The correlation of classic and experimental measurement results with quantum measurement theory

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Abstract: In classic measurement theory each measurement result has a precision which becomes infinitesimal as noise and distortion approach zero. I. e., exact repetitive measurement results are theoretically possible. This classic measurement theory is not well correlated with experimental measurement results. Because, when noise and distortion are minimized, repetitive experimental measurement results display a Gaussian distribution. This paper first correlates classic and experimental measurements by developing a new measurement function and related definitions. Then this new measurement function is shown to correlate with quantum measurement theory and resolve existing quantum measurement perplexities.

Keywords: *calibration, uncertainty, metrology, entanglement, precision, Gaussian.*

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INTRODUCTION

A BIPM [1] experimental measurement result is a quantity (defined in the International Vocabulary of Metrology [2]) which is the product of two terms: a numerical value (n) and a unit (u). In this paper u is a general unit representing each of the smallest intervals of a measurement instrument (which quantify a measurement result). Then n is a positive integer. For simplicity this paper only addresses the effects of measurement result quantization on a quantity without the effects of noise (external to the measurement system) or distortion (internal to the measurement system).

L. Euler identified that an observable's quantity is relative [3]. An observable's measurement result quantity is relative to a measurement instrument, a calibrated measurement instrument is relative to a standard unit, and a standard unit is defined. The measurement instrument's property [4] itself, as well as the numerical value and precision (see definition Section 4.0) of each u are defined by and correlated to a standard unit, or factor thereof, by calibration. See Appendix A for an example of the relative nature of a measurement instrument's property.

A precise measurement result in u (i.e., precision smaller than a u) of an observable, in theory or experiment, can only be produced by a measurement instrument calibrated into smaller states than one u. These equal calibration states are smaller than the smallest u to correlate each u to a standard unit or factor thereof (see Fig. 1).

1. A Quantity

This paper develops a new classic measurement equation (1) - (4). Currently quantity calculus [5] defines a quantity (lower case) as the product of n and u. The relationship between each, possibly not equal, u is determined by calibration and considered only empirical in the current theory of a measurement in physics [6]. Relative measurement theory (RMT) [7], which this paper expands upon, identified that all u are not theoretically equal which requires that calibration (a process that quantizes and equalizes the u) be included in measurement theory.

$$\text{measure result } \textit{Quantity } Q = \sum_{n=1}^{n} u_n \tag{1}$$

In (1) u_n represents each of the smallest intervals of an additive scale without any u_n calibration (i.e., all u_n are not defined to be equal), including during the design or construction of experimental measurement instruments. A quantity ($n \times u$) is expedient for experimental measurements and notation. A Quantity, see (1), generates a proper superset of a quantity, i.e., includes the result $n \times u$. (1) is proposed as the beginning of a formal measurement equation that applies to all measurements.

2. A relative measurement system

As first proposed in RMT, a relative measurement system (Fig. 1) consists of three measurement sub-systems: local, calibration, and non-local. This relative measurement system (without any noise or distortion) presents the descriptions (two or more additive scales), equation (4), and instruments (measure and calibrate) of a relative measurement system in theory and experiment.

In Fig. 1 the upper dashed outline (local) represents a measure process including the observable (quantum term) or measurand (metrology term), the measure instrument with an additive (but not assumed to be equal intervals) measure reference scale. A measure process is local. A measurement process includes all three subsystems. The lower dashed outline represents the calibration sub-system including the calibrate instrument with additive defined equal calibration states which correlate the local and non-local sub-systems.

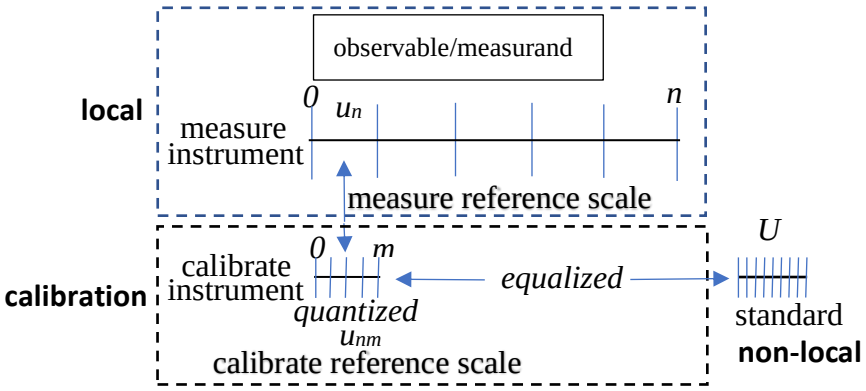

Figure 1. Relative measurement system.

The u_n intervals, where the mean interval is *1/n*, quantify the measure reference scale. The *m* equal calibration states, each defined to be *1/m*, quantify the calibrate reference scale. From Fig. 1, where the measure instrument $n = 5$, the observable/measurand has a fixed quantity of $4u_{nm}$ even though repetitive measurement results of this observable/measurand will show a Gaussian distribution centered on $4u_{nm}$.

