 lists.
- Section 4.11.3 of Ref. [1] suggests three mechanisms that might link to the pluses in the $5^{+} : 1$ ratios that Table 1 lists.
- New modeling suggests that most stars form from one-isomer clumps. DM (as in dark-matter) stuff can form one-isomer stars. OM (as in ordinary-matter) stuff does not interact directly with thermal radiation from dark-matter stars. (This notion of dark-matter stars differs from notions of dark stars that Ref. [15] discusses.)
  - New modeling suggests that thermal states of objects might link to 1x$>$ use of $1 = \Sigma = |-4+5|$, which links to $R_{/i} = 1$. (Additionally, 1x$>$ uses of the following $R_{/i} = 1$ solution-pairs might pertain: $1 = \Sigma = |-1-2+3-4+5|$, $1 = \Sigma = |+1+2-3-4+5|$, $1 = \Sigma = |-1+2-3-4+5|$, and $1 = \Sigma = |+1+2-3-4+5|$.)
- Based on the notion that atoms link to 1x$>$ and $(R_{/i})_{\mathrm{ef}} = 2$, it might (someday) be possible to detect light that isomer-three DM atoms emit recently (compared to when cosmic optical background radiation formed). The following sentences point to possible bases for such light. Some isomer-three DM atoms would exist near isomer-three DM stars. Some isomer-three DM atoms might exist in laboratory settings (and, OM light might - occasionally - excite such isomer-three DM atoms).
- Section 4.11.4 of Ref. [1] suggests that a combination of popular modeling and new modeling might comport with observations regarding the aftermath of the so-called Bullet Cluster collision of two galaxy clusters. Section 4.11.4 of Ref. [1] suggests that people might want to interpret observations regarding light from IGM (as in intergalactic medium) in the context of isomers and reaches.

3.8. Baryon asymmetry

This unit suggests an explanation for baryon asymmetry.

Popular modeling suggests that each interaction that creates a left-handed (as in matter) elementary fermion also produces a right-handed (as in antimatter) elementary fermion.

New modeling uses the notation f_{l-r} to denote the handedness of an elementary fermion. New modeling links $f_{l-r} = +1$ to left-handedness. New modeling links $f_{l-r} = -1$ to right-handedness. New modeling posits that conservation of fermion handedness - as in $0 = +1 - 1$ - pertains regarding each interaction that - in the sense of popular modeling - creates (or destroys) two fermion elementary particles.

New modeling suggests that each one of the three isomer-pairs links to its own instance of conservation of fermion handedness. New modeling suggests that an interaction can create (for example) a pair of fermion elementary particles for which the left-handed fermion elementary particle links to isomer-zero and the right-handed fermion elementary particle links to isomer-three.

New modeling suggests the following scenario.

- At sometime early in the history of the universe, for each isomer, the number of left-handed fermion elementary particles equaled the number of right-handed fermion elementary particles.
- For the isomer-pair that links to isomer-zero and isomer-three, at some time, a run-away (as in lasing-like) burst of interactions led to the dominance (in what new modeling happens to model as isomer-zero stuff) of the popular modeling notion of left-handed fermion elementary particles. Here, dominance links to particle counts. New modeling suggests that a dominance (in what new modeling happens to model as isomer-three stuff) of right-handed fermion elementary particles exists.

Examples of 1x> uses of solution-pairs that might associate with such lasing include solution-pairs for which $\Sigma = 1$ and $K = \{3, 4\}$ or $K = \{3, 4, 5, 7\}$, solution-pairs for which $\Sigma = 2$ and $K = \{1, 4, 5\}$ or $K = \{1, 4, 7\}$, and solution-pairs for which $\Sigma = 3$ and $K = \{1, 4\}$ or $K = \{1, 4, 5, 7\}$. For each one of the six (as in three values of Σ times two sets K for each value of Σ) cases, for the 1x> use, $R_{/i} = 2$ pertains, $6 \notin K$ pertains, and a 2x> use of a one-step cascade that associates with $6 \in K$ can pertain.

3.9. N-body modeling, binding energy, and entanglement

This unit suggests that new modeling can be a basis for insight regarding popular modeling multi-body modeling.

New modeling links Eq. (13) to elementary particles.

New modeling suggests that atoms might model as two-component systems, with one component being the electron cloud and the other component being the atomic nucleus.

New modeling suggests that Eq. (38) pertains regarding the above-suggested modeling for atoms and regarding modeling for other two-component systems.

$$\{5, 7\} \cap K \neq \emptyset,\ \{9, 11, 13, \cdots\} \cap K = \emptyset \tag{38}$$

One component might link to $5 \in K$ and $7 \notin K$. The other component might link to $5 \notin K$ and $7 \in K$. The system might link to $5 \in K$ and $7 \in K$. The following $0 = \Sigma$ solution-pairs would link to $S = 0$.

- For the first component, $|-1-2-5+8|$ could link to nonzero charge; $|+1-2-4+5|$, $|+1-4-5+8|$, and $|+2-3-4+5|$ could link to zero charge.
- For the second component, $|+2-3-7+8|$ could link to nonzero charge; $|-1-2-4+7|$ and $|+3-4-7+8|$ could link to zero charge.
- For the system, $|+1-3-5+7|$ could link to nonzero charge; $|+2-4-5+7|$ could link to zero charge.

