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Dark Energy as an intrinsic property of Matter

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Starting from the proviso that general relativity is the valid theory of gravitation, we invoke a novel line of thought that assigns to matter the intrinsic property of permanent space emission. With this property, dark energy is seen as a property of matter. According to these thoughts, we set up a matter model and derive its equation of state. Both matter and volume expansion remain tightly linked through the energy-momentum conservation law and the key-assumption of strict proportionality of the respective energy densities in time, associated with a two-component fluid model. In essence, the so-introduced 'Space Production Model' (SPM) posits that both 'matter' and 'dark energy' are two manifestations of the same entity. One realization of SPM leads to a fluid analogy of a scalar field matter model that is minimally coupled to gravity, and that resides in permanent virial equilibrium. This latter property of SPM implies a constant ratio of $1/3$ of matter energy and $2/3$ of dark energy in agreement with current observations. We discuss the resulting expansion and acceleration laws that occur within homogeneous cosmology, as well as some consequences for inhomogeneous cosmology. While SPM leads to expansion in the homogeneous case, it also allows for contraction in the inhomogeneous case. In both cases SPM implies a stationary state that may arise after a relaxation period of inflation. While not discussed here, the proposal that matter and space are, in the SPM sense, contingent one upon the other could have implications beyond cosmology.

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I. BACKGROUND

The world models by Friedmann and Lemaître have been established since the early 20*th* century as effective models of the Universe. The robustness of these homogeneous-isotropic models is exemplified by their consistency with the enormous collection of observational data accumulated over the last several decades, although recently an increasing number of 'tensions' have been pointed out [7, 9, 19], especially between the expansion rates as drawn from Cosmic Microwave Background observations and the late Universe. During this collection of new observational data, changes of the model have been entirely due to adaptations of its parameters. The contemporary, still accepted, model is Lemaître's coasting model, coined 'concordance model'. It starts out with a singularity, the 'Big Bang', and infinite expansion. The expansion then slows down due to the gravitational attraction of its energy content, and experiences a period of 'scale-factor acceleration' since the epoch of structure formation. This acceleration period is modeled in the simplest case by Einstein's cosmological constant—being repulsive if positive, and it dominates the expansion history since then. The cosmological constant is interpreted as 'dark energy' if put on the side of the sources of Einstein's equations, nowadays thought to make up about two thirds of the universal energy budget.

However, the description of the early stages of the Universe still enjoys a variety of scenarii, governed by the paradigm of 'inflation'. Invoking an inflationary period helps to explain the apparently causally connected visible Universe, as documented by the analysis of the uniform Cosmic Microwave Background radiation. The favoured models of inflation are mostly due to a single or several scalar fields whose potential energy density dominates at early stages, hence acting like a positive cosmological constant. Such phenomenological scalar fields are also employed to model 'dark energy' at late epochs of the cosmological evolution and coined 'quintessence' (besides other suggestions). We have observed a qualitative transition in the discussion of inflationary models since the discovery of the Higgs boson: until then, no scalar field was known to exist in nature. The Higgs field will certainly play a major role in understanding the early Universe (e.g. [21] and references therein), and the model we are presenting may allow for a new perspective on the Higgs field.

The standard model of cosmology, well after inflation, invokes the matter model of radiation, governed by an equation of state of radiation in local thermal equilibrium. As soon as the Universe becomes less dense and ceases to be opaque, the recombination of matter allows the radiation to propagate freely and matter dominates the expansion of the Universe thereafter. This matter is then thought of to be dominated by 'dark matter' that may, to a good approximation, be treated as dust, i.e., pressure-less and non-collisional matter. Certainly, these are macroscopic matter models that won't reveal much about the microscopic state.

Since the beginning of the 20*th* century, there have been numerous efforts to work out alternative cosmologies, mostly based on the hypothesis that the Universe in the large can be described by a homogeneous solution of Einstein's equations. Although there has been much effort to reconcile these alternative models with observational evidence at the time—some models needed efforts to reconcile their hypotheses with Einstein's laws of gravitation—numerous such models have been abandoned (for some overview articles, see e.g. [3, 11, 13, 18]).

II. INTRODUCTION

Contrary to the alternative cosmological thoughts mentioned above, we believe that the *Space Production Model* (SPM) we introduce here invokes an interesting way of thinking about the nature of matter. SPM exploits the largely phenomenological treatment of the energy-momentum tensor in general relativity. In cosmology and in other fields, the interpretation of Einstein's equations with regard to what is geometrical in nature and what is due to intrinsic properties of the sources, enjoys considerable flexibility. We are reminded of the geometrical freedom of the cosmological constant that is interpreted as a source of 'dark energy'. Also, the dynamical aspects of high-density objects that seem to require a component dubbed 'dark matter' that could just as well be, at least partly, a result of spatial curvature on the geometrical side of Einstein's equations. This is neglected in the standard 'concordance model' with everywhere flat space sections.

Assigning to matter the intrinsic property of space emission is on the one hand phenomenologically realized in terms of an equation of state in an otherwise standard cosmological setting, but on the other hand it allows for another way of thinking about repulsive effects. The underlying microscopic mechanism has to be left open, similar to the explanation of 'dark energy' through e.g. the vacuum energy of space (see, however, [2]).

