Quantized Newton and General Relativity Theory

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Abstract

Recent advances in understanding the Planck scale have led to a new, straightforward method for quantizing the general theory of relativity. This results in a theory of gravity that predicts the same outcomes as general relativity with greater fundamental comprehension connected directly to the Planck scale.

Key Words: general relativity theory, quantization, Planck scale, Planck length, quantum mechanics, quantum gravity.

1 Backround

Einstein's theory of general relativity [1, 2] is a fundamental cornerstone of modern physics. It has been successfully tested against a wide range of astronomical and macroscopic observations. However, its testing in relation to the atomic scale has been limited. On the other hand, quantum mechanics has been extensively tested at the atomic and subatomic scales, but it currently cannot explain or be successfully unified with gravity.

As early as 1916, Einstein proposed that the next advancement in gravity theory would involve a quantum gravity theory, which could ultimately lead to a unified theory of gravity. Despite dedicating much of his life to this pursuit, Einstein made limited progress. Other attempts, such as superstring theory and quantum loop gravity theory, have also had limited success. Despite significant progress in both experimental and theoretical gravity research, it is generally agreed that a breakthrough in gravity theory is still lacking [3–5]. As Kiefer has described in his article "*Quantum Gravity – An Unfinished Revolution*," a lot has been achieved in the pursuit of a quantum theory of gravity, but a major breakthrough is still missing. We contend that the lack of a major breakthrough is not due to a lack of brilliant researchers or mathematical abilities, but rather because we may have taken a wrong path at some point or overlooked a simple solution that has been in front of us all along. We argue that we are actually much closer to a revolutionary theory of quantum gravity than previously believed. However, before explaining why, we need to delve deeper into the history of gravity. Since the so-called Newtonian gravitational constant plays a central role in both Newtonian mechanics and general relativity, we need to take a closer look at what it actually represents. To do this, we should go back to Newton's time and examine how and when the gravitational constant, as well as the concept of gravitational mass, were introduced.

Newton [6] in his *Principia* only introduced the gravitational force formula in words that corresponded to

$$F = \frac{M_n m_n}{R^2} \tag{1}$$

We use the notation M_n and m_n to distinguish it from the kilogram mass M and m used in the modern modified Newton gravity force formula. This formula and its mass have been used successfully for more than 200 years with no knowledge of any gravitational constant. For example, in Maxwell's [7] book "*A Treatise on Electricity and Magnetism*," published early in 1873, he mentions astronomical mass, which has units of $[L^3 \cdot T^{-2}]$, and how one can find this mass. Maxwell describes that he can find this mass from Newton's gravitational acceleration formula $g = \frac{M_n}{R^2}$ or any other gravitational-related formula that includes the mass. We can easily measure gravitational acceleration on Earth without any knowledge of G or the kilogram mass M. This can be done, for example, by simply dropping a ball from a height of H above the ground. We need to measure the time it takes for the ball to hit the ground. There even exist balls with built-in stopwatches and sensors for exactly this purpose, known as a "g-ball." or more accurate apparatuses. The gravitational acceleration is then given by $g = \frac{2H}{T_d^2}$, where T_d is the time it took for the ball to hit the ground. So we simply have $M_n = \frac{2HR^2}{T_d^2}$.

Maxwell mentioned that there were multiple mass standards during his time. The so-called astronomical mass, based on Newton's original theory of gravity, was used for astronomical purposes. In France, for non-astronomical objects, Maxwell mentioned the gram and the kilogram. In the United Kingdom, the standard unit of mass was the avoirdupois pound, which was preserved in the Exchequer Chambers. Additionally, Maxwell mentioned the grain, which he defined as the 7000th part of this pound.

The original formula for the Newtonian gravitational force, therefore, had output units of $[L^4 \cdot T^{-4}]$, which is in strong contrast to the modern version of the Newton formula, which has output units of $[M \cdot L \cdot T^{-2}]$ or $kg \cdot m \cdot s^{-2}$. This may lead one to mistakenly think that the two formulas give very different output predictions and that the original Newton formula must be corrected. However, the small mass in the original Newton formula, as well as in the modern version of the formula, always cancels out during derivation when formulating equations that can actually be used to predict observable phenomena. Furthermore, it should be noted that $M_n = GM$, which means that both the original and modern formulas for the Newtonian gravitational force always give the same predictions, as analyzed in detail in [8] and that also can be seen from Table 1 that we soon will get to.

When did the Newtonian gravitational constant come into play? In an effort to standardize mass notation across countries and to use the same mass definition for small and astronomical masses, the kilogram mass was chosen during the 1875 Meter Convention meeting. This decision had been planned and discussed for several years. Therefore, we believe there is no coincidence that the gravitational constant was suggested in 1873 by Cornu and Baille [9], who proposed modifying the original Newtonian formula from $F = M_n m_n / R^2$ to $F = f M m / R^2$, where f represented the gravitational constant.

Boys [10] is likely the first to have introduced the notation G for the gravitational constant in 1894, but it took many years before G became the standard notation. Max Planck [11] used the notation f as late as 1928, while Einstein used the notation k in 1916 in his general relativity theory. Ultimately, whether one uses the notation f, G, or k for the gravitational constant is merely cosmetic. What's important to note is that the Newtonian gravitational force formula existed and was used for over 200 years without the gravitational constant.

What about Cavendish? Some books and papers incorrectly claim that Cavendish [12] was the first to measure the gravitational constant in 1798. However, as pointed out by Clotfelter [13] and Sean [14], Cavendish did not use or measure any gravitational constant. What is correct is that a Cavendish apparatus can also be used to find the gravitational constant. What Cavendish did was to measure the density of the Earth relative to a uniform clump of matter whose density was already known, and thereby, he was able to measure the density of the Earth. Newton had not been able to achieve that, even though he discussed possibilities for it in the Principia. A more accurate measuring device was needed for this, and the Cavendish apparatus was accurate enough to do this. It could also accurately measure gravitational effects from macroscopic masses of a size we can easily handle, such as the large balls in the Cavendish apparatus.

