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On Superposition and Entanglement of Polarized Photons

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Abstract

It is shown that the superposition and entanglement of polarized photons can be explained by a local realistic model without introducing a hidden parameter. The basis of the modeling is that only relations between states of photon beams are predefined and not the states themselves. The actual state before a measurement has to be selected by a polarizer.

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Introduction

This paper is a further step towards clarifying the question of whether quantum correlations in entangled photons can be explained locally or not. Quantum mechanics does not provide any information on this.

Various approaches, including Bell's, assume that for a realistic explanation a hidden parameter must be introduced, which is given to both partner particles at the source of an entangled photon pair [1]. This was also done in[2]. This allowed a model to be developed that correctly explains the quantum correlations. This is sufficient to disprove Bell's theorem, which states that a realistic local model for the prediction of quantum correlations is not possible.

However, a closer look reveals that phenomena such as entanglement swapping and teleportation cannot be explained with a common parameter. This was taken into account in [3]. A model was presented in which the hidden parameter does not necessarily have to have the same value for both sides of an entangled pair, i.e., both sides are independent of each other. The model also correctly explains the quantum correlations. In [3] it is also explained that the derivation of Bell's inequality [1], [4] does not take into account that the entangled states are not separable. Therefore, Bell's inequality cannot reproduce the predictions of quantum correlations.

In both models, [2] and [3], it is assumed that the entangled state on each side consists of a mixture of indistinguishable beams of horizontally and vertically polarized photons. If a partial beam is now selected with a polarizer, its polarization results from the mixing ratio of its horizontally and vertically polarized components. The reason for this is the indistinguishability of the two beams. This resulting polarization corresponds to the position of the applied polarizer even before the measurement. That is the actual reason for the correlations of entangled photons and this assumption was also used in this paper. Due to the coupling of the initial states on both sides, their polarizations are either equal or orthogonal depending on the Bell state. This also applies to the resulting polarizations from the selection.

What is the motivation for this paper? With the introduction of a common polarization of indistinguishable photons in [2] and [3], the question arose as to whether the inverse was also true. Namely, that a certain polarization of a photon beam can be understood as a mixture of indistinguishable photon beams. Then, a further mechanism with hidden parameters for the description of Malu's law is no longer necessary. This was realized in the present paper. The basis of the modeling is that only relations between states of photon beams are predefined and not the states themselves. The actual state before a measurement has to be selected by a polarizer. This also reflects the fact that entangled states are not separable.

Model description

For a description of the measurement setup with entangled photons, see[3].

Model assumption MA1 *(describing superposition)*

> *A photon beam with polarization α is a mixture of two indistinguishable photon beams, one with polarization β and the share $\cos^2(\alpha-\beta)$, and the other with polarization β+π/2 and the share $\sin^2(\alpha-\beta)$ for any α and β. A polarizer set to β/ β+π/2 selects those photon beams from the original beam.*

All pairs of beams are equivalent. MA1 reproduces Malu's law.

Complementary to MA1 we define

Model assumption MA2: *(describing the amount of the common polarization)*

> *Indistinguishable photon beams with fractions $\cos^2(\alpha-\beta)$ of polarization β and $\sin^2(\alpha-\beta)$ of polarization β+π/2 assume the common polarization α or -α.*

The sign of the common polarization is given for Bell states by model assumption MA3.

Model assumption MA3: *(controlling the sign of the common polarization)*

> *Each Bell state is a mixture of indistinguishable constituent photon pairs in equal shares whose components have the same polarization 0° or 90° for Φ+ and Φ- and an offset of π/2 for Ψ+ and Ψ-. The constituent photon pairs make up the initial state. From the conservation of the spin angular momentum, we obtain for Ψ- and Φ+ the same sign of the polarization of the beams, and for Ψ+ and Φ- the opposite sign in the original coordinate system.*

This is shown below. See also[3].

The four Bell states can be written in terms of linear polarizations and in terms of circular polarizations as well:

$$\Phi^+ = 1/\sqrt{2} * (|H_A> |H_B> + |V_A> |V_B>) = 1/\sqrt{2} * (|R_A> |L_B> + |L_A> |R_B>),$$

$$\Psi^- = 1/\sqrt{2} * (|H_A> |V_B> - |V_A> |H_B>) = i/\sqrt{2} * (|R_A> |L_B> - |L_A> |R_B>),$$

$$\Phi^- = 1/\sqrt{2} * (|H_A> |H_B> - |V_A> |V_B>) = 1/\sqrt{2} * (|R_A> |R_B> + |L_A> |L_B>),$$

$$\Psi^+ = 1/\sqrt{2} * (|H_A> |V_B> + |V_A> |H_B>) = -i/\sqrt{2} * (|R_A> |R_B> - |L_A> |L_B>).$$

