Possible f Quark Model of Tetraquarks and Pentaquarks

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Abstract

Selected tetraquark and pentaquark candidates are compared to meson and baryon states populated by ~2.9 GeV/c^2 f quarks postulated by Chapman. The meson and baryon states incorporating an f quark are modeled using a first-order mass formula based on the methodology developed by Zel'dovich and Sakharov. The resulting states have masses below the experimental tetraquark and pentaquark values. First-order mesons and baryons mass formula results containing an f quark improve as the mass of the postulated quark increases.

1.0 Introduction

Chapman[1,2] presumes the existence of a fourth down-type quark (f) having a mass of ~ 2.9 GeV/c^2. The basis for the proposed f quark is outlined in Ref. 1. Ref. 2 discussed additional arguments for and against the f quark. Chapman[2] also compares calculated f quark meson and baryon states to experimental tetraquark and pentaquark configurations.

A series of publications[3-17] utilized the Zel'dovich and Sakharov[18-19] approach to formulate a first-order mass formula to predict the masses and J$^\pi$ values of tetraquarks[3-14] and pentaquarks[15-17]. The first-order mass formula calculations provided a reasonable representation of the experimental tetraquark and pentaquark states.

This paper uses the approach of Refs. 18-19 and the updated quark masses[3-17,20] to calculate mesons and baryons states based on an f quark as an alternative to the experimental tetraquark and pentaquark states. These states include mesons and baryons containing f and conventional quarks.

2.0 Model Formulation

Zel'dovich and Sakharov[18,19] proposed a semiempirical mass formula that provides a prediction of mesons and baryons masses in terms of effective quark masses. Within this formulation, quark wave functions are assumed to reside in their lowest S state. The meson (m) mass (M) formula of Refs. 18 and 19 is:

$$M_m = \delta_m + m_1 + m_2 + b_m \frac{m_0^2}{m_1 m_2} \sigma_1 \cdot \sigma_2 (1)$$

where m_1 (m_2) are the mass of the first (second) quark comprising the meson, m_0 is the average mass of a first generation quark[20,21], and the σ_i (i = 1 and 2) are the spin vectors for the quarks incorporated into the meson. The parameters δ_m and b_m are 40 MeV and 615 MeV, respectively[19].

The last term in Eq. 1 represents the spin-spin interaction of the quarks and $\sigma_1 \cdot \sigma_2$ is the scalar product of the quark spin vectors. $\sigma_1 \cdot \sigma_2$ has the value -3/4 and +1/4 for pseudoscalar and vector mesons, respectively.

In a similar manner, the baryon (b) mass formula[18,19] is:

$$M_b = \delta_b + m_1 + m_2 + m_3 + Z \quad (2a)$$

$$Z = \frac{b_b}{3}\left[\frac{m_0^2}{m_1 m_2}\sigma_1 \cdot \sigma_2 + \frac{m_0^2}{m_1 m_3}\sigma_1 \cdot \sigma_3 + \frac{m_0^2}{m_2 m_3}\sigma_2 \cdot \sigma_3\right] \quad (2b)$$

where the m_i labels the three baryon quarks (i = 1, 2, and 3) and δ_b and b_b are 230 MeV and 615 MeV, respectively[19]. For a particle with a total baryon spin 1/2, the following prescription is used if the baryon (comprised of three quarks q_1, q_2, and q_3) contains two identical quarks[19] q_2 and q_3

$\boldsymbol{\sigma_2 \cdot \sigma_3 = 1/4}$ (3)

$\boldsymbol{\sigma_1 \cdot \sigma_2 = \sigma_1 \cdot \sigma_3 = -1/2}$ (4)

For completeness, the reader should note that $\sigma_i \cdot \sigma_j$ has the value +1/4 for a J= 3/2 baryon. In addition, these basic $\sigma_i \cdot \sigma_j$ relationships must be modified if the baryon contains three different quarks. The methodology is detailed, and described in Ref. 19.