As Thüring [15] pointed out, the gravitational constant was basically ad hoc inserted into the gravitational formula. This was done to make the formula consistent with the newly chosen mass standard, the kilogram. Thüring also noted that the gravitational constant cannot be directly linked to anything physical; it is a constant that must be calibrated to a gravity observation to make the kilogram mass work to predict gravity phenomena. We will argue that the gravitational constant is not currently fully understood by modern physics and, as we will soon show, a deeper understanding of what it truly represents is one of two important keys to developing a quantum theory of gravity.

For example, Hossenfelder [16] claims that "*Newton's constant (G) quantifies the strength of gravity*". However, we argue that such claims cannot be entirely correct, or at least imprecise. If G truly quantifies the strength of gravity, then why can the original Newtonian theory predict all gravitational phenomena just as well as the modern Newtonian gravity force formula $F = GMm/R^2$, despite the original formula not having a gravitational constant?

The original formula for gravitational acceleration used by Maxwell in 1873, just before the invention of the gravitational constant, is:

$$g = \frac{M_n}{R^2} \tag{2}$$

Since the original Newtonian mass has units of $[L^3 \cdot T^{-2}]$, the output here is $[L \cdot T^{-2}]$. If we use meters and seconds, it gives exactly the same result as the modern GM/R^2. This means that we must have $M_n = GM$. However, this also means $m_n = Gm$. Therefore, the original Newton formula can also be written as:

$$F = \frac{M_n m_n}{R^2} = \frac{GMGm}{R^2} = G^2 \frac{Mm}{R^2} \tag{3}$$

Based on Hossenfelder's reasoning, the strength of gravity must be different in the original Newton formula and the modern Newton formula, but this idea doesn't seem to make sense. The two gravity force formulas indeed give both different output units and numbers, but the gravity force itself has never been measured. The gravity force formulas are only used for the derivation of new formulas to predict the effects of gravity that can actually be observed and checked. If we, for example, want to derive the escape velocity based on the original Newton formula, we can use:

$$\frac{1}{2}m_n v^2 - \frac{M_n m_n}{R} = 0 \tag{4}$$

Solving this equation for v gives $v = \sqrt{2M_n/R}$. Since $M_n = GM$, we have $v = \sqrt{2GM/R}$. Note that we could have also written the derivation as:

$$\begin{aligned} \frac{1}{2}m_n v^2 - \frac{M_n m_n}{R} &= 0 \\ \frac{1}{2}Gmv^2 - G^2\frac{Mm}{R} &= 0 \end{aligned} \tag{5}$$

Solving this equation for v also gives $v = \sqrt{2GM/R}$, which is the same result as one obtains in derivations using the modern 1873 version of Newton. The fact that we can have no gravity constant but still use the Newton mass or even G^2 or G and the kilogram mass to obtain the same predictions should, at a minimum, raise the question of whether we truly understand the gravity constant. We claim that the reason for introducing the gravitational constant was to make the incomplete definition of the kilogram mass complete in relation to gravity, and this was done ad hoc without a full and deep understanding, but it worked. After G was calibrated to one observable gravity phenomena, one could use the same constant in relation to other kilogram masses (other astronomical objects) and for a series of different gravitational observations that do not need a re-calibration of G. In other words, G is indeed likely a constant, but what does it represent from a deeper perspective?

1.1 The Planck scale and the Compton scale

Max Planck [17, 18] assumed in 1899 and 1906 that there were three important universal constants: the speed of light c, the Planck constant[1] $\hbar$, and Newton's gravitational constant G. By combining these constants with dimensional analysis, he found a unique length: $l_p = \sqrt{\frac{G\hbar}{c^3}}$, time: $t_p = \sqrt{\frac{G\hbar}{c^5}}$, mass: $m_p = \sqrt{\frac{\hbar c}{G}}$, and temperature: $T_p = \frac{1}{k_b}\sqrt{\frac{\hbar c^5}{G}}$. To calculate temperature, one must also use the Boltzmann constant k_b, which is simply a constant used to convert from one unit scale to another, namely from joules to temperature.

Eddington [19] was likely the first to suggest in 1918 that a quantum gravity theory should be connected to the Planck length. This viewpoint is still widely held among most experts in the field of quantum gravity [20–22], but despite extensive efforts, no breakthrough has yet been achieved, and there is no consensus on any quantum gravity theory. In this paper, we propose a new and simple method for incorporating the Planck scale and achieving quantum gravity. It is so straightforward that it may be easy to dismiss it based on preconceptions, but we encourage the gravity community to examine it closely before rejecting it. Sometimes, the simplest solution is the best.

In 1984, Cahill [23, 24] suggested that the Planck units might be more fundamental than the gravitational constant and proposed that the gravitational constant could be expressed as a composite constant of the form $G = \frac{\hbar c}{m_p^2}$, which is simply the Planck mass formula solved with respect to G. Cohen [25] derived the same formula for G in 1987, but correctly pointed out that since no one had been able to find the Planck mass or some of the other Planck units independently of G, expressing G in terms of the Planck units would lead to a circular and unsolvable problem. This has been a view held at least to 2016; see [26]

In 2017, Haug [27] showed for the first time that the Planck length can be determined independently of any knowledge of G. He used a Cavendish apparatus and derived the following equation:

$$l_p = \sqrt{\frac{\hbar 2\pi^2 L R^2 \theta}{M T^2 c^3}}, \tag{6}$$

where $\hbar$ is the reduced Planck constant, L is the distance between the two small balls in the apparatus, R is the distance between the center of the large and small balls in the apparatus, θ is the measured angle, T is the

[1] Max Planck originally used the Planck constant h, rather than the reduced Planck constant $\hbar$, for this purpose. However, the standard now is to use the reduced Planck constant, which also appears to be more appropriate for our purposes. We plan to write a separate paper on this topic in the future, but it will not affect the conclusions of this paper.

oscillation period, c is the speed of light, and M is the mass of the large ball in kilograms. The mass of the large ball can be determined using a standard balance weight and does not require knowledge of G. Generally, we only need to know G when we need to determine the kilogram mass of astronomical objects.

