Review of: "Measurement Mechanics"

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Measurement Mechanics by Ken Krechmer. Context and Review

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The context of this manuscript is metrology, defined as the study of measurement. Within this scope, metrology focuses on the setting of standards. In 1795, French revolutionaries proposed standards for length, volume, weight, and money. In 1875, France established a permanent body to oversee the Bureau Internationale des Poids et Mesures (BIPM), which defines units of measurement.

More specifically, the context of this manuscript is nanometrology, the study of measurement at nano (less than 100 nm) scales. Nanometrology is a 21st century extension of the science of measurement (Hand 2016), necessitated by the emerging capacity to measure at these scales, together with increasing applications, notably quantum computing, nanoscale engineering, and nanoscale medicine. At present, the development of protocols and standards in nanomaterials is in its early stages. It is not yet clear how these standards should be developed (Jorio and Dresselhaus 2007). A review of current techniques treats atomic structure (electron diffraction, small angle x-ray scattering, x-ray absorption spectroscopy), with listings of limitations (Herrera-Basurto and Simonet 2013). The transition from single lab to general application is still in the early stages. For example, a one-size-fits-all metrology solution for ultra-thin 2-D structures does not yet exist (Celano *et al.* 2024).

The Measurement Mechanics manuscript addresses a problem not mentioned in current reviews of nanometrology. Measurement at nano scales necessarily disturbs the object -- it can change its state (Heisenberg 1927). Nanometrology thus faces a quandary encountered in other sciences (Hand 2004 Ch4.5). In biology, for example, multiple mark-recapture studies alter the catchability of an animal and hence alter estimates of population size and mortality rates. In demography, the presence of a question about citizenship alters the probability of completion of a form. The question potentially influences answers on the form. In ethology, the presence of an observer alters the behavior of objects of study such as birds and mammals. While these examples are not quantum entanglement, the observer effect was used by Heisenberg as a physical "explanation" of quantum uncertainty (Heisenberg 1930, p20).

These examples are relevant because the MM manuscript states its applicability across the sciences. How relevant, then, are Heisenberg's examples beyond quantum mechanics? One limitation is that in principle, quantum effects can be calculated from theory, a situation rarely found in many sciences. Such calculations are only rarely available for animals (genetics), and calculations from theory are not available for people (psychology) or economic systems (econometrics). From this, one would expect analytic approaches in nanometrology for quantum effects to differ from analytic approaches in systems where the effects of measurement must be estimated from data. Another limitation is that Heisenberg uncertainty applies only to pairs of variables that are conjugate: they can be expressed in either units of time and distance, or in units of frequency (Hz) and periodicity (cycles/m). For example, momentum and distance are conjugate, and so the variance in momentum increases as precision increases in measuring location. The same applies to energy and time. In contrast, energy and momentum are not conjugate and so are not subject to quantum uncertainty when measured simultaneously. Both are still subject to observer effects.

The MM manuscript builds on a series of previous articles. The first (Krechmer 2016) concluded that measurements based on units of constant size cannot address the quandary of indeterminant units due to Heisenberg uncertainty at nanoscales. It proposed relational measurement, defined as the product of the measurement magnitude and the standard deviation of the measurement intervals due to calibration and sampling processes. This definition appears to differ from an earlier definition of relational measurement (Mari and Sartori 2007).

In a subsequent article (Krechmer 2018), the uncertainty due to calibration and resolution is shown to be equal to the uncertainty that appears in quantum mechanics theory and experiment. This stands in contrast to error considered as due to externally imposed experimental artifacts. The article concluded that representational measure theory (Krantz *et al.* 1971) should be extended to include relational measurement theory. A third article (Krechmer 2021) applies relational measurement theory as defined above to several quantum measurement experiments and thought experiments.

The MM manuscript expands on the previously proposed solution to the quandary of quantum measurement. It treats measurement as the product of both a unit and a number, each with a component of variability. In the example given, four measurements with unit and numeric variability result in $2^4 = 16$ values that are taken as converging to a normal distribution. This product of variable solutions, called measurement mechanics, opens the way to further analytic development, such as considering the conditions leading to non-independence of number and unit. This solution opens the way for analytic approaches in systems other than nano-scale physics. An example would be biological rates based on counts of units such as genes, cells, and organisms. The solution could include money. The solution, when applied to money, recognizes error in both enumeration and in the notoriously variable relation of any currency relative to the gold standard (before the gold standard was eventually dropped). The MM manuscript would be strengthened by noting the applicability of the product of variable solution to areas of science where both the unit and the numerical reading are subject to error.