This paper is structured as follows. In Section III we explain the 'Space Production Model' and derive a matter model that is inspired by these thoughts. The defining equations are explicitly discussed in the framework of homogeneous-isotropic universe models that are sourced, and hence thought to be dominated by this matter model. We here advance an equation of state that results from an equilibrium between the postulated inherent property of matter to emit space (i.e. to act repulsively), and the gravitational attraction of matter. We discuss the resulting solution and a mapping of the SPM matter model to a minimally coupled scalar field in virial equilibrium. In section IV we put SPM into the context of inhomogeneous cosmologies, and in section V we summarize the properties of the proposed model.

III. HOMOGENEOUS COSMOLOGY

We will discuss the SPM proposal in terms of a matter model as an energy-momentum source for Einstein's equations,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \ , \tag{1}$$

with the Lorentzian 4−metric components $g_{\mu\nu}$, the 4−Ricci tensor components, $R_{\mu\nu}$, and its trace, the 4−scalar curvature R, the cosmological constant Λ, and a perfect-fluid form of the energy-momentum tensor, $T_{\mu\nu} = \mathrm{diag}(-\varepsilon, p, p, p)$, with the energy density ε and an isotropic pressure p.[1] We will concentrate on spatial properties, which implies that we assume a foliation of spacetime. In the following discussion we first specify our considerations to homogeneous-isotropic solutions of Einstein's equations, and we will employ a flow-orthogonal 3 + 1-foliation that requires the matter fluid to be irrotational.

A. The Friedmannian framework

We assume the Einstein equations to hold and specify them to locally isotropic models that obey the well-known Friedmann equations, consisting of an *expansion law*, that is the temporal change of a scale-factor $a(t)$ of the universe model, and an *acceleration law*, that is the temporal change of the expansion:

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G \epsilon_h}{3} + \frac{\Lambda}{3} - \frac{k}{a^2} \ ; \\ \left(\frac{\ddot{a}}{a}\right) &= -\frac{4\pi G(\epsilon_h + 3p_h)}{3} + \frac{\Lambda}{3} \ ; \\ \dot{\epsilon}_h + 3\left(\frac{\dot{a}}{a}\right)(\epsilon_h + p_h) &= 0 \ . \end{aligned} \tag{2}$$

In the first equation, $\dot{a}/a = H(t)$ is the relative rate of change of the scale factor $a(t)$, i.e. the expansion, often denoted by the Hubble function $H(t)$, ϵ_h is the energy density of the sources, i.e. the energy per unit volume, and p_h their pressure (where the index h stands for homogeneous); k is a constant that describes a homogeneous curvature of space. In the second equation we can observe that the cosmological constant, if it is positive, can lead to a positive second time-derivative of the scale factor, dubbed 'acceleration' of the universe model, and so can counteract gravitation. The third equation describes energy-momentum conservation. The three equations are connected in the sense that, if the third equation holds, then the second equation is just the time-derivative of the first. Note that the set of equations (2) is not closed until we specify an equation of state that relates p_h with ϵ_h.

The parameters of the model are usually written by dividing the expansion law above (the first equation of the set (2)) by the square of the Hubble function H^2. Then, one obtains a sum of three cosmological parameters:

$$\Omega_m + \Omega_k + \Omega_\Lambda = 1 \ , \tag{3}$$

with the definitions:

$$\Omega_m := \frac{8\pi G \epsilon_h}{3H^2} \quad ; \quad \Omega_k := \frac{-k}{a^2 H^2} \quad ; \quad \Omega_\Lambda := \frac{\Lambda}{3H^2} \ . \tag{4}$$

We, henceforth, omit the cosmological constant, $\Lambda = 0$.

B. The matter model of SPM

We base the construction of a matter model on the paradigmatic thought that 'matter inherits the permanent property of emitting space'. We may think of two components, whose energy densities are associated to 'matter' and 'space', respectively. A change in 'volume' as the geometric representative of 'space' is thus tightly linked to and seen as an intrinsic property of the matter model.

We furthermore invoke, to simplify the realization of such a matter model and as already stated, the energy-momentum tensor to be of the form of a perfect fluid with energy density and isotropic pressure. Having in mind that the presence of matter

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[1] The coupling constant is $\kappa := 8\pi G/c^4$. Units are $c = 1$, but we often reinsert the speed of light; the signature of the Lorentzian 4−metric is $(-, +, +, +)$.

comes along with the production of volume, we may invoke an illustration within the homogeneous point of view: distributing elements of matter evenly throughout space and assuming that they all exert jointly a uniform negative pressure that tends to push the elements apart (and that represents the intrinsic property of space emission), we are led to think that such a matter model tends to an equilibrium state of totally zero gravitational acceleration, balancing out the intrinsic properties of matter (gravitational attraction and repulsion). This state of zero acceleration in the homogeneous models is mirrored by the stationarity of the scale factor, i.e. the vanishing of the source $\epsilon_h + 3p_h = 0$ (for $\Lambda = 0$).

We note that this matter model is difficult to realize within a Newtonian picture, since pressure is not self-gravitating. The Newtonian analogy is possible, if we invoke Tolman's observation [23] of 'active mass': a clearcut line of arguments is given in [10], where the effective Newtonian source equation for the gravitational field strength $\mathbf{g}_h$ is given by:

$$\nabla \cdot \mathbf{g}_h = \epsilon_h + \frac{3}{c^2} p_h \ . \tag{5}$$

Since $\mathbf{g}_h = (\ddot{a}/a)\mathbf{x}$ in the homogeneous models, we get the same result of stationarity of the scale factor, $\ddot{a} = 0$.