Further investigations [28] have shown that the Planck length can also be determined in a Cavendish apparatus using the following equation:

$$l_p = \sqrt{\frac{\bar{\lambda} 2\pi^2 L R^2 \theta}{T^2 c^2}}, \tag{7}$$

In this case, we are no longer dependent on knowing either the kilogram mass of the large ball in the Cavendish apparatus or the Planck constant. However, we now do need to know the reduced Compton wavelength of the large ball in the apparatus. This we have demonstrated in a series of papers [28–30] can be done for any mass independent of knowing both the Planck constant or the kilogram mass of any object and also without knowing G.

The fact that we can find the Planck length independent of G means that we can express the gravitational constant in terms of Planck units. We can solve the Planck length formula for G, which gives us the equation:

$$G = \frac{l_p^2 c^3}{\hbar} \tag{8}$$

There are many ways to express the gravitational constant using Planck units. For example, we could have solved the Planck time formula for G, which would give us $G = \frac{t_p^2 c^5}{\hbar}$. Other suggestions can be found in the literature, as discussed in [31], which provides a review of the composite view of the gravitational constant.

Now that we have the composite gravity constant, we can input it into the modified 1873 Newton formula:

$$F = G\frac{Mm}{R^2} = \frac{l_p^2 c^3}{\hbar}\frac{Mm}{R^2} \tag{9}$$

However, this does not provide much new or deeper insight. While one could argue that Newton's force of gravity is now a function of both the Planck constant and the Planck length, as well as the speed of light, we need to look more closely at the mass.

As discussed in our review paper [31], there is more to the story than just the Composite constant of G and the Planck units. We need to consider the role of mass in relation to the gravitational constant and how it affects our understanding of gravity. So, while the composite view of the gravitational constant is an important step forward, it is just one of two steps to get to a quantum gravity theory.

Compton [32] formulated the Compton wavelength formula in 1923 as $\lambda = \frac{h}{mc}$. We can solve this formula for the mass m in kilograms, giving us:

$$m = \frac{h}{\lambda}\frac{1}{c} = \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \tag{10}$$

In fact, any kilogram mass can be expressed in this form. Even if a composite mass does not have a single physical reduced Compton wavelength, it has what we can call an aggregated reduced Compton wavelength see [29, 33], given by:

$$\bar{\lambda} = \frac{1}{\sum_i^n \frac{1}{\lambda_i}} = \frac{1}{\frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \frac{1}{\lambda_4} \cdots \frac{1}{\lambda_n}} \tag{11}$$

Even energy, when looked at as mass, can be described with a Compton wavelength, because:

$$\begin{aligned} \frac{E}{c^2} &= m \\ \frac{h\frac{c}{\lambda_\gamma}}{c^2} &= \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \\ \frac{\hbar\frac{c}{\bar{\lambda}_\gamma}}{c^2} &= \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \\ \bar{\lambda}_\gamma &= \bar{\lambda} \\ \lambda_\gamma &= \lambda \end{aligned} \tag{12}$$

Here, $\bar{\lambda}_\gamma$ is what we can call the reduced photon wavelength, which must be equal to the reduced Compton wavelength for the mass equivalent of the energy, for $\frac{E}{c^2} = m$ to be true. It simply means the photon wavelength can be seen as the Compton wavelength of the photon. Therefore, even such things as binding energy [34] can be treated within this framework of aggregating Compton wavelengths to get the Compton wavelength of the mass in question. So assume, for example, we have a mass consisting of three elementary particles and two types of binding energy; then, we have

$$\begin{aligned} m &= m_1 + m_2 + m_3 + \frac{E_1}{c^2} + \frac{E_2}{c^2} \\ \frac{\hbar}{\bar{\lambda}}\frac{1}{c} &= \frac{\hbar}{\bar{\lambda}_1}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_2}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_3}\frac{1}{c} + \frac{\hbar\frac{c}{\bar{\lambda}_4}}{c^2} + \frac{\hbar\frac{c}{\bar{\lambda}_5}}{c^2} \\ \bar{\lambda} &= \frac{1}{\frac{1}{\bar{\lambda}_1} + \frac{1}{\bar{\lambda}_2} + \frac{1}{\bar{\lambda}_3} + \frac{1}{\bar{\lambda}_4} + \frac{1}{\bar{\lambda}_5}} \end{aligned} \tag{13}$$

The important point here is simply that we indeed can express any kilogram mass in the form of $m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$. Next, let's get back to gravity.

If we examine the modern Newtonian gravity force formula, we can observe that any formula used to predict observable gravitational phenomena includes GM and not GMm. In two-body problems where m is not much smaller than M (i.e., m is also significantly large and important for gravitational predictions), the gravitational parameter is $\mu = G(M + m) = GM + Gm$. Even in such cases, we do not encounter GMm in any formula that predicts observable gravitational phenomena.

However, when $m \ll M$, the smaller mass m is necessary only during derivations, as it cancels out before we arrive at a formula that can be utilized to predict gravity phenomena that can be verified through observations. The gravitational force itself is not directly observable.