The MM manuscript contrasts metrology with representational measurement as defined in a three-volume set: *Foundations of Measurement*, Vol I (Krantz *et al.* 1971), Vol II (Suppes *et al.* 1989), and Vol III (Luce *et al.* 1990). FM1 established ordinal scale measurement on the axiomatic basis of an extensive library of theorems. FM1 includes ratio scale measurement (Chapter 10), the algebra of dimensional analysis, and a listing of over 100 derived units of measurement. The three-volume work specifies axioms that empirical systems must satisfy to permit representation in terms of a given numerical system, such as a number line (Hand 2004, p. 32). The first fundamental condition is that units can be ordered. The second fundamental condition is that units be additive on a real number line: 2 rods + 2 rods set end to end = 4 rods. Applying these standards, we find that representational measurement includes relative measurement (Krechmer 2018) as a special case, one requiring ratio scale measurements, with calibration. Applying FM1 standards to Figure 1 in the MM manuscript, we find two items (Quantum Mechanics and Relativistic Mechanics) that are ordered and additive. They occupy definable ranges on a log scale on both the vertical and horizontal axes. We find two items (Statistics and Metrology in $\mathbb{R}$) that are not ordered and additive. They do not occupy definable ranges on either the vertical or horizontal axis scale.

Representational measurement (FM1) is foundational to both natural and social science measurement theory. What differs between the two is the presence (or not) of physically defined unit standards. In FM1, physical standards, even when defined relative to physical constants, are taken as arbitrary (FM1 p 474). For example, in 1983, the meter was defined as the length of the path travelled by light in a vacuum during a time interval of 1/c = 1/299,792,458 of a second, where the constant c is the speed of light. The numerical value is arbitrary. It applies only to our prior definition of a meter and a second.

In its treatment of ratio scale units, FM1 listed over 100 derivative units in addition to 6 base units in the Système International (SI). FM1 (p 484) showed that laws of similitude, such as Hooke's law for forces by a spring on an object, meet the axiomatic conditions for conjoint measurement. In FM1 (p503), "fundamental" is synonymous with that same term in Campbell's (1920) text in physics. The three-volume work does not address the setting of standards, which is central to metrology. Representational theory does not, however, preclude the setting of standards. It does not preclude divisible units (p460), such as 10^9 nanometers / meter. It does not preclude variable units, provided these are transformable to equal sized units, as in FM2.

The MM manuscript argues against representational measurement in favor of measure mechanics based on relational measurement. The premises for the conclusion are not stated. The logic appears to be:

Premise: If representational measurement, then X

Definition: Metrology requires X

Conclusion 1: Representational measurements are not consistent with metrology.

This logic is valid for Conclusion 1, which concerns consistency.

However, this logic is not valid for a different conclusion, one concerning the intersection of two terms: representational measurement and metrology. It is not, of course, clear whether this is the logic intended. The logic needs to be made clear.

An alternative to Conclusion 1 is:

Conclusion 2: Representational measurement precludes metrology.

The absence of calibration from representational measurement does not preclude representational measurement. Mari and Sartori (2007) conclude that relational modeling (metrological standards, traceability, and calibration) is complementary, rather than alternative to the standard representational point of view. Metrology and relational measurement both meet the standards for a representational measurement scale. To support an argument against representational measurement, quotes from any of the three volumes of fundamental measurement are needed. These need to establish that representational measurement precludes calibration or other requirements of metrology.

The MM manuscript lists one of several definitions of measurement, that of Euler (1765): "It is not possible to determine or measure one quantity other than by … determining the ratio between the quantity being measured and that [of another] quantity" It is interesting to note that Euler, a mathematician, was unwilling to combine values from different sources, taking the view that errors increase with aggregation, rather than the statistician's view that random errors tend to cancel (Stigler 1986 p28). Euler's definition differs from Maxwell's (1873) bi-partite definition of a quantity as having a number and a unit. Subsequent measurement theory (Helmholtz 1887, Campbell 1920) justified this bi-partite definition by reference to measurement by concatenation: two rods, each of length 1 meter, = 2 meters when laid end to end in a straight line. This empirical approach was followed by definitions of the type of scale. The classification of scales began with the distinction between fundamental and derived measurement (Campbell 1920). This was extended to a classification (Stevens 1946) that has proved useful: nominal, ordinal, interval, and ratio. It has lent itself to extension to more types, based on several criteria. These two traditions, one empirical (Euler, Maxwell), the other concerning classification, were put on an axiomatic basis by representational measurement theory.

The MM manuscript is consistent with the development of model-based measurement theory in this century. This theory emphasizes the relationships between measurement and theoretical and statistical modeling (Morgan 2001), recognizing that scientific knowledge is based on a continuously iterative, adaptive practice of try-and-revise (Nelder 1999, Mari 2005). Model-based theory recognizes that previous developments abstract measurement away from the process of measurement and so are ill-suited for metrology (Frigerio *et al.* 2010). Scale construction (*e.g.*, FM1) is just one of several tasks in metrology, a body of practice that extends to the definition of measured parameters, instrument design and calibration, error detection, and uncertainty evaluation. Scale construction is a critical precondition for the execution of measurement (Mari *et al.* 2023).

