Condition for convergence of debt to GDP ratio under full employment with consumption from assets and impossibility of fiscal collapse

Yasuhito Tanaka
Faculty of Economics, Doshisha University, Kyoto, Japan
E-mail: ochibocho@gmail.com

Abstract

We give a simple proof of the Domar condition for the debt-GDP ratio not to diverge, and present conditions for the debt-GDP ratio not to diverge when full employment and consumption from assets are considered. It is a weaker condition than the Domar condition based on a simple comparison of government bond interest rates and growth rates. This condition is probably true in many cases. Also we show that fiscal collapse is impossible when there exists consumption from assets.

Key Words: condition for convergence of debt to GDP ratio, consumption from assets, impossibility of fiscal collapse

https://doi.org/10.32388/5DHSCH.2

1 Introduction

Whether or not a country's public finances are on the verge of bankruptcy is often based on whether the ratio of government debt to GDP grows or diverges over time. There is a condition called the Domar condition to determine this, which is often discussed, especially in Japan (Domar (1944), Yoshino and Miyamoto (2021)). In brief, the Domar condition states that the government debt to GDP ratio does not diverge if the nominal growth rate exceeds the interest rate on government bonds. However, this condition is not necessarily appropriate because it simply compares the accumulation of budget deficits with the increase in GDP, regardless of the economic situation. As Blanchard (2022), (2023) notes, many discussions of debt to GDP ratio use simple calculations based on comparisons of primary budget balances (budget deficit excluding interest payments to government bonds), the interest rate, and the growth rate. But is the argument not so simple? Assuming a steady state of full employment, which may or may not include inflation, the size of the budget deficit to achieve this is naturally determined, and the larger the budget deficit is, the higher the inflation rate is. On the other hand, the larger (smaller) the propensity to consume is, the smaller (larger) the budget deficit required to achieve full employment under a constant rate of price increase is.

Fiscal deficits and the accumulation of government debt correspondingly create private financial assets. As these assets increase, consumption is naturally expected to be affected. This is the so-called asset effect on consumption. In addition, fiscal policy is likely to be managed so as to achieve full employment.

Therefore, this paper seeks conditions under which the government debt to GDP ratio converges to a finite value without divergence, assuming that people's consumption depends on financial assets corresponding to government debt as well as income, and that full employment is realized and maintained.

In the next section I present a simple proof the Domar condition. In Section 3 I present the condition for convergence of the debt to GDP ratio under full employment with consumption from assets. In Section 4 I will prove impossibility of fiscal collapse. Fiscal collapse means divergence of the debt to GDP ratio.

In Appendix I present microeconomic foundation for the consumption function.

2 A simple proof of the Domar condition

G_t: fiscal expenditure in period t.
T_t: tax in period t.
B_t: government debt in period t, $B_t \geqq 0$.
γ: growth rate: it is assumed to be constant.
Y_t: GDP in period t.
Y_{t+1}: GDP in period $t+1$: $Y_{t+1} = (1+\gamma)Y_t$.
$t \geqq 0$.
Al variables are nominal values.
i: interest rate of the government bonds: it is assumed to be constant.

First,

$$B_{t+1} = (1+i)B_t + G_t - T_t.$$

Assuming that $\frac{G_t - T_t}{Y_t} = \rho$ is constant, divide this equation by GDP in period $t+1$,

$$\frac{B_{t+1}}{Y_{t+1}} = \left(\frac{1+i}{1+\gamma}\right)\frac{B_t}{Y_t} + \frac{\rho}{1+\gamma}.$$

Let $\frac{B_t}{Y_t} = b_t$. Then,

$$b_{t+1} = \left(\frac{1+i}{1+\gamma}\right) b_t + \frac{\rho}{1+\gamma}. \tag{1}$$

Assuming $i \neq \gamma$, we have

$$b_{t+1} - \frac{\rho}{\gamma - i} = \left(\frac{1+i}{1+\gamma}\right)\left(b_t - \frac{\rho}{\gamma - i}\right).$$

Therefore,

$$b_{t+1} - \frac{\rho}{\gamma - i} = \left(\frac{1+i}{1+\gamma}\right)^{t+1}\left(b_0 - \frac{\rho}{\gamma - i}\right).$$

There are several cases and sub-cases.

1. $\gamma > i$.
   1) $\rho > 0$ (primary balance deficit)

      We get

      $$\lim_{t\to\infty} b_t = \frac{\rho}{\gamma - i}.$$

      Therefore, the debt to GDP ratio converges to the finite value $\frac{\rho}{\gamma - i}$.

   2) $\rho = 0$ (primary balance equilibrium)

      We have $b_t \to 0$.