If we substitute G with $G = \frac{l_p^2 c^3}{\hbar}$ and M with $M = \frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}$, we obtain:

$$GM = \frac{l_p^2 c^3}{\hbar}\frac{\hbar}{\bar{\lambda}_M}\frac{1}{c} = c^3\frac{l_p}{c}\frac{l_p}{\bar{\lambda}_M} \tag{14}$$

Here, $\bar{\lambda}_M$ represents the reduced Compton wavelength of the larger mass M in the Newton formula, while l_p denotes the Planck length and c represents the speed of light. Several important points related to Eq. (14) deserve attention. The composite gravitational constant contains the embedded Planck constant, which cancels out with the Planck constant present in the kilogram mass. Of the components l_p, $\bar{\lambda}_M$, c, and $\hbar$, only the Planck constant has kilogram units embedded in it. Therefore, everything related to kilogram units cancels out when predicting gravitational phenomena.

This outcome should not surprise those familiar with pre-1873 Newtonian theory, which did not include anything about kilograms. In our view, multiplying G with the kilogram mass M replaces the Planck constant with the Planck length, converting an incomplete kilogram mass into a complete gravitational mass. We have in multiple papers [35, 36] claimed that the last part of Eq. (14):

$$\frac{l_p}{c}\frac{l_p}{\bar{\lambda}_M} = t_p\frac{l_p}{\bar{\lambda}_M} \tag{15}$$

can be described as the real gravitational mass, or what we have also called collision-time mass (M_t), as it has dimension time $[T]$. If we multiply the collision-time mass by c^3 and call that mass, we arrive back at the original Newton mass that also Maxwell described with units $[L^3 \cdot T^{-2}]$. Therefore, from a deeper perspective, the Newton mass is equal to

$$M_n = GM = c^3 M_t = c^3 t_p \frac{l_p}{\bar{\lambda}_M} \tag{16}$$

One can therefore argue the Newton mass as a mass where G is embedded. However, it is important to be aware that finding GM requires less information than finding G and M separately. This is because GM contains no Planck constant, i.e., no information about the kilogram, while both G and M separately contain information about the kilogram. For example, we can find GM by measuring the gravitational acceleration at the surface of Earth. We have $GM = R^2 g$, and we can find g by dropping a ball from height H and measuring how long time it took before it hit the ground, we have $g = \frac{2H}{T_d^2}$ where T_d is the time it took for the ball from it was dropped to it hit the ground. This is why the mass of the Earth (M_n) can be directly calculated from gravitational acceleration or any other gravitational observation, and GM can also be directly calculated from a gravitational acceleration or another gravitational observation. However, finding the kilogram mass (M) of the Earth requires finding the gravitational constant G, for example, from a Cavendish apparatus.

The collision-time mass can be derived by dividing the Newton mass by c^3, which in this model acts as a gravitational constant. Thus, we have $GM = M_n = c^3 M_t$. However, a deeper understanding of these values reveals that they are all linked to the Planck scale through the expression $c^3 t_p \frac{l_p}{\bar{\lambda}_M}$. This implies that even Newton's gravity is linked to the Planck scale since the Planck length can be extracted from a Cavendish apparatus without knowing G. Newton naturally did not assume this; however, the Planck scale gets embedded in the mass by measuring the mass from gravitational phenomena.

Quantum gravity theory assumes that gravity is quantized, and since quantization in quantum mechanics is associated with the Planck constant, it is reasonable to expect that the Planck constant would also play a role in quantum gravity. Some experiments involving gravity have even claimed to be linked to the Planck constant, but as we will see in Section 4, this is not correct.

The Planck constant is not present in M_n or $c^3 M_t$ and even cancels out when multiplying G with M. Nevertheless, we assert that quantization still exists within the expression $\frac{l_p}{\bar{\lambda}}$, which is equivalent to the reduced Compton frequency per Planck time. The reduced Compton frequency per second is $f = \frac{c}{\bar{\lambda}}$, but the reduced Compton frequency for the Planck time must be $f = \frac{c}{\bar{\lambda}} t_p = \frac{l_p}{\bar{\lambda}}$. This can also be understood by considering how far light travels per Planck time. In other words, we can measure the speed of light in meters per Planck time instead of per second. Since light can travel the Planck length per Planck time, we can conclude that the reduced Compton frequency per Planck time is as described above.

This indicates a connection between quantization in gravity, the reduced Compton wavelength, and the Planck length. For a particle with mass equal to the Planck mass, the reduced Compton frequency per Planck time is equal to one $f = \frac{l_p}{\bar{\lambda}} = \frac{l_p}{l_p} = 1$, as the reduced Compton wavelength of the Planck mass particle is the Planck length: $\frac{\hbar}{m_p c} = l_p$. It is natural to assume that the minimum frequency that can be observed in an observational time window is one. If the shortest possible time window is the Planck time, then the minimum observation in terms of reduced Compton frequency in such a time window is also one. Particles with a mass smaller than the Planck mass will have a frequency of less than one in the Planck time window. For instance, an electron will have a reduced Compton frequency per Planck time of only $f_e = \frac{l_p}{\bar{\lambda}_e} \approx 4.19 \times 10^{-23}$. We have suggested in multiple papers [35, 37] that this can also be interpreted as a frequency probability when it is below one, which we will discuss further later.