The strength of the MM manuscript lies in a proposal that is applicable to a wide variety of measurement scales. It is applicable in fields where the act of measurement affects the behavior of the object of measurement. Arguments against representational theory distract from that strength and do not appear to be necessary.

The manuscript has several statements that are inconsistent with material in FM1 (Krantz*et al.* 1971) and FM2 (Suppes *et al.* 1989). It has statements about FM1 that require substantiation by quotes from FM1. Statements from the manuscript are listed below in courier font.

p.3 *Representational theory - a measurement result is a distribution of numerical values on a scale (probabilistic).*

I cannot find any reference in FM1 that restricts the measurement scale to probabilistic.

p.3 *In this theory, a measurement result is a numerical value on one of three basic scales (ordinal, counted/linear, and ordered).*

FM1 includes ratio scale measurement. Chapter 10 (91 pages) is an outstandingly complete and lucid exposition of units and the algebra of dimensional analysis. Table 3 lists the dimensions of over 100 physical quantities.

p.3 *In statistics, the central limit theorem identifies that …. repetitive measurement… will have a distribution that converges on a normal distribution [4]*

The Law of Large Numbers (LLN) applies to the average of the results obtained from repeated trials. The LLN states that this average converges to the expected value. It does not state that as *n* increases, the sum of *n* results gets close to the expected value (times n). The LLN becomes irrelevant at quantum scales (Hand 2004).

For complex dynamics, we are not justified in claiming that the central limit theorem applies.

Examples are magnitude of earthquakes, fire area, and river discharges perturbed by floods. In these series, as more measurements are added, the mean drifts downward until it lurches upward after a large (and rare) value is encountered. The largest event is unknown relative to long time scale dynamics such as Milankovitch cycles, solar radiation cycles, and greenhouse gas forcing.

p.3 *Normal distribution … are treated in representational theory as distributions of errors due to noise and distortion in measurement processes [5]. In experimental measurement systems where noise and distortion are closely controlled,*

The definition in FM1 p 27 is broader than that quoted above. The definition is that error is due to "inherent features of the observational situation that cause us to fail to observe exactly what we wish to observe."

p.4 *In Fig. 2, representational theory treats a standard as arbitrary [8]*

The statement in FM1 (p 454) is more specific. It reads as follows: "All of the physical measures with which we shall deal are (or are treated as if they are) ratio scales, i.e. they are completely determined except for an arbitrarily chosen unit."

p.5 *this requires calibration, which representational theory does not include.*

The logic appears to be:

Statement 1: FM1 does not include calibration.

Statement 2: Metrology includes calibration.

Conclusion: FM1 is inconsistent with metrology.

p.5 *calibration, which representational theory does not include.*

The logic might also be:

Statement 1: FM1 does not treat calibration.

Statement 2: Metrology requires calibration.

Conclusion: FM1 could be consistent with metrology.

The logic might also be:

Statement 1: FM1 excludes calibration (quote needed from FM1, FM2, or FM3)

Statement 2: Metrology requires calibration.

Conclusion: FM1 is not consistent with metrology.

The logic differs because “not include” is not the same as “exclude.”

The distinction between “not include” and “exclude” applies to Figure 2 versus Figure 3.

Does FM1 exclude Figure 3? If so, a quotation from FM1 is needed to show this. Quotes from any of the 3-volume work are needed to support the characterization of representational measurement and the arguments against it.

p.6. *Heisenberg's uncertainty theory [11] makes the possibility of a true value measurement result erroneous (even though a very small error) and also ignores the ubiquitous central limit theorem; therefore, calibration demonstrates the fallacy of representational theory*

The logic here is not clear. From the paragraph, the premises and hypotheses appear to be:

Statement 1: HU makes the possibility of a true value measurement result erroneous

Premise 1: If HU, then the possibility of a true value measurement is erroneous.

Statement 2: HU ignores the ubiquitous central limit theorem.

Conclusion: Calibration demonstrates the fallacy of representational theory.

The relation of the conclusion to the preceding statements is not clear.

Statement 1, phrased as a premise, is not valid. A logical argument requires a valid premise.

In the conclusion, does “fallacy” refer to an invalid argument, or does it refer to a false conclusion based on an untrue premise or statement? Note that Heisenberg (1927) used the words “indeterminacy” or “uncertainty.” It was not called a theory.