In formulating the meson and baryon mass formulae, effective quark masses provided by Griffiths[20] are utilized. These effective masses for d, u, s, c, b, and t quarks are 340, 336, 486, 1550, 4730, and 177000 MeV/c^2, respectively. These masses are utilized in Eqs. 1 and 2.

These six quarks are arranged in three generations: [d(-1/3 e), u(+2/3 e)], [s(-1/3 e), c(+2/3 e)], and [b(-1/3e), t(+2/3 e)][21]. The three generations are specified by the square brackets and the quark charges are given within parenthesis in terms of the proton charge e.

The first-order meson model only permits a primitive L = 0 coupling structure between the quarks

$\mathbf{J^\pi_{meson} = J^\pi \text{ (quark 1) x L x } J^\pi \text{ (quark 2)} = 1/2^+ \text{ x 0 x } 1/2^+}$ **(5)**

where L is the angular momentum between the clusters. The allowed meson J^π values are 0^+ for pseudoscalar mesons and 1^+ for vector mesons.

In a similar manner, the first-order baryon model only permits a primitive L = 0 coupling structure between the quarks

J^{π}_{baryon} = J^{π} (quark 1) x L x J^{π} (quark 2) x L x J^{π} (quark 3) (6)

Since L = 0, the quark values utilized in Eq. 6 only permit are J = $1/2^+$ and $3/2^+$ values. In view of the limitations of the first-order mass formula model, comparison to Ref. 2 are limited to 0^+ and 1^+ mesons and $1/2^+$ and $3/2^+$ baryons.

3.0 Results and Discussion

First-order mass formulas are compared to the mesons and baryons containing the postulated f quark[2]. Calculations are provided for f d-bar and d f-bar mesons (Table 1), f u-bar and u f-bar mesons (Table 2), f s-bar and s f-bar mesons (Table 3), f f-bar mesons (Table 4), and the fuu baryon (Table 5). Both pseudoscalar 0^+ and vector meson 1^+ states are presented in Tables 1-4. For baryons, Table 5 lists both $1/2^+$ and $3/2^+$ states. Given the uncertainties in the models, further matching with f meson and baryon states is not addressed. Moreover, the discussion is limited to possible tetraquark and pentaquark states that match the possible J^{π} values derived from the first-order mass formula.

f meson and baryon masses are derived for f quark masses of 2800 MeV/c^2, 2900 MeV/c^2, 3000 MeV/c^2, and 3100 MeV/c^2. These values overlap the f quark mass of 2900 MeV/c^2 utilized in Ref. 2. Given the assumed mass of the f quark and the masses of the u, d, and s quarks, an increase in the f quark mass by 100 MeV/c^2 results in an increase of meson and baryon masses by about 100 MeV/c^2. Since the f quark mass is an estimated value, only general comments related to the first-order mass formula are possible.

3.1 f d-bar and d f-bar Mesons

Chapman proposed a number of f d-bar and d f-bar mesons[2]. Following the previous discussion, the first-order mass formula results are compared to states with J^{π} = 1^+ states (χ_{c1}(3872), Z_c^0(3900), χ_{c1}(4274), and χ_{c1}(4685)) and a J^{π} = 0^+ state (χ_{c0}(4700)) in Table 1. In general, the first-order mass formula results for f d-bar and d f-bar mesons underestimate the experimental multiquark (i.e., tetraquark and pentaquark) results summarized in Table 1. All first-order mass formula results lie below the experimental multiquark values. However, the results improve as the f quark mass is increased.

Table 1

f d-bar and d f-bar Mesons Compared to States with $J^\pi = 1^+$ (χ_{c1}(3872), Z_c^0(3900), χ_{c1}(4274), and χ_{c1}(4685)) and a $J^\pi = 0^+$ (χ_{c0}(4700))[a]

f Quark Mass (GeV/c^2)	Meson J^π	
	0^+	1^+
2.8	3125	3198
2.9	3227	3298
3.0	3328	3397
3.1	3430	3497

[a]Multiquark masses and J^π values are derived from Ref. 2.