In summary, we make the bold claim that even Newton's gravity is quantized and linked to the Planck scale and the Compton frequency when understood from a deeper perspective. We are naturally not suggesting that Newton himself hid quantization in his formula since the Planck length and the Compton wavelength were unknown during his time. Rather, we assert that this is what even Newton's gravity represents when the gravitational mass is understood at a deeper level. Moreover, it should be possible to determine the reduced Compton frequency per Planck time from gravitational observations. For instance, by measuring the gravitational acceleration on Earth, we can determine the reduced Compton frequency of the Earth per Planck time. Equation 17 shows that the reduced Compton frequency per Planck time of the Earth can be determined by knowing the gravitational acceleration, the Planck length, the speed of light, and the radius of the Earth.

$$\frac{l_p}{\bar{\lambda}} = \frac{gR^2}{l_p c^2} \tag{17}$$

By comparing the value obtained from this formula with traditional methods for finding the Compton wavelength and frequency, we can verify its accuracy. Additionally, we can determine the reduced Compton wavelength of the Earth by knowing its kilogram mass, which requires knowledge of the gravitational constant in standard theory. However, once we have the Earth's mass, we can use $\bar{\lambda}_E = \frac{\hbar}{Mc}$ to find the reduced Compton frequency per Planck time, given by $f = \frac{c}{\bar{\lambda}_e} t_p = \frac{l_p}{\bar{\lambda}_e}$. Equation 17 works for any astronomical object and provides the correct prediction for the reduced Compton frequency per Planck time.

The key point to understand so far is that, at a deeper level, Newton's gravity exhibits quantization of gravity and a link to the Planck scale and what we could call the Compton scale or Compton frequency. Next, we will try to see how this fits in with Einstein's general relativity theory.

Table 1 shows that both the original Newtonian gravitational force formula with mass M_n and no gravitational constant, as well as the modern 1873 Newtonian gravitational force formula, predict the same results. From a deeper perspective, they are exactly the same formulas.

2 General relativity theory

Moving on to the theory of general relativity, Einstein's [1] field equation is given by

Prediction	Formula:
Gravity acceleration	$g=\frac{GM}{R^2}=\frac{M_n}{R^2}=\frac{c^2 l_p}{R^2}\frac{l_p}{\bar{\lambda}_M}$
Orbital velocity	$v_o=\sqrt{\frac{GM}{R}}=\sqrt{\frac{M_n}{R}}=c\sqrt{\frac{l_p}{R}\frac{l_p}{\bar{\lambda}_M}}$
Orbital time	$T=\frac{2\pi R}{\sqrt{\frac{GM}{R}}}=\frac{2\pi R}{\sqrt{\frac{M_n}{R}}}=\frac{2\pi R}{c\sqrt{\frac{l_p}{R}\frac{l_p}{\bar{\lambda}_M}}}$
Velocity ball Newton cradle	$v_{out}=\sqrt{2\frac{GM}{R^2}H}=\sqrt{2\frac{M_n}{R^2}H}=\frac{c}{R}\sqrt{2Hl_p\frac{l_p}{\bar{\lambda}_M}}$
Frequency Newton spring	$f=\frac{1}{2\pi R}\sqrt{\frac{GM}{x}}=\frac{1}{2\pi R}\sqrt{\frac{M_n}{x}}=\frac{c}{2\pi R}\sqrt{\frac{l_p}{x}\frac{l_p}{\bar{\lambda}_M}}$

Table 1: The table shows a series of gravity predictions that typically are linked to Newton's gravity theory, which is the weak field approximation of general relativity theory.

$$R_{\mu v}-\frac{1}{2}g_{\mu v}R+\Lambda g_{\mu v}=\frac{8\pi G}{c^4}T_{\mu v} \tag{18}$$

In 2016, our study [38] replaced the gravitational constant G with its composite form $G=\frac{l_p^2c^3}{\hbar}$ and obtained the following equation[2] (see also [39]):

$$R_{\mu v}-\frac{1}{2}g_{\mu v}R+\Lambda g_{\mu v}=\frac{8\pi l_p^2}{\hbar c}T_{\mu v} \tag{19}$$

However, at the time, we did not discuss the potential implications of this result, as we had not yet determined the validity of the equation. Furthermore, in 2016, we had not found a way to calculate the Planck length independently of G, which could lead to a circular problem, as previously noted by Cohen in 1987, but as discussed in the section above, the circular problem here has been resolved in recent years. We believe that the rewritten field equation is fully valid; it does not alter any prediction from general relativity at the macroscopic scale, but as we will see, it gives a new interesting quantized quantum gravity theory that is consistent with general relativity theory.

One could take different stances on it and argue that it is just a trivial creative rewriting of G that provides no new insights and is not even valid. We believe that such a view would be a mistake and would stem from a prejudice against looking at general relativity from a new perspective. Alternatively, one could speculate on the implications of changing G to its composite. The Planck constant is now visible in the field equation, but one should not conclude that general relativity theory based on getting the Planck constant into the formula implies quantization. The Planck constant we obtain from the composite view of G will cancel out with the Planck constant embedded in the stress-energy tensor. The stress-energy tensor contains the energy density, and the Planck constant is therefore embedded there and will cancel out with the Planck constant now visible in the rewritten field equation.

That the Planck constant in the rewritten field equation cancels out can be best seen if we study one of the solutions of the field equation that can actually be used to predict observable gravity phenomena. The most famous and widely used solution to Einstein's field equation is the Schwarzschild metric [40, 41], which is given by:

$$c^2d\tau^2 = \left(1-\frac{2GM}{c^2R}\right)c^2dt^2-\left(1-\frac{2GM}{c^2R}\right)^{-1}dR^2-R^2(d\theta^2+\sin^2\theta d\phi^2) \tag{20}$$

Now by replacing G with its composite form $G=\frac{l_p^2c^3}{\hbar}$ and the kilogram mass with its composite form $M=\frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}$ and this gives

$$\begin{aligned} c^2d\tau^2 &= \left(1-\frac{2\frac{l_p^2c^3}{\hbar}\frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}}{c^2R}\right)c^2dt^2-\left(1-\frac{2\frac{l_p^2c^3}{\hbar}\frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}}{c^2R}\right)^{-1}dR^2-R^2(d\theta^2+\sin^2\theta d\phi^2) \\ c^2d\tau^2 &= \left(1-\frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}\right)c^2dt^2-\left(1-\frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}\right)^{-1}dR^2-R^2(d\theta^2+\sin^2\theta d\phi^2) \end{aligned} \tag{21}$$