For each $S = 0$ solution-pair, one series of one-step cascades starts by adding 6 to K (to provide a solution-pair that links to $S = 0.5$) and continues to fill out the sequence $S = 0.5$, $S = 1.5$, and so forth. For each $S = 0$ solution-pair, one series of one-step cascades fills out the sequence $S = 1$, $S = 2$, and so forth.

Discussion above assumes that two components (for each of which $S \geq 0$) exist. New modeling suggests the possibility that - for purposes of modeling - some solution-pairs might link to the notion of uninferred (as in not inferred or as in not relevant within the relevant modeling; for example, some popular modeling classical mechanics modeling does not necessarily include the notions of boson and fermion). For one component, $|-2-3+5|$, $|-1-4+5|$, or $|-1-5+6|$ might link to the notion of uninferred. For the other component, $|-3-4+7|$ or $|-1-6+7|$ might link to the notion of uninferred. For the two-component system, $|-2-5+7|$ might link to the notion of uninferred.

For a pair of objects, one can consider changes in modeling as modeling transits from considering each of the two objects as independent of the other object to considering that each of the two objects is a component of one system (as in one two-component object).

- During the transition between models, some momentum (as in the 2x> $2 = \Sigma = |-2+4|$ property) of an object can transit to become angular momentum (as in the 1x> $2 = \Sigma = |-2+4|$ property) of the system. Before the modeling transition, the 2x> $2 = \Sigma = |-2+4|$ inferred reduction to energy (as in the 1x> $2 = \Sigma = |+2|$ property) links to $R_{/i} = 6$. After the modeling transition, the 1x> $2 = \Sigma = |-2+4|$ inferred reduction to energy (as in the 1x> $2 = \Sigma = |+2|$ property) links to $R_{/i} = 2$. New modeling suggests that the inferred increase (from before the modeling transition to after the modeling transition) in energy links to the popular physics notion of (for the two-component system) an increase (possibly from zero) in binding energy.

- During the transition between models, some aspects of magnetism transit to potentials that link to (in the sense of new modeling) higher values of n_k and to (in the sense of popular modeling) higher (than dipole) poles (as in, quadrupole, octupole, and so forth). A step that links to 1x> use of $1 = \Sigma = |-1-2+4|$ - and to a popular physics notion of quadrupole - can pertain. Regarding 1x> use of $1 = \Sigma = |-1-2+4|$, $R_{/i} = 6$ pertains and contrasts with $R_{/i} = 1$ for each one of 1x> use of $1 = \Sigma = |+1|$ and 1x> use of $1 = \Sigma = |-1+2|$.

New modeling suggests that, as modeling considers larger and larger objects, aspects that link to momentum can transit to linking to angular momentum and binding energy. (The following sentences might pertain regarding transiting from momentum to angular momentum. Ref. [16] discusses - in a context of space-time coordinates - concepts of de-compactified - as in linear and somewhat flat - dimensions and compactified - as in curled-up - dimensions. Popular modeling associates momentum with velocity. New modeling associates velocity with 2x>. Popular modeling associates angular momentum with angular velocity. New modeling associates angular velocity with 1x>. Transiting a modeling focus from momentum toward angular momentum might link to a notion of a process that links to the curling up - or the mathematically compactifying - of a three-dimensional space that links - when not at all compactified - to velocity.)

This paper does not explore the notion that modeling might usefully parameterize transiting from momentum to angular momentum via a parameter l_{cu} for which $1 \leq l_{cu} \leq 2$, $l_{cu} = 2$ links to the two in 2x>, and $l_{cu} = 1$ links to the one in 1x>.

To the extent that notions above link to useful modeling for two-component systems, nested use of the notions might link to useful modeling for some N-component systems for which N is an integer power of two (with the integer exceeding one). To the extent that modeling can consider that some components link to notions of empty states, such modeling might pertain to N-component systems for which N is any nonnegative integer. While such modeling might prove useful regarding spin states of N-component systems, such modeling might not necessarily extend to broader popular modeling notions regarding N-body dynamics. (This paper does not explore notions that modeling for N-component systems - for which N is at least three - might link to $\{9, 11, 13, \cdots\} \cap K \neq \emptyset$.)

Popular modeling suggests that each object and each excited mode of a field associates with positive energy. Thus, popular modeling suggests that each object and each excited mode of a field links to gravity. New modeling suggests that an object O could infer entanglement within any inferred multi-component object I. New modeling suggests that - with an appropriate definition of entanglement - an object I entangles with the fields (such as the gravitational field) that object O links to object I. New modeling suggests that modeling can consider that entanglement is a universally applicable notion and that inferences (by an object O) of an object I might disturb - in a sense of modeling - aspects of the notion of universal entanglement.

3.10. Uncertainty

This unit discusses the notion that new modeling might point to the notion that popular modeling classical physics modeling for objects could include notions of uncertainty.

