

RESEARCH ARTICLE

Decay Characteristics of Neutron Excess Vanadium Nuclei

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Abstract

Neutron excess vanadium nuclei are investigated using a single-particle model. The model predicts that $A = 65 - 80$ neutron excess vanadium systems are bound and have half-lives in the range of 0.277 – 8.53 ms. Model results for these nuclei are about a factor of 2 to 3 smaller than the calculations summarized in the Japanese Nuclear Data Compilation. The model calculations include the alpha, beta, positron, electron capture, and spontaneous fission decay modes. Short lived decay modes involving neutron emission are not evaluated. Omission of these modes suggests that the model results could overestimate the half-lives of neutron excess vanadium nuclei.

J. J. Bevelacqua*Bevelacqua Resources, 7531 Flint Crossing Circle SE, Owens Cross Roads, AL 35763 USA bevelresou@aol.com***Keywords:** Nucleosynthesis, Neutron Excess Vanadium Nuclei, Beta Decay, Nuclear Structure.

1. Introduction

Interest in neutron excess nuclei has intensified with the construction of facilities and advances in experimental and theoretical physics [\[1\]\[2\]\[3\]\[4\]\[5\]\[6\]\[7\]\[8\]\[9\]\[10\]\[11\]\[12\]\[13\]\[14\]\[15\]\[16\]\[17\]\[18\]\[19\]\[20\]\[21\]\[22\]\[23\]\[24\]\[25\]\[26\]\[27\]\[28\]\[29\]](#). Many of these systems are produced from r-process nucleosynthesis, and their study also provided insight into various astrophysical phenomena including neutron star and black hole mergers [\[1\]\[2\]](#).

This paper investigates neutron excess vanadium systems, and supplements previous work that addressed neutron excess systems having $Z = 9 - 22, 26,$ and 30 [\[8\]\[9\]\[10\]\[11\]\[12\]\[13\]\[14\]\[15\]\[16\]\[17\]\[18\]\[19\]\[20\]\[21\]\[22\]\[23\]](#). Vanadium systems are a topic of continuing interest, and these studies assist in determining the evolution of the structure of nuclear systems as the neutron number increases [\[25\]\[26\]\[27\]](#).

2. Computational Methodology

The calculational method for investigating neutron excess nuclei is provided in Refs. 8-23, and utilizes a basic single particle model approach. This model applies the methodology of Lukasiak and Sobiczewski [28] and Petrovich et. al. [29]. The numerical methods of Refs. 30 and 31 are utilized to determine the single particle energies.

The radial Schrödinger Equation determines the binding energy E_{NLSJ} of a neutron or proton in the field of a nuclear core [8][9][10][11][12][13][14][15][16][17][18][19][20][21][22][23]

$$[(\hbar/2\mu) (d^2/dr^2 - L(L + 1)/r^2) - E_{NLSJ} - V_{LSJ}(r)] U_{NLSJ}(r) = 0 \quad (1)$$

where r is the radial coordinate. In Eq. 1, $V_{LSJ}(r)$ is the model interaction and $U_{NLSJ}(r)$ is the radial wave function. The orbital, spin, and total angular momentum quantum numbers are represented by L , S , and J , respectively. The remaining terms in Eq. 1 are the radial quantum number (N) and the reduced mass (μ).

3. Nuclear Interaction

The nuclear potential is based on the Rost interaction [30] with a central strength

$$V_0 = 51.6 [1 \pm 0.73 (N - Z)/A] \text{ MeV} \quad (2)$$

where the positive (negative) sign is assigned to protons (neutrons). Parameters defining the interaction are provided by Rost [30]. The strength of the spin-orbit interaction V_{so} is defined in terms of a parameter γ [30]:

$$V_{so} = \gamma V_0 / 180 \quad (3)$$

Model calculations also include the Blomqvist and Wahlborn [31] pairing correction interaction.

Difficulties in defining an appropriate interaction were noted by Ray and Hodgson [32] and Schwierz, Wiedenhöfer, and Volya [33]. Refs. 35 and 36 demonstrate that adjustments to the nuclear interaction must be made to individual nuclei to ensure a proper fit to the observed experimental energy levels and decay characteristics.

Following the guidance provided in Refs. 35 and 36, modifications are made to the base Rost central interaction strength (V_A)

$$V_A = V_0 \lambda [1 \pm a(A)] \text{ MeV} \quad (4)$$

Eq. 4 includes a potential strength multiplier (λ), and a factor $[a(A)]$ that is used to adjust the potential strength with varying A value [32][33]. A value of $\lambda = 1.5$ is utilized for vanadium that is consistent with previous calculations [8][9][10][11][12][13][14][15][16][17][18][19][20][21][22][23], and to ensure agreement with available data [34][35][36].