The matter model of SPM thus assumes, within the perfect fluid approximation, an equilibrium equation of state (setting again $c = 1$):

$$p_h = -\frac{1}{3}\epsilon_h \ . \tag{6}$$

A more general realization of a matter model would transform a component with energy density ϵ_h^m into a component with energy density ϵ_h^e, and this would obey the, in general time-dependent, ansatz for the corresponding homogeneous sources:

$$\epsilon_h = \lambda \epsilon_h^m + (1-\lambda)\epsilon_h^e \ , \tag{7}$$

where the function $\lambda(t)$ would (phenomenologically) determine the transition. Similar transitions or interactions between dark matter and dark energy have been considered, see e.g. [1], [15], compare also suggestions such as in [12]. SPM also includes the assumption that both 'matter' and 'space' are two manifestations of a single entity inheriting an associated volume production. Literally, we will not consider a transformation, but instead a permanent constant fraction between the two within a two-component view, and we write:

$$\epsilon_h = \epsilon_h^m + \epsilon_h^e \ ; \quad \epsilon_h^e = \mu \epsilon_h^m \ , \tag{8}$$

with a constant fraction μ between the two energy densities. This should not be viewed as a simplification, rather it is a key element of the paradigm. Accordingly, the total pressure is split into $p = p_h^m + p_h^e$ with *a priori* individual equations of state for the two components, $p_h^m = \alpha \epsilon_h^m$ and $p_h^e = \beta \epsilon_h^e$. Hence, according to (6) and (8), $\epsilon_h^m(1 + \mu + 3\alpha + 3\beta\mu) = 0$, which enjoys some freedom in the properties that we wish to assign to the individual components. We will later invoke a fluid model for a scalar field that suggests $\alpha = 1$ ('stiff equation of state') and $\beta = -1$ ('dark energy equation of state') implying $\mu = 2$. If we invoke a dust equation of state for matter, $\epsilon_h^m = \varrho c^2$, i.e. $p_h^m = 0$, then $\alpha = 0, \beta = -1$ leads to $\mu = 1/2$, or alternatively keeping $\mu = 2$ implies $\beta = -1/2$. The first suggestion is favoured based on observations: in terms of the energy balance within the Friedmannian framework (for reasons of simplifying the model, with no curvature and no cosmological constant), we have:

$$\Omega_m + \Omega_e = 1 \ ; \quad \Omega_e = \mu\, \Omega_m \ , \tag{9}$$

with a common Hubble expansion $H(t)$, where $\Omega_m := 8\pi G \epsilon_h^m / 3H^2$ and $\Omega_e := 8\pi G \epsilon_h^e / 3H^2$. Specifying the constant fraction to be $\mu = 2$, we have $\epsilon_h^e = 2\epsilon_h^m$. This choice can be considered to comply, if normalized to the standard model at present time, with a total matter density parameter of $33,3\%$ and a dark energy density parameter of $66,6\%$ (which is only slightly off the currently accepted best-fit values to observations [20]), but fits exactly the values quoted in the recent analysis [4].

We summarize the elements underlying the SPM model:

i. The energy-momentum tensor is modeled by a perfect fluid source with a gravitational equilibrium equation of state.

ii. The energy density of the source can be thought of as being made up of a constant fraction between a component that models matter energy density, obeying a 'stiff' equation of state in the scalar field model, and a component that models energy density associated with volume production, with a 'dark energy' equation of state in the scalar field model [17].[2]

iii. In the homogeneous-isotropic framework this amounts to the energy density $\epsilon_h(t) = \epsilon_h^m(t) + \epsilon_h^e(t)$, $\epsilon_h^e = \mu \epsilon_h^m$, with a constant μ that is determined in the scalar field model through the equilibrium equation of state (6): $\mu = 2$.

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[2] For details on the scalar field properties, see section III D.

We now look at the conservation equation (the last equation of the set (2)), and write the total energy density as $\epsilon_h = \epsilon_h^m + \epsilon_h^e = 3\epsilon_h^m$, with total pressure $p_h = p_h^m + p_h^e = -\epsilon_h^m$, where we took $\mu = 2$. Using the equations of state and the assumption above, the conservation equation and its integral becomes, e.g. for the m-component:

$$\dot{\epsilon}_h^m + 2H\epsilon_h^m = 0 \quad ; \quad \epsilon_h^m = \frac{\epsilon_h^m(t_i)}{a^2} \, . \tag{10}$$

The solution for the Friedmannian scale factor follows from the expansion law for the matter component,

$$H^2 = 8\pi G \frac{\epsilon_h^m(t_i)}{a^2} \, , \tag{11}$$

to yield the solution

$$a(t) = a(t_i) \pm \sqrt{8\pi G\epsilon_h^m(t_i)}\, (t - t_i) \, , \tag{12}$$

which we write for $a(t_i) = \sqrt{8\pi G\epsilon_h^m(t_i)t_i^2} \equiv 1$ as follows:

$$a(t) = \pm \left(\frac{t}{t_i} \right) \, , \tag{13}$$

i.e. the scale factor behavior of this model is the same as a stationary solution with $\ddot{a} = 0$.[3] This can be readily confirmed with regard to (2) since $\epsilon_h + 3p_h = 0$.