That is, there is no Planck constant in the Schwarzschild metric, but the Planck length and the reduced Compton wavelength are there. Again, the term $\frac{l_p}{\bar{\lambda}_M}$ is the reduced Compton frequency per Planck time, so this can indeed be

Prediction	Formula:
Gravity acceleration	$g=\frac{GM}{R^2}=\frac{c^2l_p}{R^2}\frac{l_p}{\bar{\lambda}_M}$
Orbital velocity	$v_o=\sqrt{\frac{GM}{R}}=c\sqrt{\frac{l_p}{R}\frac{l_p}{\bar{\lambda}_M}}$
Orbital time	$T=\frac{2\pi R}{\sqrt{\frac{GM}{R}}}=\frac{2\pi R}{c\sqrt{\frac{l_p}{R}\frac{l_p}{\bar{\lambda}_M}}}$
Velocity ball Newton cradle	$v_{out}=\sqrt{2\frac{GM}{R^2}H}=\frac{c}{R}\sqrt{2Hl_p\frac{l_p}{\bar{\lambda}_M}}$
Frequency Newton spring	$f=\frac{1}{2\pi R}\sqrt{\frac{GM}{x}}=\frac{c}{2\pi R}\sqrt{\frac{l_p}{x}\frac{l_p}{\bar{\lambda}_M}}$
Gravitational red shift	$z=\frac{\sqrt{1-\frac{2GM}{R_1c^2}}}{\sqrt{1-\frac{2GM}{R_2c^2}}}-1=\frac{\sqrt{1-\frac{2l_p}{R_1}\frac{l_p}{\bar{\lambda}_M}}}{\sqrt{1-\frac{2l_p}{R_2}\frac{l_p}{\bar{\lambda}_M}}}-1$
Time dilation	$T_R=T_f\sqrt{1-\frac{2GM}{Rc^2}}=T_f\sqrt{1-\frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}}$
Gravitational deflection (GR)	$\theta=\frac{4GM}{c^2R}=4\frac{l_p}{R}\frac{l_p}{\bar{\lambda}_M}$
Advance of perihelion	$\sigma=\frac{6\pi GM}{a(1-e^2)c^2}=\frac{6\pi l_p}{a(1-e^2)}\frac{l_p}{\bar{\lambda}_M}$
Schwarzschild radius	$R_s=\frac{2GM}{c^2}=2l_p\frac{l_p}{\bar{\lambda}}$

Table 2: The table shows a series of gravity predictions given by general relativity theory, not only on their traditional form but also from their deeper level, when G is replaced by its composite form and the mass is understood from a deeper level. All the predicted gravity phenomena contain the factor $\frac{l_p}{\bar{\lambda}}$, which is the reduced Compton frequency of the mass M per Planck time. We will boldly claim that this gives both the link to the Planck scale and the quantization of gravity without altering general relativity theory.

seen as the quantization of gravity. The appendix shows a few more coordinate systems where the same methodology is applied. Table 2 shows the prediction from general relativity for a series of gravity phenomena.

Our approach has led to a quantized theory of gravity that is consistent with both Newton's theory and general relativity. However, this does not amount to a complete unification of quantum gravity and quantum mechanics. We suspect that the necessary modification lies within quantum mechanics itself, which relies on the kilogram as a unit of mass and the Joule as a unit of energy. These units, also Joule energy, are inadequate for describing gravity, just as a kilogram of mass without being multiplied by the gravitational constant G is insufficient for the task.

We have achieved a type of quantization within general relativity. This was done by simply replacing G and M with their composite forms, which were obtained by solving the Planck length formula for G and the Compton wavelength formula for M. This method is fully mathematically valid. Additionally, the obstacle that was previously pointed out, which stated that we had to know G to find the Planck length and that the composite of G was therefore not valid, has been resolved in recent years.

We could have ended the paper at this point and left it to other researchers to interpret our findings, just as there are various interpretations in standard quantum mechanics. However, we are intrigued by the deeper question of why the reduced Compton frequency per Planck time is embedded in general relativity theory. There could be multiple interpretations of this, ranging from it being a coincidence that provides no deeper insight to it being the key to quantum gravity. We personally believe that the latter is more likely. Below, we will provide our interpretation. However, we are open to other interpretations as well. The most important point of this paper has already been made: there is a very simple and easy method to quantize the general relativity theory and also get in the Planck scale that has not been discussed before. This method has been addressed to some extent in our other papers, but only with a focus on Newtonian gravity and only briefly in relation to the general relativity theory.

3 Possible interpretation

Pure photon energy is given by

$$E=hf=h\frac{c}{\lambda_\gamma}=\hbar\frac{c}{\bar{\lambda}_\gamma} \tag{22}$$

It is an arbitrary choice that the frequency typically is given per second. If we change the frequency per Planck time rather than per second, then we have that

[2]In that paper, we had also set the cosmological constant to zero.

$$E = \hbar \frac{c}{\bar{\lambda}_\gamma} t_p = \hbar \frac{l_p}{\bar{\lambda}_\gamma} \quad (23)$$