Ref. [17] suggests that, regarding popular modeling, notions of uncertainty regarding position may pertain, even in the absence of trying to measure momentum.

Discussion above suggests - regarding multi-component systems - that modeling regarding angular momentum (as in 1x> $2 = \Sigma = |-2+4|$) might link to useful inferences even if modeling regarding N-body dynamics within multi-component systems might not link to an adequately complete set of useful inferences.

One might consider the following thought-experiment. One considers popular modeling Newtonian dynamics notions about angular momentum for a rigid multi-component system that models as cylindrically symmetric with respect to an axis of spin. One considers that the system is a collection of "objects" that revolve - with the same angular velocity - around the axis. The system models as having nonzero size.

One can consider the system to be an object I. One can explore inferences that link to an object O. One might assume (from discussion about N-body systems) that object O does not (as in possibly cannot) infer a complete and completely accurate set of values for the trajectories of all the objects. One can compare two possible sets of inferences. The sets share one value of system angular momentum. The sets differ with respect to the notion of an inferred radius, which here links to notions regarding two dimensions (that are orthogonal to the axis of revolution). One can suppose that the second set of inferences links to a smaller (than does the first set) radius. One can link a scale factor F to the notion of smaller, with $F = 1$ linking to the notion of same size (and to the notion of the first set of inferences), $F = 0$ linking to a notion of zero radius (as in a three-dimensional straight-line-like system), and $0 < F < 1$ pertaining (for this discussion) to the second set of inferences. For each object, the mass m does not vary between the two sets of inferences. For each object, the lever arm l scales based on the factor F. Thus, the angular velocity scales based on a factor F^{-1}.

Object O has an option to change the modeling basis - regarding the trajectories of the objects - from 1x> $2 = \Sigma = |-2+4|$ (and angular velocities) to 2x> $2 = \Sigma = |-2+4|$ (and linear velocities). In the new modeling basis, the following notions pertain. For each object, the mass m does not vary (between the two modeling bases and, within the new modeling basis ...) between the two sets of inferences. The relevant linear distances (as in position coordinates x that associate with two dimensions that are perpendicular to the axis of system rotation) scale based on the factor F. The linear velocities v of the objects scale (based on the scaling that pertained to angular velocity and hence ...) based on the factor F^{-1}. The momenta P of the objects are mv and thus scale based on the factor F^{-1}.

Per remarks above, object O does not infer a complete set of completely accurate values regarding trajectories for the objects. Popular modeling quantum modeling (and popular modeling wave modeling) suggest notions of uncertainty principles. For example, regarding one object and one dimension, Eq. (39) pertains. Here, σ_x is the square root of an average value for the square of x (as in position, in one dimension) minus the square of an average value for x. σ_P is the square root of an average value for the square of P (as in the x-related component of momentum) minus the square of an average value for P.

$$\sigma_x \sigma_P \geq \hbar/2 \tag{39}$$

New modeling suggests that (based in part on new modeling notions of pairs - each of an object O and an object I - of objects and new modeling notions regarding inferences that an object O can make about an object I) that position-and-momentum uncertainty can link to popular modeling classical N-body physics. (New modeling links directly to popular modeling notions of S and $S\hbar$. New modeling does not directly link to spins that link to axes of rotation. One might assume that new modeling notions regarding popular modeling position-and-momentum uncertainty extend to - in the sense of popular modeling - to treat the three spatial dimensions equally.)

Popular modeling includes an uncertainty relationship of the form that Eq. (40) shows. Here, σ_t is the square root of the difference between an average value for the square of t (as in time) and the square of an average value for t. σ_E is the square root of the difference between an average value for the square of E (as in energy) and the square of an average value for E.

$$\sigma_t \sigma_E \geq \hbar/2 \tag{40}$$

New modeling notes the following two analogies. E (as in energy) is to 1x> as P (as in momentum) is to 2x>. t (as in time) is to (a would-be) 0x> as x (as in position) is to 1x>. Via these analogies, new modeling suggests that time-and-energy uncertainty can link to popular modeling classical N-body physics.

This paper mentions here (but does not further discuss) the following notions. Given notions (above) about uncertainty, some seemingly successful popular modeling that has bases in point-like objects might not necessarily be adequately accurate or adequately useful. For such circumstances, uses of the notions

Table 6: Possible matches between some new modeling aspects and the popular modeling general relativity notion of a stress-energy tensor. Regarding components of a stress-energy tensor, the equations $1 \leq a \leq 3$, $1 \leq b \leq 3$, and $a < b$ pertain. T^{00} associates with energy density. T^{0a} associates with momentum density. T^{a0} associates with energy flux. T^{aa} associates with pressure and with angular momentum density. T^{ab} associates with shear stress. T^{ba} associates with momentum flux. The first four rows in the table associate with $1 \leq n_k \leq 3$. The last three rows in the table associate with $4 \leq n_k$. For each $\Sigma = |\cdots|$ to which the table alludes, $\Sigma = 2$. The symbol C-1 links to the notion of the use of the solution-pair $2 = \Sigma = |+2|$ and of uses of solution-pairs that cascade from $2 = \Sigma = |+2|$. The symbol C-2 links to the notion of the use of the solution-pair $2 = \Sigma = |+1-2+3|$ and of uses of solution-pairs that cascade from $2 = \Sigma = |+1-2+3|$.