4. Model Limitations

Most decay modes (i.e., alpha, beta, positron, and electron capture transitions, and spontaneous fission [8][9][10][11][12][13][14][15][16][17][18][19][20][21][22][23][28][29][37]) are reasonably represented by spherical single-particle energy level calculations. However, single-particle models are not the optimum methodology to determine neutron emission half-lives. Since these decay modes tend to be shorter than the previously noted decay modes, omission of the neutron emission decay modes could lead to model results that would overestimate the decay half-lives.

5. Results and Discussion

Table 1 summarizes the complete set of $80 \geq A \geq 56$ vanadium isotopes evaluated in this paper. The $80 \geq A \geq 56$ vanadium isotopes occupy the $1f_{5/2}$ ($^{56}\text{V} - ^{61}\text{V}$), $2p_{1/2}$ ($^{62}\text{V} - ^{63}\text{V}$), $1g_{9/2}$ ($^{64}\text{V} - ^{73}\text{V}$), $2d_{5/2}$ ($^{74}\text{V} - ^{79}\text{V}$), and $3s_{1/2}$ (^{80}V) neutron single-particle levels. Data summarized in Refs. 37 - 39 indicate that ^{64}V is the heaviest observed vanadium system. Extrapolations beyond $A > 64$ become more uncertain because data is not available to guide the calculations.

5.1. $56 \geq A \geq 64$ Vanadium Isotopes with Experimental Half-Life Data

Table 1 lists the half-life of the limiting decay transition (i.e., the transition that has the shortest decay half-life). For example, the ^{56}V model predicts eight beta decay transitions (i.e., allowed $1f_{7/2}(n)$ to $1f_{7/2}(p)$ [2.22 s], allowed $1f_{7/2}(n)$ to $1f_{5/2}(p)$ [17.8 h], allowed $2p_{3/2}(n)$ to $2p_{3/2}(p)$ [3.89 s], allowed $2p_{3/2}(n)$ to $2p_{1/2}(p)$ [25.2 s], allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ [216 ms], allowed $1f_{5/2}(n)$ to $1f_{5/2}(p)$ [3.44 s], first forbidden $1d_{3/2}(n)$ to $1f_{7/2}(p)$ [96.3 d], and first forbidden $1f_{5/2}(n)$ to $1g_{9/2}(p)$ [5.19 yr]). For ^{56}V the limiting beta decay mode is the allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ [216 ms] transition.

Table 1. Calculated Single-Particle and Experimental Decay Properties of Vanadium Nuclei with $56 \leq A \leq 80$

| Nuclide | a(A) | Half-Life (Decay Mode) | |
|-----------------|---------|-----------------------------|---|
| | | Experiment ^{a,b,c} | This Work |
| ⁵⁶ V | -0.0001 | 216 ms ^c | 216 ms (β ⁻) ^d |
| ⁵⁷ V | -0.0214 | 320 ms ^c | 320 ms (β ⁻) ^d |
| ⁵⁸ V | -0.0180 | 191 ms ^c | 191 ms (β ⁻) ^d |
| ⁵⁹ V | -0.0066 | 97 ms ^c | 96.9 ms (β ⁻) ^d |
| ⁶⁰ V | ----- | e | ----- |
| ⁶¹ V | +0.0000 | 48.3 ms ^c | 48.3 ms (β ⁻) ^d |
| ⁶² V | +0.0062 | 33.6 ms ^c | 33.6 ms (β ⁻) ^d |
| ⁶³ V | +0.0216 | 20 ms ^c | 20.0 ms (β ⁻) ^d |
| ⁶⁴ V | +0.0155 | 19 ms ^c | 19.0 ms (β ⁻) ^d |
| ⁶⁵ V | +0.0524 | f, g | 8.53 ms (β ⁻) ^d |
| ⁶⁶ V | +0.0678 | f,h | 5.94 ms (β ⁻) ^d |
| ⁶⁷ V | +0.0832 | f, i | 4.27 ms (β ⁻) ^d |
| ⁶⁸ V | +0.0986 | f,j | 3.15 ms (β ⁻) ^d |
| ⁶⁹ V | +0.1140 | f,k | 2.39 ms (β ⁻) ^d |
| ⁷⁰ V | +0.1294 | f,l | 1.84 ms (β ⁻) ^d |
| ⁷¹ V | +0.1448 | f,m | 1.45 ms (β ⁻) ^d |
| ⁷² V | +0.1602 | f,n | 1.15 ms (β ⁻) ^d |
| ⁷³ V | +0.1756 | f,o | 0.931 ms (β ⁻) ^d |
| ⁷⁴ V | +0.1910 | f | 0.761 ms (β ⁻) ^d |
| ⁷⁵ V | +0.2064 | f | 0.630 ms (β ⁻) ^d |
| ⁷⁶ V | +0.2218 | f | 0.525 ms (β ⁻) ^d |
| ⁷⁷ V | +0.2372 | f | 0.443 ms (β ⁻) ^d |
| ⁷⁸ V | +0.2526 | f | 0.376 ms (β ⁻) ^d |
| ⁷⁹ V | +0.2680 | f | 0.322 ms (β ⁻) ^d |
| ⁸⁰ V | +0.2834 | f | 0.277 ms (β ⁻) ^d |
| | | | |