If the logic above is not what was meant, that confirms the need for the logic leading to “fallacy of representational measurement” to be laid out clearly.

p.8 Fig.4 *identifies that when an observable's property is ordered, additive, and has a zero point*

Figure 4 meets FM1 criteria (ordered, additive). These criteria do not exclude the zero point.

p.8 Eq 3 which converges to a normal distribution

The LLN applies to the average of the results obtained from repeated trials. It is these averages that converge to the expected value.

p.8 *measurement result quantities that converge to a normal distribution of ɥ.*

The LLN does not state that as *n* increases, the sum of *n* results converges to the expected value (times *n*).

p.8 *Such normal distributions of ɥ occur in all repetitive physical measurement results of unchanged observable*

Measurements of complex phenomena (such as phase transitions) do not converge. Examples of physical quantities that do not converge are earthquake and flood magnitudes.

p.9 *Eq.(4) is the measurement mechanics measurement result function that applies to all formal and experimental measurement systems without noise or distortion.*

The statement applies only to experiments that meet Eq. 4. Not all experiments meet the requirement of divisibility in Eq.4. Such experiments are not invalid.

p.11 *The ubiquitous nature of a normal distribution of repetitive measurement comparisons, caused by the summing of the quantized precision, strongly supports MM.*

Physical measurements are often non-normally distributed (river basin area, stream discharges, *etc*), often with errors that are non-normal unless a realistic error model is used instead of a normal distribution.

p.12 *quantum mechanics (QM) has applied representational theory by applying the ratios of quantities with common units which have invariant numerical values. That is, in such a ratio of quantities, the numerical value ratios remain the same when the common units in the ratio change numerical values.*

The statement is unsubstantiated. It requires a citation. The citation needs to be an example that cites FM1 criteria correctly: ordinal placement, additive (or divisible) units.

p.15 . *The EPR paper, which is based upon representational theory…*

The statement is unsubstantiated. It requires quoted material from EPR, along with a citation from FM1 relevant to a quote from EPR.

p.15 *Representational theory does not recognize a quantity [33]; assumes measurement result comparisons occur without a calibrated scale or standard; assumes units are equal [34],requires any calibration to be empirical [35]; and indicates that all measurement result quantity deviation is due to noise and distortion in the measurement system [36].*

Notes 33 through 36 refer to FM1. As above, FM1 recognizes quantities, does not assume absence of calibrated scale (*viz* Chapter 10), and does not require calibration to be empirical. FM2 does not indicate that all deviation is due to noise and distortion. Quotes from FM1 or FM2 are needed to support all three references to FM1.

p.16 *Metrology and representational theory have been applied successfully for a long time. Measurements of population distributions, often to a reference, are very useful in the social sciences*.

This statement can be extended to some parts of chemistry, many parts of biochemistry, and much of biology.

p.16 *However, as Euler explained in 1765, the EPR paper was formally developed in 1935, and Bell refined in 1989, the inconsistencies across measurement theories and experimental measurement results beg to be resolved.*

The quote from Euler on p.3 is as follows.:

L. Euler (1765) made this clear: "Now, we cannot measure or determine any quantity, except by considering some other quantity of the same kind as known and pointing out their mutual relation.

The quote from Euler combines a restrictive statement ("cannot measure….except by") with a vague term ("mutual relation"). As a result, the next paragraph displays a logically valid conclusion (affirming the antecedent) from an unsupported premise.

Premise: (from Euler). If no mutual relation, then not a measurement.

Statement 1: Mutual relation is the presence of a reference standard

Modified Premise: If no reference standard, then not a measurement

Statement 2: Statistics has no reference standard.

Conclusion Statistics is not a measurement.

The conclusion applies to a modified premise, not to the Euler quote.

Note that statement 2 is false for nested likelihood ratios, in the case of a denominator that is a binomial likelihood based on theory and has units on a ratio type of scale.

The premise excludes most of the measurements covered by Hand (2004).

The quote from Euler combines a restrictive statement ("cannot measure….except by") with a vague term ("mutual relation"), which compromises any conclusion. The EPR citation needs to be substantiated by an example that cites FM1 criteria: ordinal placement, additive (or divisible) units.

A counterexample to this restrictive definition of measurement is the success of population biology in predicting short-term COVID infection and morbidity rates. The units are those of Stahl (1962), organism counts. Using a physical unit (viral mass) instead of organism counts omits the biology of infection. The use of mass would be needlessly complex and prone to irrelevant error due to variance in mass/organism.

p.16 *Measurement Mechanics establishes one formal measurement function consistent across the physical sciences. When this is applied, the inconsistencies across all the measurement theories and experimental measurement results can be resolved.*

This statement applies to ratio scale measurement of physical quantities. It is inconsistent with the relational theory of measurement based on traceability (Mari and Sartori 2007), which entails a statement of the type of measurement scale. In physical oceanography and meteorology, simple and ranked categorical variables are regularly used. Examples are water masses, mixed layer, Beaufort wind scales, and the Saffir-Simpson Hurricane Wind Scale. All are valid by representational measurement standards.

In summary, the manuscript would be strengthened by focusing on the novel proposal, rather than dwelling on arguments against representational measurement theory.

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