Components	Force	C-1 solution-pair	C-1 $R_{/i}$	C-2 solution-pairs	C-2 $R_{/i}$
T^{00}	Attracting	1x> $|+2|$	6	-	-
T^{0a}, T^{a0}	Repelling	2x> $|-2+4|$	2	-	-
T^{aa}	Repelling	1x> $|-2+4|$	2	-	-
T^{ab}, T^{ba}	Attracting	2x> $|-2-4-8|$	2	1x> $|+1-2+3|$	1
?	Repelling	-	-	2x> $|\pm 2-1-3+4|$	1
?	Repelling	-	-	1x> $|\pm 2-1-3+4|$	1
?	Attracting	-	-	2x> $|\pm 2-1-3-4+8|$	1

of point-like objects and the techniques of Fourier transforms (for example, uses that link position and momentum) may lead to suboptimal results. (Ref. [18] discusses the notion that linking images of objects to notions of Airy disks can provide - regarding resolving images of objects - bases for higher resolution results than results that stem from linking images to notions of point-like objects and using Fourier transforms.)

3.11. Vacuum energy and long-range-interaction-boson ground states

This unit discusses the notion that popular modeling might want to include the possibility that the total - regarding electromagnetism, regarding gravity, and regarding the combination of electromagnetism and gravity - vacuum energy can be zero.

New modeling suggests that popular modeling for the ground state of a photon could link to the ground state of a $D = 2$ (as in two-dimensional) isotropic quantum harmonic oscillator. In effect, $D = 2$ links to the sum of $D = 1$ for the $s = +1$ (as in left-circular polarization) mode and $D = 1$ for the $s = -1$ (as in right-circular polarization) mode.

New modeling suggests that the ground state for a photon (and for any other LRI boson) links to $D = 2$ and $\nu = -1$. (Section 5.10.1 of Ref. [1] provides more detail regarding relevant mathematics.) Per Eq. (7), each ground state links to $E = 0$. The sum over all such ground states (even without invoking notions of limiting the ranges of frequencies) is $E = 0$.

Possibly, popular modeling might want to consider the possibility that photons (or electromagnetism) and gravitons (or gravity) might contribute nothing to a cosmological constant Λ.

3.12. General relativity and quantum gravity

This unit suggests limits regarding the applicability of general relativity, discusses interpretations of general relativity, and provides insight that might have uses regarding the notion of quantum gravity.

Table 6 discusses possibly useful notions of matches (that new modeling suggests) between some new modeling aspects and the popular modeling general relativity notion of a stress-energy tensor.

New modeling suggests bounds on the appropriateness of general relativity. For example, popular modeling based on general relativity would not be appropriate to the extent that some relevant objects include stuff that links to more than one isomer-pair and at least one such relevant object models as having significant angular momentum.

To date, possibly no precision test of general relativity has involved objects that link to more than one isomer-pair.

New modeling might not disturb popular modeling notions of so-called equivalence principles - regarding mass or energy - that popular modeling links to general relativity. However, new modeling points to the notion that - regarding angular momentum (as in 1x> $|-2+4|$) and other possibly relevant properties (such as moments of inertia, as in 1x> $|+1-2+3|$) - no similar principle would pertain.

Popular modeling discusses two interpretations of modeling based on general relativity. One interpretation links to notions of forces. The other interpretation links to notions of geodesic motions (of small-mass objects) and a curved space-time. New modeling suggests that (even if modeling ignores electromagnetic and other non-gravitational interactions) geodesic motion does not pertain, (at least) because the reach $R_{/i}$ regarding angular momentum is (two and thus) less than six.

Section 4.9.1 of Ref. [1] suggests (qualitatively) that (based on the interpretation that links general relativity to forces) use of popular modeling and an appropriate equation of state might link (quantitatively) to some closing regarding the gap (regarding increases in the rate of expansion - of the universe - over the most recent some billions of years) between data and popular modeling.

Possibly, popular modeling inabilities to develop models for would-be quantum gravity link to the notion that general relativity is not a basis for adequately accurate modeling regarding gravitational phenomena.

Popular modeling regarding electromagnetic interactions with properties such as charge might use classical physics notions. Popular modeling regarding electromagnetic interactions with properties such as atomic states use quantum physics notions.

New modeling might (regarding gravitational interactions) suggest the following notions.

- For gravitational aspects related to properties such as energy, momentum, and angular momentum, classical modeling might suffice.
- For gravitational aspects related to properties that link to 1x> uses of solution-pairs such as the two solution-pairs $2 = \Sigma = |\pm(-1-3+4)-5+7|$ and the two solution-pairs $2 = \Sigma = |\pm(-1-2+3)-5+7|$, modeling based on quantum gravity might be useful.

In this sense, new modeling might suggest that future popular modeling regarding quantum gravity could be as easy as - and as hard as - successful popular modeling quantum electrodynamics regarding multi-component systems.