^a Ref. 37. ^bRef. 38. ^cRef. 39.

^d Allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ beta decay transition.

^e Only excited state data available.

^f No data provided in Ref. 37 - 39.

^g The Japanese data compilation ^[36] notes a calculated value of 14.3 ms for ⁶⁵V.

- ^h The Japanese data compilation^[36] notes a calculated value of 10.0 ms for⁶⁶V.*
- ⁱ The Japanese data compilation^[36] notes a calculated value of 6.67 ms for⁶⁷V.*
- ^j The Japanese data compilation^[36] notes a calculated value of 5.51 ms for⁶⁸V.*
- ^k The Japanese data compilation^[36] notes a calculated value of 3.80 ms for⁶⁹V.*
- ^l The Japanese data compilation^[36] notes a calculated value of 3.24 ms for⁷⁰V.*
- ^m The Japanese data compilation^[36] notes a calculated value of 2.30 ms for⁷¹V.*
- ⁿ The Japanese data compilation^[36] notes a calculated value of 3.24 ms for⁷²V.*
- ^o The Japanese data compilation^[36] notes a calculated value of 2.30 ms for⁷³V.*

As noted in Table 1, the model predicts the proper decay mode for the known $80 \geq A \geq 56$ vanadium^{[34][35][36]} systems. The model half-lives are also consistent with data^{[34][35][36]}.

⁵⁶V – ⁶¹V ($1f_{5/2}$ neutron shell) decay via beta emission through allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ transitions. Model predictions for ⁵⁶V – ⁶¹V are within about 0.1% of the experimental half-lives^[36]. The calculated decay modes for ⁵⁶V – ⁶¹V are in agreement with Ref. 39.

⁶²V – ⁶³V ($2p_{1/2}$ neutron shell) also decay via beta emission through allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ transitions. Model predictions for the ⁶²V – ⁶³V half-lives are also in agreement with the experimental half-lives^[36]. The calculated decay modes for ⁶²V – ⁶³V are consistent with Ref. 39.

⁶⁴V partially fills the $1g_{9/2}$ neutron shell. The decay mode and half-life for ⁶⁴V is consistent with the data^[36]. ⁶⁴V decays via beta emission through an allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ transition.

5.2. $80 \geq A \geq 65$ Vanadium Isotopes without Experimental Half-Life Data

The $a(A)$ values for $65 \geq A \geq 80$ vanadium isotopes were derived from a linear fit based on the half-lives of ⁶²V – ⁶³V. These extrapolated $a(A)$ values are provided in Table 1.

The ⁶⁵V – ⁷³V systems ($1g_{9/2}$ neutron shell) have beta decay half-lives between 0.931 – 8.53 ms. These nuclei decay through an allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ beta decay transition. Japanese Data Compilation calculations^[36] for ⁶⁵V – ⁷³V are a factor of 2 to 3 larger than the model results.

The ⁷⁴V – ⁷⁹V systems ($2d_{5/2}$ neutron shell) decay through an allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ beta decay transition. The ⁷⁴V – ⁷⁹V half-lives decrease from 0.761 to 0.322 ms, respectively. Japanese Data Compilation calculations^[36] do not predict any of the ⁷⁴V – ⁷⁹V systems.

The ⁸⁰V system partially fills the $3s_{1/2}$ neutron shell. This system decays through an allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ beta decay transition. The ⁸⁰V half-life is 0.277 ms. ⁸⁰V is not predicted by the Japanese Data Compilation calculations^[36].

No vanadium systems with $A > 80$ are predicted by the model or the Japanese Data Compilation calculations^[36]. This

model limitation occurs because only 57 neutrons are bound in the vanadium system.

7. Conclusions

Single-particle level calculations suggest that neutron excess vanadium isotopes terminate with ^{80}V . The $65 \leq A \leq 80$ vanadium systems have predicted beta decay half-lives in the 0.277 – 8.53 ms range, decay through allowed $1f_{5/2}(n)$ to $1f_{7/2}(p)$ beta decay transitions, and the model likely overestimate the actual half-life values.

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