This looks somewhat similar to $l_p f = l_p \frac{l_p}{\bar{\lambda}}$ that we have in the rewritten Schwarzschild metric and also in all the formulas derived to predict gravitational phenomena that can be observed (Table 2). The main difference is that $\hbar$ is swapped with the Planck length. Is there any good interpretation of this? We think so. Let's go back to the Newtonian mass; it is equal to

$$M_n = GM = c^3 t_p \frac{l_p}{\bar{\lambda}_M} \quad (24)$$

We have, in a series of papers, argued to define mass only as the last part of this, namely $t_p \frac{l_p}{\bar{\lambda}}$, and called it collision-time mass that we can use the symbol M_t for. Then, we again have a gravity constant, but it is c^3. We have also argued that energy then is a collision length of the form

$$E_g = l_p \frac{l_p}{\bar{\lambda}} \quad (25)$$

This energy we have called collision-length energy and also quantized gravitational energy [35, 36]. This is exactly the term we observe in the rewritten Schwarzschild metric and in all formulas we can use to predict observable gravity phenomena. However, the relation between collision-length energy and collision-time mass must then be

$$E_g = M_t c = l_p f = l_p \frac{l_p}{\bar{\lambda}_M} \quad (26)$$

One might mistakenly think that this is inconsistent with $E = Mc^2$, but in fact, it is fully consistent with Einstein's energy-mass relation. The standard unit for energy is the Joule, and for mass, it's the kilogram. There is nothing mathematically wrong with introducing a new quantity, which we could call "energy type-two," equal to the Joule energy divided by c. We could then write $E_2 = E/c$, which gives us $E_2 = Mc$. This doesn't violate any mathematical or logical principles, although it would require us to adjust other formulas. Essentially, it's just a matter of choosing units.

However, this particular unit of energy, E_2, doesn't provide any new insight. It's simply Joules times meters divided by seconds. To introduce a new terminology for energy or mass, it should give us a better understanding of the underlying physics. In contrast, we claim that "*collision-length energy*" and "*collision-time mass*" provide new and deeper insights. These concepts tie space and time together more closely than the standard interpretation in general relativity does, because now even mass and energy are related to time and space. So, curving of space-time is curving of mass and energy, and curving of mass and energy is curving of space-time at the deepest level?

In the standard relativistic energy-momentum relation, we have

$$\begin{aligned} E &= \sqrt{p^2c^2 + M^2c^4} \\ E &= \sqrt{(Mv\gamma)^2c^2 + M^2c^4} \end{aligned} \quad (27)$$

We can always divide or multiply by a constant on each side. If we multiply by $\frac{G}{c^4} = \frac{l_p^2}{\hbar c}$ on each side we get

$$\begin{aligned} \frac{G}{c^4}E &= \frac{G}{c^4}\left[(Mv\gamma)^2c^2 + M^2c^4\right] \\ E_g &= \sqrt{M_g^2v^2\gamma^2 + M_g^2c^2} \end{aligned} \quad (28)$$

When working with photons, the result is slightly different as the momentum then is the photon momentum, except that the same procedure can be used. We think that there is no coincidence that there is $\frac{G}{c^4}$ as a part of Einstein's gravitational constant that needs to be multiplied by the stress-energy tensor. Einstein indirectly converts energy to collision-length energy, but unknowingly so, as he likely never thought of G as a composite constant. Therefore, our new energy and mass definition, which contains both the Planck scale and quantization, is fully consistent with the standard relativistic energy-momentum relation. It is, however, more than a simple change of notation. Because if we do not multiply M or E with G, then we are not getting the Planck length into the mass and energy and the Planck constant out, which is needed for gravity. Therefore, we believe that it is the mass and energy in quantum mechanics that require modification. In the case of mass in gravity, it is already modified by multiplication with G, or the Planck length was actually already embedded in the original Newton mass M_n, not by assumption, but by calibration to gravity phenomena. Understanding that this leads to a deeper understanding is first seen when we understand G is a composite constant and M can be written on the form $M = \frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}$.

3.1 Schwarzschild radius

For masses smaller than half the Planck mass, the Schwarzschild radius becomes much smaller than the Planck length, and it has been suggested that particles such as the proton or the electron, therefore, cannot have a Schwarzschild radius. In our modified general relativity theory, the Schwarzschild radius can be expressed as:

$$R_s = \frac{2GM}{c^2} = 2l_p \frac{l_p}{\bar{\lambda}_M} \tag{29}$$

For macroscopic masses, the value of $\frac{l_p}{\bar{\lambda}}$ is a large integer number with a negligible fraction. This ratio can be understood as the reduced Compton frequency per Planck time, and it quantizes the Schwarzschild radius.

In the case when $\frac{l_p}{\bar{\lambda}} < 1$, we interpret this factor as a probability. This means that even protons and electrons have a Schwarzschild radius of $2l_p$, but it comes in and out of existence with a frequency of $\frac{c}{\bar{\lambda}}$ per second. Therefore, masses smaller than the Planck mass (or half the Planck mass) will have a probabilistic Schwarzschild radius, rather than having no Schwarzschild radius at all. This interpretation is also discussed in [35]. However, it is important to note that this is just one possible interpretation, and we welcome other possible interpretations as well.

4 The claims that the Planck constant plays a role in some observed gravity phenomena is likely wrong when understood from a deeper perspective

Already in 1975, Colella, Overhauser and Werner [42] had observed what is known as gravitationally induced quantum interference using neutrons. They claimed this was related to both g and the Planck constant. The observational study was repeated and done more precisely [43, 44]. Abele and Leeb [45] have done a similar experiment with neutrons that they claim "*the outcome depends upon both the gravitational acceleration g and the Planck constant $\hbar$*". Further, they claim that the Colella, Overhausser and Werner experiment

> "*First of all, they demonstrate the validity of quantum theory in a gravitational potential on the 1% level. The gravity-induced interference is purely quantum mechanical, because the phase shift is proportional to the wavelength λ and depends explicitly on Planck's constant $\hbar$, but the interference pattern disappears as $\hbar \to 0$. The effect depends on $(m_n/\hbar)^2$ and the experiments test the equivalence principle.*"

The authors claim that both papers present accurately derived formulas that predict gravitationally induced quantum interference, which closely aligns with observations. However, they argue that the interpretation that this effect is related to the Planck constant is likely wrong. The researchers failed to recognize that the kilogram mass in their formula is, at a deeper level, simply $m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$. The formulas they used contain $m_n^2/\hbar^2$, meaning the Planck constant is necessary to cancel out with the Planck constant embedded in the neutron mass. To approach $\hbar$ close to zero, they must also modify the Planck constant in the mass. Since the two Planck constants cancel out, this has no effect on the predictions. The formula for gravitationally induced quantum interference also includes g, which we have already discussed at a deeper level, containing both the Planck length and quantization in the form of the reduced Compton frequency per Planck time. This is likely the cause of the observed interference pattern. Setting the Planck length to zero would remove the predicted interference pattern. At the very least, we encourage the gravity research community to investigate this possibility further.