3.2 f u-bar and u f-bar Mesons

Table 2 summarizes the results of first-order mass formula calculations for f u-bar and u f-bar mesons. The results are similar to the results of Table 1. First-order mass formula calculations underestimate the masses of $J^\pi = 1^+$ $Z_c^\pm$(3900) and $Z_c^\pm$(4430) states[2]. Model results approach the experimental $Z_c^\pm$(3900) and $Z_c^\pm$(4430) masses as the f quark mass increases.

Table 2

f u-bar and u f-bar Mesons Compared to States with $J^\pi = 1^+$ ($Z_c^\pm$(3900) and $Z_c^\pm$(4430))[a]

f Quark Mass (GeV/c^2)	Meson J^π	
	0^+	1^+
2.8	3120	3195
2.9	3222	3294
3.0	3324	3393
3.1	3425	3493

[a]Multiquark masses and J^π values are derived from Ref. 2.

3.3 f s-bar and s f-bar Mesons

Table 3 provides a summary of first-order mass formula results for f s-bar and s f-bar mesons. The predicted first-order mass formula results are below the values for the $\chi_{c1}(4140)$, X(3960) and $\chi_{c0}(4500)$ states[2]. Consistent with the f d-bar and d f-bar (Table 1) and f u-bar and u f-bar (Table 2), the first-order mass formula underestimates the experimental values summarized in Ref. 2. The model results approach the experimental values as the assumed f quark mass increases.

Table 3

f s-bar and s f-bar Mesons Compared to States with $J^\pi = 1^+$ ($\chi_{c1}(4140)$) and

$J^\pi = 0^+$ (X(3960) and $\chi_{c0}(4500)$)[a]

f Quark Mass (GeV/c^2)	**Meson J^π**	
	0^+	**1^+**
2.8	3287	3339
2.9	3389	3438
3.0	3490	3538
3.1	3591	3638

[a]Multiquark masses and J^π values are derived from Ref. 2.

3.4 f f-bar Mesons

f f-bar meson projections using the first-order mass formula are summarized in Table 4. Chapman[2] provides f f-bar meson results for the X(6900) 0^+ state. First-order mass formula results approach the experimental mass value[2] as the assumed f quark mass increases beyond the 2900 MeV/c^2 value utilized by Chapman.

Table 4

f f-bar Mesons Compared to the State with $J^{\pi} = 0^+$ (X(6900))[a]

f Quark Mass (GeV/c^2)	Meson J^{π}	
	0^+	1^+
2.8	5633	5642
2.9	5834	5842
3.0	6034	6042
3.1	6235	6242

[a]Multiquark masses and J^{π} values are derived from Ref. 2.

3.5 fuu Baryon

In addition to mesons containing f quarks, Ref. 2 also addressed the fuu baryon and compared it to $J^{\pi} = 1/2^+$ P_c^+ (4380) and $J^{\pi} = 3/2^+$ (P_c^+ (4457) states. The baryon results are similar to meson calculations. In particular, the first-order mass formula underestimates the experimental values[2]. In addition, the first-order mass formula results approach the experimental mass values[2] as the assumed f quark mass increases beyond the 2900 MeV/c^2 value utilized by Chapman.

Table 5

fuu Baryon Compared to States with the

J^{π} = 1/2$^+$ State (P_c^+ (4380)) and J^{π} = 3/2$^+$ State (P_c^+ (4457))[a]

f Quark Mass (GeV/c^2)	Baryon J^{π}	
	1/2$^+$	3/2$^+$
2.8	3729	3766
2.9	3830	3866
3.0	3931	3965
3.1	4031	4065

[a]Multiquark masses and J^{π} values are derived from Ref. 2.

4.0 Conclusions

The f quark assumption proposed by Chapman[2] offers an alternative explanation for tetraquark and pentaquark structures. Chapman proposes a 2900 MeV/c^2 mass for the f quark.

First-order mass formula calculations incorporating these f quark meson and baryon structures yield values below the experimental results. The results approach the experimental tetraquark and pentaquark masses as the f quark mass increases beyond the 2900 MeV/c^2 value suggested by Chapman[2].

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