Introducing the (positive) *Hubble length* (reintroducing the speed of light here), we obtain (with the initial time t_i normalized as above):

$$L_h := c/H(t) = \frac{ct_i}{\sqrt{8\pi G\epsilon_h^m(t_i)}}\, a(t) = c\, t \, , \tag{14}$$

i.e., we have that it increases along the (here by assumption flat) light cone.

For the flat universe model considered here we have a simple geometry of an Euclidean sphere surrounding any observer with proper radius R, surface area $A = 4\pi R^2$ and enclosed volume $V = 4/3\pi R^3$. Inserting the Hubble length, we obtain a relation of the volume to surface fraction of a Hubble sphere to the sources:

$$\frac{V_h}{L_h A_h} = \frac{1}{3} = -\frac{p_h}{\epsilon_h} \, . \tag{15}$$

C. Considerations on the equations of state

The missing piece for our intuition is to explain the constant fraction of 2 between the two energy density components. We can work in the Euclidean case and think of the geometric situation of pressure exerted on a surface area $A_h = 4\pi R^2$ of a Euclidean ball with radius R and volume $V_h = 4\pi R^3/3$. Let us distinguish the interior energy density within the ball, $\epsilon_h^{\rm int}$, and the external energy density, $\epsilon_h^{\rm ext}$. Both densities are equal in the homogeneous situation, but we redistribute the homogeneous energy density inside the ball and put all total energy $\epsilon_h^{\rm int} V_h$ onto the surface of the bubble with surface tension γ. Note that this does not change the expansion law of the bubble according to Newton's iron sphere theorem that also holds for a general-relativistic spherically symmetric redistribution of energy density for spatially vanishing Ricci tensor, as proved in [5]. Hence, we can compute $\gamma = (V_h/A_h)\epsilon_h^{\rm int} = \epsilon_h^{\rm int} R/3$. Since with this redistribution we have a transition from a vacuum bubble to the outside homogeneous energy density $\epsilon_h^{\rm ext}$, we invoke the law of Young-Laplace to compute the pressure difference due to the jump across the bubble's surface. The Laplace pressure is given by $p_h^{\rm int} - p_h^{\rm ext} = 2\gamma/R = 2/3\epsilon_h^{\rm int}$. Thus, the internal pressure is larger than the external one leading to expansion, and the pressure p_h at the interface of the bubble is calculated to be $-p_h^{\rm int}\mathbf{e}_R = -p_h \mathbf{e}_R$ and $p_h^{\rm ext}\mathbf{e}_R = p_h \mathbf{e}_R$, with the unit normal to the bubble, $\mathbf{e}_R$, pointing inside towards the higher pressure, i.e. $-2p_h = (2/3)\epsilon_h$. The equation of state follows to be $p_h = -1/3\epsilon_h$ in both phases (note that $\epsilon_h^{\rm int} = \epsilon_h^{\rm ext} = \epsilon_h$). The equilibrium condition thus forces the total pressure of matter and space emission to obey the equation of state of the stationary state.

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[3] This behaviour is also shared by the Milne model of FLRW cosmology with negative constant curvature and vanishing cosmological constant.

The total equation of state, $p_h = -1/3\epsilon_h$ is the opposite to the equation of state for a trace-free energy-momentum tensor, i.e. radiation with $p_\gamma = 1/3\epsilon_\gamma$. The trace of the energy-momentum tensor is commonly called 'gravitational mass', which amounts to $-2\epsilon = 6p$ in our case and to 0 in the radiation case. It is interesting to put into perspective the equations of state in the two-component view. Individually, say in the case of absence of one of the components, the mass obeys a 'stiff' equation of state with $p_h^m = \epsilon_h^m$, so that the conservation equation for vanishing second component returns:

$$\dot{\epsilon}_h^m + 6\left(\frac{\dot{a}}{a}\right)\epsilon_h^m \;=\; 0 \;\; , \tag{16}$$

i.e., the energy density of mass decays in proportion to the square of the volume. For the second component with 'dark energy' equation of state, $p_h^e = -\epsilon_h^e$, for vanishing first component, we would have:

$$\dot{\epsilon}_h^e \;=\; 0 \;\; , \tag{17}$$

i.e., the energy density is a constant in time, which represents a fluid model of the cosmological constant.

Hence, the tight coupling of the two components changes the behavior of the total system drastically, leaving a single entity where both energy densities evolve at the same rate.

Let us compare the scale factor evolution in this model (a) with the evolution in a pure matter model (b), and the standard concordance or ΛCDM model (CDM for Cold Dark Matter) (c):

$$a(t) = \sqrt{\frac{8\pi G\epsilon_h(t_i)t_i^2}{3}}\left(\frac{t}{t_i}\right) = (H_i t_i)\left(\frac{t}{t_i}\right) = \left(\frac{t}{t_i}\right) \quad (a)\;; \tag{18}$$

$$a(t) = (3/2H_i t_i)^{2/3}\left(\frac{t}{t_i}\right)^{2/3} \quad (b)\;; \tag{19}$$

$$a(t) = \left(\frac{\Omega_h^m(t_i)}{\Omega_\Lambda(t_i)}\right)^{1/3}\sinh^{2/3}\left(\frac{3H_i t_i}{2}\sqrt{\Omega_\Lambda(t_i)}\left(\frac{t}{t_i}\right)\right) \quad (c)\;. \tag{20}$$

For the illustration below, Figure 1, we normalize the scale factor such that the numerical value of the constant $H_i t_i$ is set for each model (knowing the Hubble function). For model (a) we have $H_i t_i = 1$, for model (b) $H_i t_i = 2/3$, and for model (c) $H_i t_i = 2/3\sqrt{\Omega_\Lambda}$ (For the ratio we have $\Omega_m/\Omega_\Lambda = 1/2$).