   3) $\rho < 0$ (primary balance surplus)

      Again we have $b_t \to 0$ since we assume $b_t \geqq 0$.

2. $i > \gamma$

1) $\rho > 0$ (primary balance deficit)

Since $b_0 \geqq 0$, if $\rho > 0$ then $b_0 - \frac{\rho}{\gamma - i} > 0$. Since $\frac{1+i}{1+\gamma} > 1$, b_t diverges to infinity.

2) If $\rho = 0$ (primary balance equilibrium)

If $b_0 > 0$, b_t diverges to infinity similarly to 1. If $b_0 = 0$, $b_t = 0$ for all t.

3) $\rho < 0$ (primary balance surplus)

Then, $\frac{\rho}{\gamma - i} > 0$.

(i). If $b_0 - \frac{\rho}{\gamma - i} > 0$, b_t diverges to infinity similarly to 1.

(ii). If $b_0 - \frac{\rho}{\gamma - i} = 0$, $b_t = \frac{\rho}{\gamma - i}$ is constant.

(iii). If $b_0 - \frac{\rho}{\gamma - i} < 0$, $b_t \to 0$ since we assume $b_t \geqq 0$.

3. $\gamma = i$

From (1)

$$b_{t+1} = b_t + \frac{\rho}{1+\gamma}.$$

Thus, b_t is an arithmetic progression, and if $\rho > 0$ it diverges to infinity. If $\rho = 0$, $b_t = b_0$ is constant. If $\rho < 0$, $b_t \to 0$.

3 The condition for convergence of debt to GDP ratio under full employment with consumption from assets

G_t: fiscal expenditure in period t.
I_t: investment in period t.
C_t: consumption in period t.
B_t: government debt in period t, $B_t \geqq 0$.
γ: growth rate: it is assumed to be constant.
Y_t: GDP in period t.
Y_{t+1}: GDP in period $t+1$: $Y_{t+1} = (1+\gamma)Y_t$.
$t \geqq 0$.
Al variables are nominal values.
τ: income tax rate: it is assumed to be constant.
i: interest rate of the government bonds: it is assumed to be constant.

Suppose full employment. The price of the goods is constant or rises at the constant rate.

Assume that the consumption function is

$$C_t = c[(1-\tau)Y_t - I_t] + dB_t, \tag{2}$$

taking into account that government debt creates private financial assets, which affect consumption. d is the propensity to consume from the assets. Then,

$$Y_t = c(1-\tau)Y_t + dB_t + (1-c)I_t + G_t, \tag{3}$$

and so

$$G_t - \tau Y_t = (1-c)(1-\tau)Y_t - dB_t - (1-c)I_t.$$

On the other hand, we have

$$B_{t+1} = (1+i)B_t + (1-\tau)Y_t - C_t - I_t = (1+i)B_t + (1-c)(1-\tau)Y_t - dB_t - (1-c)I_t, \tag{4}$$

and so

$$B_{t+1} = (1+i)B_t + G_t - \tau Y_t,$$

or

$$G_t - \tau Y_t + iB_t = B_{t+1} - B_t. \tag{5}$$

Now assume

$$(1-\tau)Y_t - I_t > 0. \tag{6}$$

Let $Y_{t+1} = (1+\gamma)Y_t$, and assuming that $\frac{I_t}{Y_t} = \eta$ is constant, divide (4) by Y_{t+1}, then

$$\frac{B_{t+1}}{Y_{t+1}} = \left(\frac{1+i-d}{1+\gamma}\right)\frac{B_t}{Y_t} + \frac{(1-c)(1-\tau-\eta)}{1+\gamma}.$$

Let $\gamma \neq i-d$. We have

$$\frac{B_{t+1}}{Y_{t+1}} - \frac{(1-c)(1-\tau-\eta)}{\gamma-i+d} = \left(\frac{1+i-d}{1+\gamma}\right)\left[\frac{B_t}{Y_t} - \frac{(1-c)(1-\tau-\eta)}{\gamma-i+d}\right],$$

and then

$$\frac{B_{t+1}}{Y_{t+1}} - \frac{(1-c)(1-\tau-\eta)}{\gamma-i+d} = \left(\frac{1+i-d}{1+\gamma}\right)^{t+1}\left[\frac{B_0}{Y_0} - \frac{(1-c)(1-\tau-\eta)}{\gamma-i+d}\right].$$

When $\frac{1+i-d}{1+\gamma} \lesseqgtr 1$, $\gamma \gtreqless i-d$.

There are three cases.