$$|R> = 1/\sqrt{2} * (|H> + i|V>) \text{ and } |L> = 1/\sqrt{2} * (|H> - i|V>)$$

($|R>$, $|L>$ denote right and left polarization)

$$|H> = 1/\sqrt{2} * (|R> + |L>) \text{ and } |V> = -i/\sqrt{2} * (|R> - |L>)$$

The left sides of the first two equations above concerning the linear polarization are rotational invariant as they apply to any orientation of the coordinate system. The right sides show the conservation of the spin angular momentum as right and left-oriented circular polarizations cancel. Thus, rotational invariance and conservation of the spin angular momentum are equivalent and denote the same physical situation.

The above written is also true for equations (3) and (4) if the coordinate system is changed so that the photons move in opposite directions ($|H_B>$ -> $-|H_B>$).

Rotational invariance means that the signs of the polarizations of the photon beams at both sides are the same. This leads to the relations for the common polarizations on each side for different Bell states.

Table 1. polarization of partner photons 2 at wing B for different Bell states for a selection of photons 1 with a polarizer set to α at wing A.

Bell state	A	B
ψ^-	α	α + π/2
Φ^+	α	α
ψ^+	α	-α -π/2
Φ^-	α	-α

From these results, entanglement swapping and teleportation can be derived, see[3].

The model also explains the Mach-Zehnder interferometer (MZI) with polarizing beam splitters (PBS) without interference, see Figure 1. In a Mach Zehnder interferometer, a photon beam of polarization α is split into a beam of horizontally polarized photons with a fraction of $\cos^2(\alpha)$ and a beam of vertically polarized photons with a fraction of $\sin^2(\alpha)$ [5].

Model assumption MA4: *(controlling the sign of the output of a MZI without interference)*

> *The vertically polarized photon carries the sign of α. When generating the common polarization α, the sign of α is retained. This is achieved by maintaining the sign of the phase difference between left and right polarized components of a linearly polarized photon beam.*

The relation between the state of linearly polarized photons and the state of circular polarized photons is:

$$cos(\alpha) * |H> + sin(\alpha) * |V> = (exp(-i * \alpha) * |R> + exp(i * \alpha) * |L>)/\sqrt{2} \text{ where}$$

$$|H> = 1/\sqrt{2} * (|R> + |L>) \text{ and}$$

$$+|V> = -i/\sqrt{2} * (|R> - |L>) = (exp(-i * \;/2) * |R> + exp(i * \;/2) * |L>)/\sqrt{2}$$

$$-|V> = -i/\sqrt{2} * (|L> - |R>) = (exp(-i * \;/2) * |L> + exp(i * \;/2) * |R>)/\sqrt{2}$$

From equation (7) we obtain the phase difference between the left and right polarized components to be 2α. Thus, α and the phase difference have the same sign. This sign is retained by the vertically polarized photon eqs. (9) and (10). For α > 0 eq. (9) applies, for α < 0 eq. (10) applies.