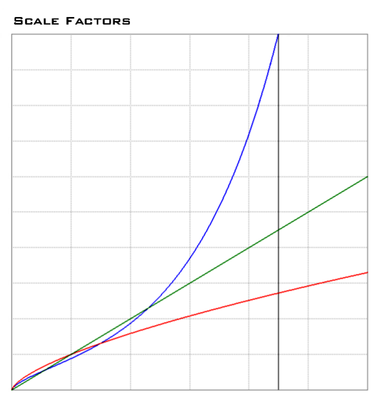

FIG. 1. The scale factors for the three models as a function of $x = (t/t_i)$: the SPM model (a) in green (middle graph), the pure matter model (b) in red (lower graph), and the standard model with cosmological constant (c) in blue (upper graph). For all models the initial stages of inflation and radiation-domination are not covered.

We see that we expect more present-day volume $V = V(t_i)a^3$ in the standard model compared with the volume of the SPM model, both of which produce more space than in a model with pure matter content.

It is important to remark here that we have made idealizing assumptions for the base-parameters of the model. We did not, though, make any effort to optimize the model in order to comply with observational cornerstones, and yet the constant fraction 2 between the energy densities complies with present-day observations. However, we notice that our realization of SPM is quite robust: introducing a non-vanishing negative curvature would only increase the effective volume production in the matter-dominated model, but would not change the expansion history in the SPM case. This can be easily seen by comparing the curvature behaviour in a Friedmannian model which has the same dependence $\propto a^{-2}$ as the energy density in the SPM model, so that including a curvature component would only change the initial values where the energy density would share the curvature contribution. However, including inhomogeneity effects, see section IV, will lead to the emergence of a negative average curvature, i.e. an additional dark energy-like effect that increases the volume production [6]. Deviations from observation could then be attributed to the fact that we are attempting to make conclusions about the inhomogeneous Universe through a homogeneous model that does not correspond to its average properties. In other words, the question whether the SPM model is compatible with observations must be considered and answered within the inhomogeneous framework.

We are now going to interpret the above solutions within the SPM picture by defining a 'total matter energy' E_h^m and a 'total space energy' E_h^e within a homogeneous volume V_h:

$$E_h^m := \epsilon_h^m V_h \quad ; \quad E_h^e := \epsilon_h^e V_h \ . \tag{21}$$

We obtain (with $E_h^m(t_i) = \epsilon_h^m(t_i) V_h(t_i)$ and the same for the $e-$component):

$$E_h^m = E_h^m(t_i)\, a(t) \quad ; \quad E_h^e = E_h^e(t_i)\, a(t) \ , \tag{22}$$

confirming what we expect in the SPM picture: matter energy is permanently converted into space energy at a rate that is exactly equal to the Hubble rate $H(t) = \dot{a}(t)/a(t)$:

$$\frac{\dot{E}_h^m}{E_h^m} = H(t) = \frac{\dot{E}_h^e}{E_h^e} \ . \tag{23}$$

Equation (23) is the fundamental equation of SPM for homogeneous cosmology.

D. Mapping SPM to a minimally coupled scalar field

A natural representative of the SPM matter model is to look at the energy density of 'only matter' as to be in the form of a fluid model for a free scalar field component—obeying a so-called 'stiff' equation of state, $p_h^m = \epsilon_h^m$, i.e. an oscillatory character of matter represented by a fluid model. The energy density of 'only space' is assumed to be directly associated to volume production—obeying a so-called 'dark energy' equation of state, $p_h^e := -\epsilon_h$, i.e. its energy density manifests itself as a negative pressure that augments the volume of space. These assumptions directly motivate invoking a scalar field nature of matter while, within the SPM interpretation, 'both' matter and space are tightly connected. The interesting aspect of the above analogy is that we can represent the SPM matter model in terms of a scalar field that evolves in a potential. Explicitly, we can write in terms of a scalar field $\Phi(t)$:

$$\epsilon_h^m = \frac{1}{2}\dot{\Phi}^2 \quad ; \quad p_h^m = \frac{1}{2}\dot{\Phi}^2 \ , \tag{24}$$

and

$$\epsilon_h^e = V(\Phi) \quad ; \quad p_h^e = -V(\Phi) \ . \tag{25}$$

We see that the equation of state for 'matter' is $p_h^m = \epsilon_h^m$, and that for 'space' is $p_h^e = -\epsilon_h^e$ in accordance with our previous two-component view. The total density and pressure are thus written:

$$\epsilon_h = \frac{1}{2}\dot{\Phi}^2 + V(\Phi) \quad ; \quad p_h = \frac{1}{2}\dot{\Phi}^2 - V(\Phi) \ , \tag{26}$$

with the total equation of state $p_h = \epsilon_h - 2V(\Phi)$. Using the assumption $\epsilon_h^e = 2\epsilon_h^m$, we get the relation $V(\Phi) = \dot{\Phi}^2$, or directly for the equation of state, $p_h = \epsilon_h - 4\epsilon_h^m = \epsilon_h - 4/3\epsilon_h = -1/3\epsilon_h$. We remark that the relation $V(\Phi) = \dot{\Phi}^2$ is reminiscent of the virial equilibrium condition, $\epsilon_{\text{kin}} + 2\epsilon_{\text{pot}} = 0$, $(\epsilon_{\text{pot}} = -\epsilon_h^e)$.