3.13. Properties, magnitudes of properties, forces, and conservation laws

This unit discusses a catalog of properties of objects, minimum magnitudes for nonzero values of some properties, interaction forces, and conservation laws.

New modeling suggests considering the following regarding a value of k.

- The pair $s_k = +1$ and $s_k = -1$ links to modeling for the ground state of a $D = 2$ isotropic quantum harmonic oscillator. (This notion echoes the notion that successful popular modeling links aspects of excitations of each one of the $s = +\Sigma$ mode and the $s = -\Sigma$ mode to mathematics for a one-dimensional harmonic oscillator.) The ground state of a $D = 2$ isotropic quantum harmonic oscillator links to $SU(2)$ symmetry. The notion of three generators links to the group $SU(2)$. For new modeling, the notion of three generators can link to three popular modeling (spatial, velocity, angular-velocity, or other) dimensions or to three new modeling discrete values.
- $s_k = 0$ links to modeling for excitations of a $D = 1$ quantum harmonic oscillator. Excitations of a $D = 1$ quantum harmonic oscillator link to $U(1)$ symmetry. The notion of one generator links to the group $U(1)$. For new modeling, the notion of one generator can link to one popular modeling (temporal or other) dimension or to one new modeling discrete value.

New modeling suggests extrapolating from the notion of k (as in a member of a set K) to a larger (than the notion of K) set of integers k'.

- For a k that is a member of a set K, $k' = k$ is a member of the larger set.
- For $k = 1$ (and for $k = 2$) and $n_k = 1$, popular modeling Newtonian dynamics suggests - regarding an object that models as point-like - the spatial dependence r^{-n_k} for the related potential.
- Popular modeling can embrace potentials that link to r^0. Here, a force would have no effect. Popular modeling can embrace potentials that link to r^1. Here, notions of asymptotic freedom link to the strong interaction. Popular modeling can embrace potentials that link to r^2. Here, modeling links to notions of three-dimensional isotropic harmonic oscillators.
- Related to the statements just above, new modeling posits (respectively) the following.
  - $k' = 0$ has relevance. The pair $s_0 = +1$ and $s_0 = -1$ links to the notion of three isomer-pairs. $s_0 = 0$ links to the notion that LRI boson elementary particles do not necessarily conceptually link to single isomer-pairs and can have components for which $R_{/i} = 6$.
  - $k' = -1$ has relevance. The pair $s_{-1} = +1$ and $s_{-1} = -1$ links to the notion of three color charges. $s_{-1} = 0$ links to the notion that elementary particles other than quarks and gluons do not link to the three color charges. Popular modeling links all known objects and all known elementary particles (other than quarks and gluons) to notions of no color charge or to notions of a white (or clear) color charge.

Table 7: Aspects of new modeling and aspects of popular modeling. The leftmost column lists notions that have bases in new modeling uses of integers and integer-arithmetic equations. The symbol ‡ links to the notion that 1x> $1 = |\pm(-1-2+3)-4+5|$ might pertain (also or instead of 1x> $1 = |-4+5|$). The symbol CBEPF abbreviates the six-word term candidate basis for elementary particle families. Regarding CBEPF, $0 = \Sigma$, $\{1, 3\} \subset K$, and $\{5, 7\} \cap K = \emptyset$ pertain. EP abbreviates elementary particle. The next two columns feature notions - from new modeling - of instances and reaches. The fourth column names properties. Popular modeling includes each property, except isomer-pair, precessing magnetic moment, and specific sub-items for the 1t0 and 2t0 notions. The symbol EF abbreviates the two-word term elementary fermion. The letter-strings precess, mag, and mom abbreviate (respectively) precessing, magnetic, and moment. The symbol 2CO abbreviates the two-element phrase two-component object. LRI abbreviates long-range interaction. Interact abbreviates interactions. In the symbol column, each symbol (except for f_{l-r}) echoes popular modeling notation. Regarding the column with label A, the following four sentences pertain. The symbol A denotes that LRI interactions do not transmit the related property and that the property adds across objects. The symbol A' denotes that LRI interactions transmit the related property and that the property adds across objects and LRI fields. New modeling suggests that - for each A property - n_i instances of a conservation law pertain. New modeling suggests that - for each A' property - one instance of a conservation law pertains. The following sentences pertain regarding the column with the one-word label note. The symbol † links to the notion of $|Q| = |q|/|q_e|$ and to the triad - which pertains for nonzero-charge elementary particles of $|Q| = 1/3$, $|Q| = 2/3$, and $|Q| = 1$. Rows that note three rational numbers point to values that elementary particles exhibit. Regarding the property of energy, popular modeling includes the notion of one temporal CV (as in complementary variable) - time. Regarding the property of momentum, popular modeling includes the notion of three spatial CV - three components of a position 3-vector. The three letter-strings uninfer, bos, and ferm abbreviate (respectively) uninferred, boson, and fermion. The three letter-strings temp, d, and spat abbreviate (respectively) temporal, dimension (or dimensions), and spatial.