So the effect is predicted by the following formula first suggested by Colella and Overhauser and Werner [42]

$$q_{grav} = 4\pi\lambda g h^{-2} m_N^2 d(d + a\cos\theta)\tan\theta \tag{30}$$

When we replace G with its composite form $G = \frac{l_p^2 c^3}{\hbar}$ and the neutron mass m_N with $m_N = \frac{h}{\lambda_n}\frac{1}{c}$ (where $\bar{\lambda}_n$ is the Compton wavelength of the neutron), we can rewrite Eq. (30) as:

$$
\begin{aligned}
q_{grav} &= 4\pi\lambda_n g h^{-2} m_N^2 d(d + a\cos\theta)\tan\theta \\
q_{grav} &= 4\pi\lambda_n \frac{GM}{R^2} h^{-2}\left(\frac{h}{\lambda_n}\frac{1}{c}\right)^2 d(d + a\cos\theta)\tan\theta \\
q_{grav} &= 4\pi\lambda_n \frac{c^2 l_p^2}{\bar{\lambda}_E R^2}\left(\frac{1}{\lambda_n^2 c^2}\right) d(d + a\cos\theta)\tan\theta \\
q_{grav} &= \frac{4\pi}{\bar{\lambda}_E R^2}\frac{l_p^2}{\lambda_n} d(d + a\cos\theta)\tan\theta \\
q_{grav} &= \frac{2}{R^2}\frac{l_p}{\bar{\lambda}_E}\frac{l_p}{\bar{\lambda}_n} d(d + a\cos\theta)\tan\theta
\end{aligned}
\tag{31}
$$

Here, $\bar{\lambda}_E$ represents the aggregated reduced Compton wavelength of the Earth. This formula predicts the same results as the original one. At the deepest level, the gravitationally induced quantum interference is related to the reduced Compton wavelength of the neutron, the Earth, and the Planck length, as well as the radius of the Earth. The Planck constant does not play a role. This is consistent with our quantized form of general relativity theory, where quantization is related to the reduced Compton wavelength and the Planck length.

5 Conclusion

We have demonstrated a straightforward way to connect Newton's theory and general relativity to the Planck scale and how this approach leads to the quantization of gravity through the reduced Compton frequency per Planck time in the gravitational mass. Despite its extreme simplicity, we believe this potential path to quantum gravity should be taken seriously. After all, the simplest solutions are often the best. However, before drawing conclusions, a series of researchers should investigate this methodology over time. This new way of looking at both Newton's theory and general relativity from a deeper, more fundamental level may reveal the need for other small changes to the general relativity theory. For example, how are singularities interpreted when we suddenly have the Planck scale incorporated into general relativity theory? We have also suggested that standard quantum mechanics needs modification to achieve unification of quantum gravity and quantum mechanics. The Planck length must be somehow incorporated into standard quantum mechanics.

Declarations

Ethical Approval

not applicable as a pure theoretical study.

Competing interests

The author declares no conflict of interest

Authors' contributions

Espen Gaarder Haug has done all the research and written the full paper.

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Appendix

In this appendix, we quickly show that our methodology can also be used for a series of coordinate systems described in the literature for general relativity theory.

The Eddington–Finkelstein Coordinates

The Eddington–Finkelstein coordinates are typically given as (see, for example, [46]):

$$-c^2 d\tau^2 = \left(1 - \frac{2GM}{c^2 R}\right) c^2 du^2 - 2dudr - R^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{32}$$

The Schwarzschild metric expressed in this coordinate form avoids singularity when $R = \frac{2GM}{c^2}$. To incorporate the Planck scale and achieve quantization, we need only replace G with $G = \frac{l_p^2 c^3}{\hbar}$ and M with $M = \frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}$, yielding:

$$-c^2 d\tau^2 = \left(1 - \frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M}\right) c^2 du^2 - 2dudr - R^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{33}$$

Once again, the term $\frac{l_p}{\bar{\lambda}}$ represents the reduced Compton frequency per Planck time, and therefore provides quantization of gravity. The fact that the Planck length can be easily incorporated without any knowledge of G also indicates a direct link between gravity and the Planck scale.

The Lemaître Coordinates

The Lemaître coordinates consist of a transformation of the Schwarzschild coordinate system from $\{t, r\}$ to the new coordinates $\{\tau, \rho\}$, and are typically given as:

$$ds^2 = c^2 d\tau^2 - \frac{R_s}{R} d\rho^2 - R^2(d\theta^2 - \sin^2\theta d\phi^2) \tag{34}$$

The Planck-quantized version is:

$$ds^2 = c^2 d\tau^2 - \frac{2l_p}{R}\frac{l_p}{\bar{\lambda}_M} d\rho^2 - R^2(d\theta^2 - \sin^2\theta d\phi^2) \tag{35}$$

Note that the Lemaître coordinates transform the time and radial distance coordinates into τ and ρ, respectively. The metric tensor in these coordinates is expressed in terms of the Schwarzschild radius R_s, the radial coordinate R, and the angular coordinates θ and ϕ. The Planck-quantized version includes the reduced Compton frequency per Planck time, $\frac{l_p}{\bar{\lambda}}$, to provide quantization of gravity.