As is well-known, by inserting (26) into the conservation equation, we obtain the Klein-Gordon equation: we calculate $\dot{\epsilon}_h$ from ϵ_h in (26), $\dot{\epsilon}_h = \dot{\Phi}(\ddot{\Phi} + V'(\Phi))$, where V' means derivative of the potential with respect to Φ; we obtain for $\dot{\Phi} \neq 0$:

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0 \ . \tag{27}$$

This equation describes the dynamics of the scalar field in the given potential. The formal integral of the Klein-Gordon equation reads:

$$\epsilon_h = \epsilon_h(t_i) \exp\left(-6 \int \left[1 - \frac{2V}{\dot{\Phi}^2 + 2V}\right] \frac{da}{a}\right) \ . \tag{28}$$

The potential itself is given in terms of Φ, but with our assumption it is determined that $V(\Phi) = \dot{\Phi}^2$. The integral (28) then simplifies, and we confirm the law $\epsilon_h = \epsilon_h(t_i)a^{-2}$. Since we have the solution for $\epsilon_h^e = V(\Phi)$, which is $\epsilon_h^e = \frac{2}{3}\epsilon_h(t_i)/a^2$, we can solve the equation for the virial equilibrium to obtain $V(\Phi)$; by changing the variable t to a we first have:

$$\left(\frac{d\Phi}{dt}\right)^2 = \left(\frac{d\Phi}{da}\right)^2 \dot{a}^2 = \frac{C}{a^2} \quad ; \quad C := \frac{2\epsilon_h(t_i)}{3} , \tag{29}$$

and with the Friedmann equation $H^2 = \frac{8\pi G}{3}\frac{3}{2}\epsilon_h^e = 4\pi G\epsilon_h^e$ and $V = \epsilon_h^e = C/a^2$, we get:

$$\left(\frac{d\Phi}{da}\right)^2 = \frac{1}{4\pi G a^2} . \tag{30}$$

Solving for $\Phi(a)$ we find

$$\Phi = \pm \ln(\sqrt{4\pi G}\, a) , \tag{31}$$

and with $V(a) = C/a^2$ we can express a in terms of V to be $a = \sqrt{C/V}$, where only the positive sign is taken since we assume $V > 0$. Inserting this expression into $\Phi(a)$ above we obtain two solutions for Φ in terms of V:

$$\Phi_1 = \ln\left[\sqrt{4\pi G}\sqrt{\frac{C}{V}}\right] \quad ; \quad \Phi_2 = \ln\left[\sqrt{4\pi G}\sqrt{\frac{C}{V}}\right]^{-1} . \tag{32}$$

Inverting these solutions provides two solutions for the potential, the first for negative Φ and the second for postive Φ:

$$V_1 = 4\pi G C \left(e^{\Phi}\right)^{-2} \quad ; \quad V_2 = 4\pi G C \left(e^{\Phi}\right)^{2} . \tag{33}$$

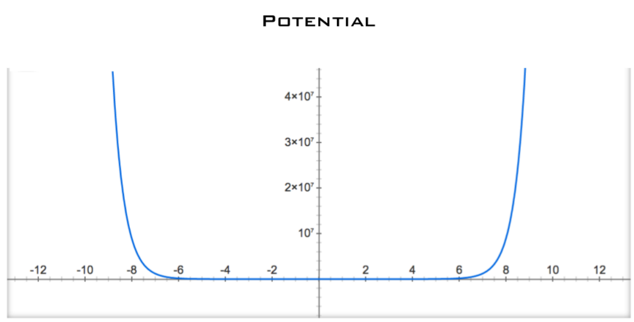

FIG. 2. The two solutions for the potential added to a total potential $V(\Phi)$.

A scalar field source can produce inflation if the kinetic term is subdominant to the potential term. In the case of a virial equilibrium both are of the same order and the situation is different. At equilibrium, $\dot{\Phi} = 0$, the potential has a minimum, $V(\Phi) = 0$. The state may oscillate around this minimum up to the "boundaries" in Figure 2, but the state is determined by the value of the total density $\epsilon_h = 1/2\dot{\Phi}^2 + V = 3/2V$. This suggests that the SPM state may arise from a pre-equilibrium, dynamical phase of cosmic evolution that is best described as an exit state after an inflationary phase.

IV. INHOMOGENEOUS COSMOLOGY

Our considerations of homogeneous cosmologies gave us an interpretation of the phenomenology of the energy-momentum tensor in general relativity: we assign to matter the simultaneous properties of 'mass' and 'space emission'. Both are synonymous within the hypothesized SPM paradigm, mass being a function of space emission per unit time. We also learned that the tight link of both 'mass' and 'space' manifestations of a single entity allowed us to map the energy-momentum tensor to a minimally coupled scalar field with its 'energies' in virial equilibrium as a conserved property in time. We may say that SPM advances a scalar field model of matter with the non-dynamical ingredient of a preserved interaction between kinetic and potential energy densities. We speculated here that this 'equilibrium' could be the result of a dynamical process at the exit from an epoch of inflation. Overall, a simplified description of the Higgs field stands out as a candidate for realizing such a scenario [8, 16, 21].