New modeling notion	n_i	$R_{/i}$	Property	Symbol	A	Note
1x> $1 = \|+1\|$	6	1	Charge	q	A	†
1x> $2 = \|+2\|$	1	6	Energy	E	A'	1 temporal CV
2x> $2 = \|-2+4\|$	1	6	Momentum	P	A'	3 spatial CV
1x> $2 = \|-2+4\|$	3	2	Angular momentum	$S\hbar$	A'	Uninfer., bos., ferm.
1x> $3 = \|+3\|$	3	2	EF handedness	f_{l-r}	A	$+1$, 0, -1
1x> $4 = \|+4\|$	6	1	EF baryon number	B	A	$+1$, 0, -1
1x> $4 = \|+4\|$	6	1	EF lepton number	L	A	$+1/3$, 0, $-1/3$
1x> $4 = \|+4\|$	6	1	$B-L$	$B-L$	A	-
1x> $6 \in K$	6	1	EF flavour	-	-	3 flavours
$k' = 0$	3	2	Isomer-pair	-	-	3 isomer-pairs
$k' = -1$	6	1	Quark color charge	-	-	3 colors
$k' = -2$	-	-	Modeling dimensions	-	-	1 temp. d., 3 spat. d.
1x> $1 = \|-1-2+4\|$	6	1	Precess. mag. mom.	-	-	-
1x> $1 = \|-4+5\|$ ‡	6	1	Surface temperature	-	-	-
1x> $1 = \|\cdots 5-7+8\|$	3	2	2CO atomic state	-	-	-
1t0 CBEPF $16 \notin K$	6	1	Non-LRI EP	-	-	-
2t0 CBEPF $16 \notin K$	-	-	Non-LRI EP interact.	-	-	-
1t0 CBEPF $16 \in K$, $S \leq 3$	1	6	LRI EP	-	-	-
1t0 CBEPF $16 \in K$, $S = 4$	6	1	LRI EP	-	-	-
1t0 $\{5, 7\} \cap K \neq \emptyset$	-	-	2CO spin	-	-	Uninfer., bos., ferm.
EP, even $2S$, 2t0 aspects	-	-	Gauge symmetries	-	-	-

– $k' = -2$ might have relevance. The pair $s_{-2} = +1$ and $s_{-2} = -1$ links to the notion of three dimensions. In popular modeling, the notion of three dimensions can link to three spatial dimensions (which, in turn, in new modeling link to K-related aspects of 1x>), three velocity-centric dimensions (which, in turn, in new modeling link to K-related aspects of 2x>), three angular-velocity dimensions (which, in turn, in new modeling link to K-related aspects of 1x>), and so forth. $s_{-2} = 0$ links to the notion of one dimension (which, in turn, in new modeling links to K-related aspects of 1t0 and to K-related aspects of 2t0).

Table 7 points to some links between aspects of new modeling and aspects of popular modeling.

The following notions pertain regarding Table 7.

- New modeling suggests linking EF handedness to 1x> $3 = \Sigma = |+3|$, based on the notion that - for 1x> $3 = \Sigma = |+3|$ - $n_i = 3$.
- New modeling suggests linking EF baryon number and EF lepton number to 1x> $4 = \Sigma = |+4|$, based on the notion that - for 1x> $4 = \Sigma = |+4|$ - $n_i = 6$.
- New modeling suggests that each property that associates with A or A' associates with a conservation law.

- For each item (except the $2 = \Sigma = |\cdots|$ items) for which one of A or A' pertains, the relevant LRI field does not transmit the property from one object to another object. n_i instances of the property pertain. n_i instances of conservation of the property pertain.
- For each one of the $2 = \Sigma = |\cdots|$ items for which A' pertains, the gravitational field exhibits and can transmit the property. Independent of the notion that $n_i = 3$ pertains for 1x> $2 = |-2+4|$, one instance of conservation of the property pertains across the gravitational field and all six isomers.

- New modeling suggests the following regarding properties that associate with A or A'.
  - For each property other than energy and momentum, a nonzero minimum absolute value exists for nonzero values. For example, for charge the minimum is $|Q| = |q|/|q_e| = 1/3$. The minimum for spin (as in angular momentum divided by $\hbar$) is 0.5.
  - Energy (a 1x> property) and momentum (the 2x> property that associates with energy) are the only two properties for which popular modeling notions of complementary variables (as in, respectively, time and position) pertain and for which some popular modeling notions of uncertainty pertain.

- New modeling suggests (but might not necessarily imply) that 1x> $3 = \Sigma = |+3|$ links to an elementary particle (which would be a LRI $S = 3$ boson elementary particle).

- New modeling suggests (but might not necessarily imply) that 1x> $4 = \Sigma = |+4|$ links to an elementary particle (which would be a LRI $S = 4$ boson elementary particle).

4. Perspective about present and possible future modeling

This unit suggests perspective about new modeling, about possibilities for developing principles that would underlie new modeling, and about relationships between popular modeling and new modeling.

4.1. New modeling

This unit suggests perspective about new modeling and about possibilities for developing principles that would underlie new modeling.

Strengths that new modeling might seem to exhibit include the following notions. New modeling - when combined with popular modeling - explains data that popular modeling alone seems not to explain. New modeling branches from and seems not to conflict with successful popular modeling. New modeling provides possibly useful perspective regarding unverified aspects of popular modeling. New modeling seems to link notions that associate with (sometimes seemingly) minimally overlapping areas of physics.