1. $\gamma > i-d$ We obtain

$$\lim_{t\to\infty}\frac{B_t}{Y_t} = \frac{(1-c)(1-\tau-\eta)}{\gamma-i+d}.$$

   Therefore, $\frac{B_t}{Y_t}$ converges to the finite value.

2. $\gamma < i-d$

   Since $-\frac{(1-c)(1-\tau-\eta)}{\gamma-i+d} > 0$, $\frac{B_t}{Y_t}$ diverges to infinity.

3. $\gamma = i-d$

   From

$$\frac{B_{t+1}}{Y_{t+1}} = \frac{B_t}{Y_t} + \frac{(1-c)(1-\tau-\eta)}{1+\gamma},$$

   $\frac{B_t}{Y_t}$ an arithmetic progression, which diverges to infinity.

If consumption depends not on the asset itself but on the interest income derived from the asset, the same argument holds by changing d to di or ci.

4 Impossibility of fiscal collapse

From (3)

$$Y_t = c(1-\tau)Y_t + dB_t + (1-c)I_t + G_t.$$

Divide this by Y_t, then

$$1 = c(1-\tau) + d\frac{B_t}{Y_t} + (1-c)\eta + \frac{G_t}{Y_t}. \tag{7}$$

$c(1-\tau)$, $(1-c)\eta$ and $\frac{G_t}{Y_t}$ are positive and smaller than 1. The sum of them is also positive and smaller than 1. On the other hand, if the debt to GDP ratio diverges to infinity and $d > 0$, $d\frac{B_t}{Y_t}$ also diverges to infinity. However, since the left-hand side of (7) is 1. It is a contradiction. Therefore, the debt to GDP ratio cannot diverge to infinity. In this case, more consumption from assets would lead to higher prices and a larger nominal growth rate so that the convergence condition would be satisfied.

5 Conclusion

In conclusion, it has been proved that the debt to GDP ratio does not diverge but converges to a finite value when the following conditions hold

$$\begin{aligned}\text{Nominal growth rate} > & \text{ interest rate on government bonds} \\ & - \text{ propensity to consume from the assets.}\end{aligned}$$

It is weaker than the Domar condition.

$$\text{Nominal growth rate} > \text{interest rate on government bonds.}$$

This condition is probably true in many cases.

Appendix: Microeconomic foundation for the consumption function

C_t: consumption in period t
B_{t+1}: asset holding in period $t+1$
P_t: price level in period t

The utility function of a representative consumer is

$$U = c\ln\frac{C_t}{P_t} + (1-c)\ln\left(\frac{B_{t+1} - B_t}{P_t}\right).$$

I assume that the utility of a consumer depends on his/her consumption and an increase in asset holding from period t to $t+1$. The budget constraint is

$$C_t + B_{t+1} = (1-\tau)Y_t - I_t + (1+i)B_t,$$

or

$$C_t + B_{t+1} - B_t = (1-\tau)Y_t - I_t + iB_t.$$

The Lagrange function is

$$\mathcal{L} = c\ln\frac{C_t}{P_t} + (1-c)\ln\left(\frac{B_{t+1}-B_t}{P_t}\right) - \lambda[C_t + B_{t+1} - Y_t + I_t - (1+i)B_t].$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial \frac{C_t}{P_t}} = c\frac{P_t}{C_t} - \lambda P_t = 0,$$

and

$$\frac{\partial \mathcal{L}}{\partial \left(\frac{B_{t+1}-B_t}{P_t}\right)} = (1-c)\frac{P_t}{B_{t+1}-B_t} - \lambda P_t = 0.$$

From them

$$C_t = c[(1-\tau)Y_t - I_t + iB_t],$$

and

$$B_{t+1} = (1-c)[(1-\tau)Y_t - I_t + iB_t] + B_t.$$

Then, d in (2) is ci.

If $C_t > 0$, $B_{t+1} > B_t$. From (5)

$$G_t - \tau Y_t + iB_t = B_{t+1} - B_t > 0.$$

References

O. Blanchard. Deciding when debt becomes unsafe,International Monetary Fund. `https://www.imf.org/en/Publications/fandd/issues/2022/03/Deciding-when-debt-becomes-unsafe-Blanchard#:~:text=When%20does%20the%20level%20of,to%20default%20at%20some%20point`, 2022.

O. Blanchard. *Fiscal Policy under Low Interest Rates*, The MIT Press, 2023.

E. Domar. The Burden of Debt and the National Income, *American Economic Review*, vol. 34, No. 4, pp. 798-827, 1944.

N. Yoshino and H. Miyamoto. Reconsideration of the “Domar condition” to check sustainability of budget deficit, *Public Policy Review*, Policy Research Institute, Ministry of Finance, Japan, vol.17, No.3, pp. 1-12, 2021.