The Friedmannian kinematics of this (flat) universe model is the simplest realization of a scale factor that traces the (flat) light cone. Arguing from an observational cosmology perspective, this universe model appears to lie in between a matter-dominated evolution (only 'mass') and a dark energy-dominated evolution (only 'space'), see figure 1, where we used the wording of the standard cosmological model that in its evolution interpolates in time from the matter-dominated era to the dark energy-dominated era. As already discussed, it is not the point of the present analysis to optimize this universe model to comply best with current observational constraints. We could do so by invoking a nonvanishing cosmological constant also in this model. Or, as we showed, allowing for a negative constant curvature in this model would not change the behaviour towards the dark energy-dominated era. A further important ingredient comes from inhomogeneous cosmologies that would help in this optimization process.

Inhomogeneous cosmologies will reveal important consequences of SPM that go beyond a mere effective description in the world of homogeneous cosmologies. It will be interesting to ask whether manifestations like 'motion', 'inertia', 'gravitational acceleration' or 'curvature' would also allow for different interpretations.

A. Newtonian thoughts about SPM in inhomogeneous models

Recalling our Newtonian motivation leading to stationarity in the homogeneous models, Eq. (5), we may repeat this consideration in the inhomogeneous case. First, the field equation (5) is also valid for inhomogeneous fields,

$$\nabla \cdot \mathbf{g} = \epsilon + \frac{3}{c^2} p \,, \tag{34}$$

as well as the equation of state (setting again $c = 1$),

$$p = -\frac{1}{3}\epsilon \,. \tag{35}$$

The stationarity condition $\mathrm{d}\mathbf{v}_h/\mathrm{d}t = \mathbf{g}_h = (\ddot{a}/a)\mathbf{x} = \mathbf{0}$ for a homogeneous velocity field $\mathbf{v}_h$ translates into a stationarity condition of the inhomogeneous velocity field $\mathbf{v}$, now invoking pressure gradients: Euler's equation, written in an inertial coordinate system, provides for a stationary velocity field an equilibrium relation between the gravitational acceleration and the pressure gradient, or energy density gradient, respectively:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{v} = \mathbf{0} \quad \Rightarrow \quad \mathbf{g} = \frac{1}{\epsilon}\nabla p = -\frac{1}{3}\frac{\nabla p}{p} = -\frac{1}{3}\frac{\nabla \epsilon}{\epsilon} \,, \tag{36}$$

where in the last two equations we have inserted (35). Since energy density is positive and pressure negative, we conclude that both for negative (positive) energy density gradient and negative (positive) pressure gradient we have positive (negative) gravitational acceleration in this stationary situation. Thus, the same process is occurring in both a homogeneous and an inhomogeneous setting: in a homogeneous environment there is no net gravitational force. However, in an inhomogeneous setting there is motion and acceleration.

B. Interpretation of space emission patterns

The interpretation within SPM associates to the negative pressure the phenomenology of 'emission of space'. According to this, a fluid element (henceforth called 'object') can be said to be moving if it has a non-uniform space emission pattern. SPM states that gravitational acceleration can be defined as any change in an object's space emission pattern and is a consequence of the partial inhibition of an object's space emission due to negative pressure caused by the space emission of surrounding matter. If the pressure around an object is not uniform due to matter inhomogeneities surrounding the object, the object will emit more space in the direction of the least pressure resulting in an increase in local expansion (acceleration) in the direction of the smaller pressure/less matter. If the pressure around an object is uniform, it is still partially inhibiting the object's emission of space but in a uniform manner. In this case, there is no change in the object's movement. However, if the surrounding uniform pressure is equal to the average pressure of the Universe, the result is more space being emitted in all directions. This represents global expansion. Based on this, effective space emission represents what in the standard model would be dark energy, but here it is not an independent component. Pressure/acceleration resulting from this space emission represents energy associated with the overall gravitational manifestations of matter. If there were only mass, the resulting gravitational effect would lead to deceleration, while SPM leads to an exact compensation of this gravitational effect through volume production.

V. DISCUSSION

In this paper we have put forward a paradigm that allows us to think differently about the nature of matter and its relation to volume expansion, arguing with respect to the interpretation freedom allowed for the energy-momentum source in Einstein's equations. SPM assigns to matter the permanent property of space emission. We have introduced this concept on the level of an equation of state that governs the matter model. We abbreviated the actual process of volume production through the presence of matter in terms of an already established energetic equilibrium between a 'stiff' component (matter) and a 'dark energy-like' component (volume production). This particular choice was motivated by equilibrium considerations resulting from the paradigm. The so-defined equilibrium is best illustrated in the homogeneous situation, but as we showed, can be generalized to the inhomogeneous situation. The phenomenological realization of this matter model is furnished through the fluid analogy of a scalar field with an equation of state in 'virial equilibrium'.

Conservatively speaking, we have in this paper analyzed a scalar field fluid model in 'virial equilibrium' that features interesting aspects in between a matter-dominated and a dark energy-dominated cosmology. Although, taken at face value, the resulting expansion law falls short of matching observational cornerstones (it does fit with the energy content, but not with the model-dependent age of the Universe), we argued that the corresponding building stones can be translated to an averaged inhomogeneous cosmology that would allow us to optimize the SPM model to achieve a better match to observations by including a cosmological constant or cosmological backreaction. This we did not attempt here.