Weaknesses that new modeling might seem to exhibit include the following notions. For each one of some individual aspects of new modeling for which some relevant data exists and the aspect seems not to conflict with data, some of the following notions might pertain. The amount of relevant data might seem small. The aspect points to possible (non-contradictory) predictions, but - for predictions for which there seems to be no approximate data - the aspect does not necessarily link to a likelihood that the prediction will prove true.

Table 7 suggests patterns and models that organize properties of objects.

The following questions point to possible opportunities for improving modeling or providing principles.

- To what extent do other values of k' or other ...x> uses of solution-pairs point to properties of objects or properties of LRI fields that people can infer or otherwise find useful?

- To what extent do known notions (such as Eqs. (11), (15), and (16)) constitute principles or point to principles that underlie modeling regarding the notion of properties of objects or the notion of fields?

- To what extent are there (for properties that do not directly link to LRI) useful analogs to ...x> uses of solution-pairs? And what (from aspects above or from new aspects) principles might pertain?

- To what extent might modeling point to notions of links between (or inseparability regarding) electromagnetism and gravity?

- To what extent might the exponent six in Eq. (5) link to the number, six, of isomers?

- To what extent might people work from a (possibly broader - than to which this paper might allude above) set of principles to produce a comprehensive catalog of properties?
- To what extent might the following notions help people develop understanding about, principles for, or other aspects regarding new modeling?
  - Relationships within and regarding solution-pairs seem to have importance. Relationships link aspects such as Σ; K; n_k; the various k that are (or are not) elements of K; interpretations that seem to associate with such k; the notion of same-Σ cascades (and the notion of other cascades); notions of uses (such as 1x> use or 1t0 use) of solution-pairs; the extent to which sums (as in Σ) or sub-sums (as in, for $2 = \Sigma = |\pm(-1-3+4)-5+7|$, the sub-sum $-1-3+4$) add to zero; relationships to data, properties of objects, fields, and objects; and relationships to popular modeling notions regarding conservation laws, changes regarding motions of objects, and changes regarding internal states of objects.
  - Associations between cascades, popular modeling notions of degrees of freedom, and popular modeling notions regarding minimal magnitudes of nonzero value might have importance.
  - Associations between new modeling notions of positive integers n_k, new modeling notions of 2^{n_k-1} solution-pairs, popular modeling notions of spatially n_k-pole components of multipole expansions, and popular modeling notions of (n_k-1)-dimensional spatial arrays of 2^{n_k-1} similar objects might have importance.
  - New insight regarding relationships between the notion of an object, the notion of object-related fields, notions of properties that associate with an object, notions of properties that associate with object-related fields, and notions of possible modeling-related boundary regions between and an object and its fields might prove to have importance.

Table 3 suggests patterns and models that organize elementary particles.

The following questions point to possible opportunities for improving modeling or providing principles.

- To what extent does nature include new elementary particles that the table suggests? (Examples of such possible new elementary particles include an inflaton, a jay boson, a graviton, bosons that link to higher (than $S = 2$) spin LRI, higher (than $S = 1$) spin relatives of gluons, and higher (than $S = 0.5$) spin relatives of the known fermion elementary particles.)
- To what extent does nature include new elementary particles to which Table 3 (or the notion of Eq. (13)) does not point?
- To what extent do notions such as Eqs. (15) and (16) constitute principles or point to principles that underlie the notion of elementary particles?
- To what extent (regarding modeling or principles) might the notion of fractionally charged elementary particles (as in quarks and 1t0 uses of the solution-pairs $|-1+2-3-6+8|$ and $|-1+2-3-4+6|$) and the notion of 2t0 use of a pair of solution-pairs that link to the same K (as in $|\pm(-1-3+4)-2-6+8|$ and $K = \{1, 2, 3, 4, 6, 8\}$ for quarks or gluons) link to each other?
- To what extent do data, modeling, or principles suggest that elementary particles might (from the standpoint of popular modeling or from the standpoint of new modeling) include component objects or exhibit structure? (For example, section 6.1.6 of Ref. [1], suggests that known nonzero anomalous magnetic moments - for the electron and the muon - might link to notions of oval or oblate spatial distributions of charge or mass.)
- To what extent might Eqs. (22) and (23) (and the possibilities of analogous equations regarding properties of fermion elementary particles) point to new physics?

4.2. Relationships between popular modeling and new modeling

This unit suggests perspective about relationships between popular modeling and new modeling.

New modeling stems from successful popular modeling and from data that popular modeling does not explain. New modeling does not necessarily disturb successful popular modeling.

Key aspects of new modeling include models (based on integer-arithmetic equations) related to properties of objects and modeling that has basis in popular modeling (for which integer-arithmetic equations pertain) related to excitations of fields and particles.

New modeling does not necessarily (within itself) feature popular modeling notions such as the principle of stationary action, trajectories of objects, and uses of coordinate systems that feature continuous coordinates.

The following questions seem reasonable.