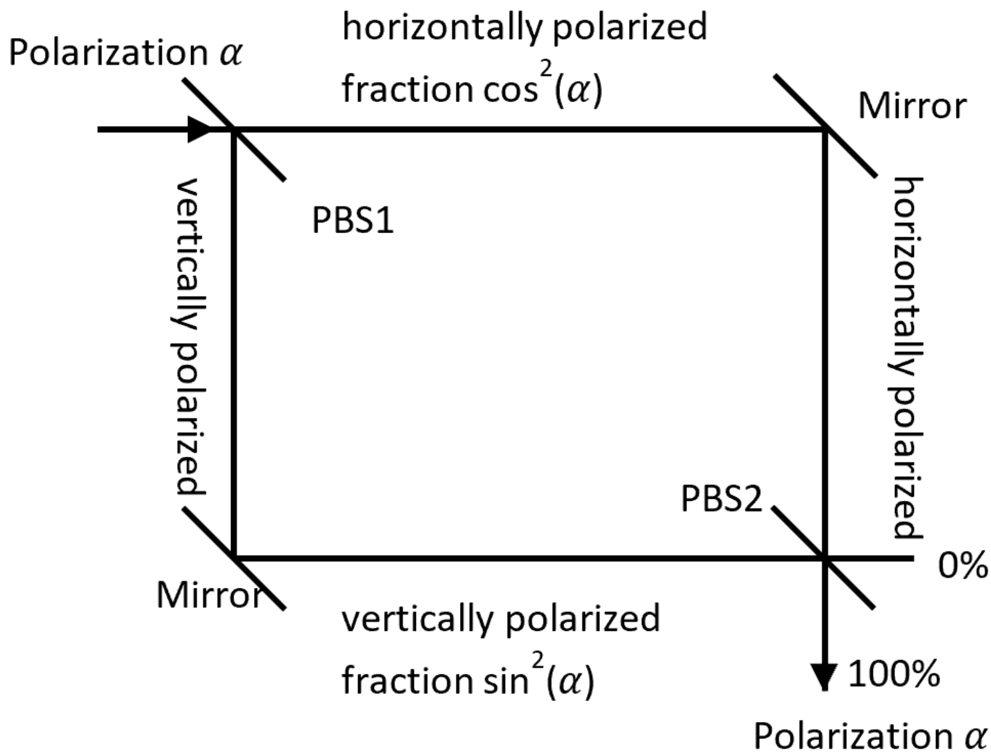

Figure 1. Beam paths at a Mach-Zehnder Interferometer with polarizing beam splitters without interference

The input beam on the first PBS (PBS1) has the polarization direction α. According to MA1, this beam of photons with polarization α is a mixture of indistinguishable beams of photons of horizontal polarization with a fraction $\cos^2(\alpha)$ and vertical polarization with a fraction $\sin^2(\alpha)$. Due to MA4, the sign of α is retained by the vertically polarized photons. Photons with horizontal polarization are transmitted at PBS1 while photons with vertical polarization are reflected by PBS1. The mirrors don't change the polarization. Input of the second PBS (PBS2) are horizontally polarized photons with the fraction $\cos^2(\alpha)$ and vertically polarized photons with the fraction $\sin^2(\alpha)$. The horizontally polarized photons are transmitted, while the vertically polarized photons are reflected by PBS2. Therefore, these photons go both to the same output of PBS2 and are indistinguishable. Thus, they have a uniform polarization direction α according to MA2 and MA4.

Calculating probabilities of matching events with entangled photons

We set PA to α. From MA1, we obtain that the contribution of horizontally polarized photons to a photon beam with polarization α is $\cos^2(\alpha-0)$. The contribution of vertically polarized photons to a photon beam with polarization α is $\cos^2(\alpha-\pi/2) = \sin^2(\alpha)$. From MA2, we obtain that the common polarization is α. All photons with the common polarization hit the polarizer set to α.

For the singlet state Ψ-, we obtain the corresponding beam of the partner photons on side B from the initial conditions. The fraction of horizontally polarized photons on side B matches the fraction of vertically polarized photons on side A, which is $\sin^2(\alpha)$; the fraction of vertically polarized photons on side B is $\cos^2(\alpha)$, matching the fraction of horizontally polarized photons on side A. From MA2 and MA3, we obtain that the common polarization of the partner photons on side B is α+ π/2. See also Table 1.

Now we set PB to β. From MA1 we obtain that the fraction of photons with polarization β contributing to the photon flux with polarization α+π/2 is $\cos^2(\alpha+\pi/2-\beta) = \sin^2(\alpha-\beta)$. This is the probability for matching events at polarizer PA and polarizer PB. The expectation value for a joint measurement with photon 1 detected behind detector PA at α and partner photon 2 detected behind detector PB at β is as obtained from ([2], equation (13))

$$E(\alpha, \beta) = -cos(2(\alpha - \beta))$$

This matches the predictions of quantum mechanics.

Conclusion

In the current paper, a model was presented that manages entirely without the assumption of hidden parameters and still correctly describes the quantum correlations with entangled photons. This can also explain how the polarization of the input state reappears at the output in a Mach-Zehnder interferometer. Malus' law also follows from this in a straightforward manner. The question arises as to why such a model can make exact predictions without defined parameters.

The answer is that the model does not define states but only relations between states. Measured states are then determined by the position of the measuring polarizer. The model does not require any additional hidden parameters. The selection on one side precisely determines the selection on the other side. The non-separability of entangled photons is taken into account in the model by the fact that the polarization of the photons is not independent but depends on the position of a polarizer.

The question of whether quantum mechanics is complete cannot be answered in the negative from the perspective of the model. However, this does not mean that the arguments from the EPR paper are invalid [6]. The EPR paper was not based on entangled photons, for which a statistical interpretation of the states is possible.

There are opinions that say that the wave function is ontic and not epistemic[7]. This means that the wave function or quantum state represents a physical fact and not just a probability amplitude. If there is no interference, the polarization derived from the presented model and the quantum state denote the same physical state of affairs, one in R3 local space and the other in Hilbert space. As the model does not cover any interference it cannot replace the quantum mechanical formalism.

Mixtures of indistinguishable states in local space are equivalent to superposition in Hilbert space. This is because the proportions of different components are the same in both representations. In this way, Born's rule is then also modelled without constraint.

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