The presented framework relies on an already established equilibrium between 'presence of matter' and 'volume production' which hints to the possibility of a dynamical process that leads to this equilibrium. Hence a link to dynamical pre-phases such as an inflationary phase is suggested. The natural candidate for this scalar field may derive from the Higgs boson.

We do not invoke or discuss microscopic thoughts about such a property of matter. This would be highly speculative, and comparative to efforts of explaining 'dark energy' from the vacuum energy of space. Possible routes of understanding microscopic properties will immediately lead to questions about those (quantum or classical) properties of matter that would manifest themselves as 'space-emitting', *cf.* [14] and [22]. We leave it to the reader to be inspired by the phenomenological scenario that we presented, being guided by the new interpretation possibilities of the *Space Production Model* without contradicting fundamental concepts of general relativity.

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[1] O. Bertolami, P. Carrilho and J. Páramos, Two-scalar-field model for the interaction of dark energy and dark matter. *Phys. Rev. D* **86** 103522 (2012) [arXiv:1206.2589]

[2] E. Bianchi and C. Rovelli, Why all these prejudices against a constant? [arXiv:1002.3966]

[3] H. Bondi and T. Gold, The Steady-State Theory of the Expanding Universe. *Mon. Not. Roy. Astron. Soc.* **108** 252 (1948)

[4] D. Brout *et al.*, The Pantheon+ Analysis: Cosmological Constraints. *The Astrophys. J.* **938** 110 (2022) [arXiv:2202.04077]

[5] T. Buchert, Toward physical cosmology: focus on inhomogeneous geometry and its non-perturbative effects. *Class. Quant. Grav.* **28** 164007 (2011) [arXiv:1103.2016]

[6] T. Buchert, Is Dark Energy Simulated by Structure Formation in the Universe? In *2nd World Summit on Exploring the Dark Side of the Universe, 25-29 June 2018, University of Antilles, Guadeloupe* PoS(EDSU2018)**038** (2019) [arXiv:1810.09188]

[7] T. Buchert, A. Coley, H. Kleinert, B.F. Roukema and D.L. Wiltshire, Observational challenges for the standard FLRW model. *Int. J. of Mod. Phys. D* **25** 1630007 (2016) [arXiv:1512.03313]

[8] C. Joana, Gravitational dynamics in Higgs inflation: Preinflation and preheating with an auxiliary field. *Phys. Rev. D* **106** 023504 (2022) [arXiv:2202.07604]

[9] E. Di Valentino, O. Mena, S. Pan, L. Visinelli, W. Yang, A. Melchiorri, D.F. Mota, A.G. Riess and J. Silk, In the realm of the Hubble tension—a review of solutions. *Class. Quant. Grav.* **38** 153001 (2021) [arXiv:2103.01183]

[10] J. Ehlers, I. Ozsvath and E.L. Schücking, Active mass under pressure. *Am. J. Phys.* 74, 607 (2006) [arXiv:gr-qc/0505040]

[11] R.G. Giovanelli, A fluctuation theory of cosmology. *Mon. Not. Roy. Astron. Soc.* **127** 461 (1964)

[12] M. Gogberashvili and A. Sakharov, Supplying dark energy from scalar field dark matter. *Int. J. Mod. Phys. D* **27** 1850100 (2018) [arXiv:1702.05757]

[13] H. Kragh, Quasi-Steady-State and related cosmological models: a historical review. [arXiv:1201.3449] (2012)

[14] L.M. Krauss and J.B. Dent, Higgs Seesaw Mechanism as a Source for Dark Energy. *Phys. Rev. Lett.* **111** 061802 (2013) [arXiv:1306.3239]

[15] W. Khyllep, J. Dutta, S. Basilakos and E.N. Saridakis, Background evolution and growth of structures in interacting dark energy through dynamical system analysis. *Phys. Rev. D* **105** 043511 (2022) [arXiv:2111.01268]

[16] Yu.G. Ignat'ev and D.Yu Ignat'ev, Short-wave approximation for Macroscopic Cosmology with Higgs scalar field. *Gravit. and Cosmol.* **26** 249 (2020) [arXiv:2007.04392]

[17] M.S. Madsen, A note on the equation of state of a scalar field. *Astrophys. & Space Sci.* **113** 205 (1985)

[18] W.H. McCrea, Relativity theory and the creation of matter. *Proc. Roy. Soc. A* **206** 562–575 (1951)

[19] L. Perivolaropoulos and F. Skara, Challenges for ΛCDM: An update. *New Astron. Rev.* **95** 101659 (2022) [arXiv:2105.05208]

[20] Planck Collaboration: Planck 2018 results. VI. Cosmological parameters. *Astron. Astrophys.* **641** A6 (2020) [arXiv:1807.06209]
[21] J. Rubio, Higgs Inflation. *Front. Astron. Space Sci.* **5** 50 (2018) [arXiv:1807.02376]
[22] I. Steinbach, Quantum-Phase-Field Concept of Matter: Emergent Gravity in the Dynamic Universe. *Z. Naturforsch.* **72** 51 (2017) [arXiv:1703.05583]
[23] R.C. Tolman, On the use of the energy-momentum principle in general relativity. *Phys. Rev.* **35** 875 (1930)