1. To what extent might new modeling point to opportunities to improve (or to provide useful alternatives to or to point to limitations regarding) popular modeling aspects that link to uses of the principle of stationary action, notions of trajectory mathematics that has bases in coordinate systems, and so forth?
2. To what extent might the combination of (present and future) data, (present and future) popular modeling, and (present and future) new modeling provide opportunities to improve notions that Table 7 discusses?
3. To what extent might it be useful for people to consider the notion that popular modeling might stem from new modeling?
4. To what extent might the combination of (present and future data), (present and future) popular modeling, and (present and future) new modeling provide opportunities to apply - to areas of physics that this paper does not feature - notions that Table 7 discusses?

Regarding the first question, the following notions might pertain.

- Discussion above suggests that inferences - by an object O - regarding an object I link to a variety of notions that object O defines. For example, regarding trajectories, regarding the relevant time (to which an inference pertains), the position (of object I), the velocity (of object I), and so forth, object O can make its own choices regarding 1x>-related coordinates and 2x>-related coordinates. For example, regarding transitions (such as decays), regarding the relevant time (to which an inference pertains) and the rate of progression of time, object O can make its own choices regarding a 1t0-related coordinate value (as in time) and a 2t0-related coordinate (as in a reciprocal of rate of decay). Such choices by object O should be consistent with other inferences that object O makes about, for example, data. Possibly, the only major invariant (as in unchanging across modeling that object O uses) assumption that object O needs to make involves some notion of a proper time τ that pertains to object O. (Relative to object O, object O does not move. Also, object O might link aspects of τ with, for example, the reciprocal of a frequency related to an atomic clock.) Thus, regarding making inferences regarding object I, object O might use (instead of a widely - or "universally" - applicable coordinate system) a notion of a coordinate "patch" that pertains to little more than the vicinity of object I, an appropriate swath the connects object I and object O, and (perhaps) some innards of object O.

- Popular modeling that features a (somewhat) universal coordinate system and two (or more than two) objects might have limitations that parallel limitations that link to popular modeling regarding three-body (or more than three body) systems. In effect, the coordinate system links to a new object O and the two (or more than two) original objects link to two (or more than two) inferred objects (as in object I objects).

Regarding the second question, the following questions might pertain.

- To what extent might one improve Table 7? Improvements might include clarifications, additions, deletions, extensions, and unifying principles. For example, improvements might include the better linking of properties with designations of A or A' (that is, properties for which conservation laws pertain) to symmetries (such as translational symmetries) that popular modeling discusses. Or, people might find value in the notion that uses of solution-pairs for which 1x> and $n_k = 1$ pertain might cascade (in the sense of non-same-Σ cascades) from a "solution-pair" for which $K = \emptyset$ and $\Sigma = 0$. (Except for 1x> $2 = |+2|$, for each such 1x> use of a solution-pair for which $n_k = 1$, the rightmost column in Table 7 notes three numbers that pertain regarding elementary particles.)

- To what extent might a combination of popular modeling and new modeling provide a means to gain insight regarding - or to change - the notion of an object? One might consider a spectrum that spans (at one end) the notion of an object that is essentially not entangled with the rest of the universe and (at the other end) the notion of an object that is sufficiently subsumed in a multi-component system that people cannot infer anything about the object. Perhaps, neither end of the spectrum links to entirely satisfactory modeling, especially compared to spectrum ranges that lie between the two ends. Can modeling link models to ranges in that spectrum? Can modeling clarify (and perhaps quantify) extents of usefulness (as in strengths and weaknesses) for models that link to various points on the spectrum?

Regarding the third question, the following opportunities might pertain.

- Posit principles regarding aspects that Table 7 discusses.
- Based on principles that underlie popular modeling and principles that underlie aspects that Table 7 discusses, recreate (starting from aspects that Table 7 discusses), refine, and possibly extend (at least) successful popular modeling.
- Explore the extent to which popular modeling seemingly needs to link conservation laws to popular modeling notions of symmetries.

Regarding the fourth question, the following opportunities might pertain.

- Produce analogs - to Table 7 - for other aspects - such as lattice dynamics, systems of quantum dots, properties of crystals, properties of amorphous solids, properties of superconductors, and so forth - of known or possible physics.
- Develop principles that underlie producing and using Table 7 and useful analogs to Table 7.
- Develop new perspective regarding links between aspects (such as data, modeling techniques, coordinate patches or systems, notions of objects and fields, notions of consistency and/or accuracy, and so forth) that associate with making inferences and with inferences.

5. Conclusion

This paper suggests so-called new modeling (as in new modeling compared to popular modeling) for some areas of physics.

New modeling branches from successful popular modeling. New modeling also suggests possibly useful reuses (as in isomers of non-long-range-interaction elementary particles) of some popular modeling.

This paper suggests that one can also consider that new modeling might stand on its own and provide a basis from which popular modeling branches.

This paper suggests that co-use of and co-evolution of popular modeling and new modeling might beneficially impact modeling, cataloging, and explaining aspects that associate with (at least) the following topics: properties of objects and fields, elementary particles and their properties, dark-matter phenomena, gravitational and dark-energy phenomena, and general physics.

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