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# Dark Matter as Dimensional Condensate

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# Dark Matter as Dimensional Condensate

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## Abstract

*Fractional statistics* (FS) is a generalization of the spin-statistics theorem and mixes bosons and fermions in a non-trivial way. Mixing is controlled by a continuous parameter  $0 \leq q \leq 1$  and the ordinary statistics is recovered in the limit  $q = 1$ . We have argued some time ago that the onset of FS occurs in a spacetime endowed with minimal fractality, whose ground state is the *Cantor Dust*, an early Universe phase created by topological condensation of continuous dimensions. Recent studies on  $q$ -bosons reinforce the hypothesis that Dark Matter is the relic of Cantor Dust left over from the early stages of cosmological evolution. The take-away point of this brief note is the growing support for the minimal fractality of spacetime and its ramifications in foundational physics.

**Key words:** Dark Matter, minimal fractal manifold, Cantor Dust,  $q$ -bosons, fractional statistics, dimensional condensate.

### Cautionary Remark:

*We caution from the outset that the sole intent of this report is to lay the groundwork for further exploration of the topic. Exclusively presented in a draft form, our analysis is far from completion and far from meeting the quality standards of a formal research project. The style and presentation do not comply with traditional academic standards. Independent work is needed to develop, validate, or reject the ideas presented here. Readers unfamiliar with the topic are encouraged to carefully review the enclosed references prior to drawing premature conclusions.*

The *spin-statistics theorem* is a fundamental principle of quantum physics and reflects the contrasting behavior of bosons and fermions in three-dimensional space. There are various extensions of the theorem enabling bosons and fermions to overlap and they are referred to as fractional statistics, anyon statistics and quantum groups [4]. These extensions have found a broad range of applications from deformed algebras of  $q$ -bosons and

$q$ -fermions to non-commutative field theory, cosmic strings, and Black Holes, to fractional quantum Hall effect and anyonic states of matter [1, 4-6].

The algebra of  $q$  - particles is specified by the following set of commutation relationships for the ladder operators  $a, a^\dagger$  and the number operator  $N$  [2]

$$aa^\dagger - q^{\pm 1}a^\dagger a = q^{\pm N} \quad (1)$$

$$[N, a^\dagger] = a^\dagger \quad (2a)$$

$$[N, a] = -a \quad (2b)$$

By (1) and (2), the Fock eigenstates  $|n\rangle$  are built as in

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{[n]!}}|0\rangle, \quad a|0\rangle = 0 \quad (3)$$

where the  $q$ -basic number and factorial are defined as, respectively,

$$[x] = \frac{q^x - q^{-x}}{q - q^{-1}} \quad (4)$$

$$[n]! = [n][n-1]\dots[1] \quad (5)$$

Ordinary numbers  $x$  correspond to the limit  $q \rightarrow 1$ , that is,

$$\lim_{q \rightarrow 1} [x] = x \quad (6)$$

The action of the operators on the state  $|n\rangle$  is given by

$$a^\dagger |n\rangle = [n+1]^{1/2} |n+1\rangle \quad (7a)$$

$$a |n\rangle = [n]^{1/2} |n-1\rangle \quad (7b)$$

$$N |n\rangle = n |n\rangle \quad (7c)$$

The Hamiltonian operator of a  $q$ -deformed harmonic oscillator is shown to take the form

$$H = \frac{\hbar\omega}{2} (a a^\dagger + a^\dagger a) \quad (8a)$$

leading to the following spectrum of eigenvalues on the basis  $|n\rangle$

$$E(n) = \frac{\hbar\omega}{2} ([n] + [n+1]) \quad (8b)$$

A close relationship exists between *fractional differential operators* and  $q$ -deformed algebras [2]. To fix ideas, consider the power function

$$f(x, \alpha) = x^{n\alpha} \quad (9)$$

in which  $\alpha$  is the index of fractional differentiation. Setting

$$\alpha = 1 - \varepsilon, |n, \varepsilon\rangle = f(n, \varepsilon) = x^{n(1-\varepsilon)} \quad (10)$$

yields the following expression of the Caputo fractional derivative of (10)

$$D_x^{1-\varepsilon} |n, \varepsilon\rangle = \frac{\Gamma[1+n(1-\varepsilon)]}{\Gamma[n(1-\varepsilon)+\varepsilon]} x^{\varepsilon-1} |n, \varepsilon\rangle; \quad n > 0 \quad (11)$$

which, in turn, leads to

$$[n]_{1-\varepsilon} |n, \varepsilon\rangle = D_x^{1-\varepsilon} |n, \varepsilon\rangle x^{1-\varepsilon} \quad (12)$$

and

$$\lim_{\varepsilon \rightarrow 0} [n]_{1-\varepsilon} = n \quad (13)$$

It follows from (6) and (13) that the direct identification

$$\boxed{q = 1 - \varepsilon, 0 \leq q \leq 1} \quad (14)$$

connects *fractional statistics* to field theory built on fractional differential operators (called *fractional field theory* [7]). Moreover, *minimal fractal manifold* (MFM) describes a scale-dependent spacetime equipped with low-level

fractality, where the continuous deviation from integer dimensionality assumes the form [Appendix, 13]

$$\varepsilon(\mu) = 4 - d(\mu) = O\left[\frac{m^2(\mu)}{\Lambda_{UV}^2}\right] \ll 1 \quad (15)$$

Here,  $\mu, m, \Lambda_{UV}$  denote the running scale, mass parameter and ultraviolet cutoff, respectively. At the far ultraviolet end of the energy scale  $m = O(\Lambda_{UV})$  both  $q$  and spacetime dimensionality drop to zero, a condition akin to the Planckian regime of *spacetime singularities*.

Remarkably, recent modeling [3] shows that  $q$ -bosons offer an intriguing picture of Dark Matter (DM), as  $q$ -bosons freeze in a condensed phase, regardless of temperature. We have suggested some time ago that the onset of fractional statistics naturally develops on the minimal fractal manifold (MFM), whose ground state is the *Cantor Dust*, an early Universe phase generated by *topological condensation of continuous dimensions*. These findings reinforce the conjecture that DM represents an exotic relic of Cantor Dust left over from the early stages of cosmological evolution [7-9, 12]. It is also instructive to note that the concept of Cantor Dust may enable an unforeseen

unification of DM and Dark Energy [10], as well as a platform for reconciling the particle physics and gravitational interpretations of DM [11].

## **APPENDIX**

Relation (15) reflects a regime that 1) develops somewhere near or far above the Fermi scale of electroweak interactions and 2), it is manifestly *nonintegrable* and prone to *transition to chaos*. It is only natural to posit that the two sides of (15) converge to the same behavior in the low-energy limit of particle physics, in a way consistent with the Decoupling Theorem. In a nutshell, (15) states that the low-energy limit of a fractal spacetime defined by a vanishing dimensional deviation  $\varepsilon(\mu) \rightarrow 0$  corresponds to the continuum spacetime of classical physics ( $\Lambda_{UV} \rightarrow \infty$ ). A relation similar to (15) exists in the  $\Phi^4$  model of Statistical Physics and it corresponds to the *nontrivial fixed point* of the Renormalization Group flow (the so-called Wilson – Fisher point) [13].

## **References**

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