Fig. 1 illustrates the successively smaller intervals, between the small vertical lines, of each horizontal scale that are required for a functional measurement system. That is, the measure reference scale mean intervals, *1/n* > calibration states, *1/m* > *U* reference scale resolution. Where the resolution is the smallest identifiable change of the standard. *n* and *m* are integers (representing counts) when *1/n* and *1/m* represent the smallest intervals or states of their respective reference scales.

The calibrate reference scale quantizes and equalizes each u_n which determines the precision of each u_n to *U*. When each u_n is quantized and equalized to *U*, all u_n are *calibrated*:

$$\text{calibrated } u_n = u_n \pm 1/m = u_{nm} \qquad (2)$$

Although the numerical value of *U* does not appear in (2), the numerical value of *U* can only be considered arbitrary in the first use. The numerical value of *U* determines the precision of each u_{nm} in *1/m* calibration states when there is noise or distortion (i.e., all experimental measurements). Changing u_n in (1) to u_{nm} from (2), produces the next measurement equation:

$$\text{measurement result Quantities} = \sum_{n=1}^{n} u_{nm} \qquad (3)$$

The *±1/m precision*, which changes each u_n individually, when summed over many repetitive measurement results establishes a Gaussian measurement result distribution. In rare statistical cases the distribution established by each *±1/m* becomes increasingly dispersed (see the examples: 5.1, 5.2 and 5.3 below). For a measurement function to represent this

Gaussian statistical effect, a Quantity (summation) must be used. When a quantity (product) is used, these statistical effects are not treated and perplexities appear.

The differences that have appeared between classic, experimental and quantum measurement are based upon the different definitions of a unit and a state in the different disciplines. Understanding these differences requires a close look at the definitions.

3. Units have multiple definitions

Currently accepted:

- In representational measure theory (all physics except metrology) *u* is defined to be *U*.
- In metrology (assumed only experimental), *u* is commonly the mean *u* which is calibrated with a precision to a standard unit or a factor of a standard unit.
- In statistics, a mean *u* or *U* may be the standard.
- In bra-ket notation (common QM notation) a state is a ket vector representing, *u* or *U*, and treated as unity [8].

In this paper:

- u_n identifies each of the smallest *u* of a measure reference scale before calibration to *U*. u_n represents each uncalibrated *u* which has a local property, local size and undetermined precision. Local means correlated only to other u_n on the same reference scale.
- u_{nm} is the numerical value of each *u* calibrated to *U* expressed in *1/m*. Each u_{nm} has a non-local property, non-local size and precision relative to a standard unit or factor thereof. The *1/m* intervals are the smallest states of the calibrate reference scale (calibration states).
- *U standard*, *U* (capitalized), is a non-local standard with a defined physical property and a defined numerical value. *U* represents one of the seven different BIPM base properties, their units or derivations. *U* may be defined without ± precision (i.e., exact). A *U*'s property and unit are required as a standard for relative measurements and *U*'s numerical value is required to determine the precision of independent measurement instruments relative to *U*.

4. Additional definitions used in this paper

The u_n, u_{nm} and *U* definitions proposed in this paper necessitate revising other definitions from the International Vocabulary of Metrology (VIM). The definitions below are related, where possible, to VIM definitions. These definitions apply in measurement theory and experiment.

Quantity may be a product (*q*) or a sum (*Q*) as developed by equations (1) - (4). In VIM a quantity is a product, because instrument calibration is assumed to establish sufficient u_{nm} precision when $n>>m$.

Reference scale represents the usually additive local property of the measure or calibrate reference instrument. This paper only addresses additive scales. A reference scale must be local to the observable (i.e., properties are locally comparable) to establish a measure. Both the size and the relative precision of the reference scale intervals relative to *U* are not defined.

A reference scale may have a zero point. VIM applies the term *reference measurement standard* to both scales and standards.

A *measurement instrument* (calibrated in theory and experiment) has a reference scale and may have a transducer which converts an observable's property to a property represented on this reference scale.

A *measure instrument* (uncalibrated in theory and experiment) has a reference scale and may have a transducer which converts an observable's property to a property on this reference scale. A measure instrument determines the *n* of an observable/measurand.

A *calibrate instrument* (in theory and experiment) has a defined equal scale which, when referenced to a *U*, defines the property of the measure and calibrate instrument reference scales. The calibrate instrument determines the numerical value of u_{nm}.

Calibration applies to both physics theory and metrology and is includes quantization and equalization (see Fig. 1) of each measure instrument to a non-local unit standard (or factor thereof). Calibration also defines the property of an observable/measurand relative to a standard. Calibration as defined in VIM is termed *instrument calibration* in this paper, and commonly generates a mean u_n. Instrument calibration may also include adjusting the *n* numerical value as adjusting the many u_n of an instrument is often not practical.

Calibration state is the smallest defined equal state of a relative measurement system. A calibration state quantifies u_n but is time independent of u_n.

Precision is the ± deviation of the unit u_{nm} of a measurement result Quantity relative to a *U* standard. Precision is determined by calibration. This definition is more rigorous than the VIM definition.

Accuracy is the ± deviation of the numerical value (*n*) of a measurement result Quantity relative to the mean numerical value. In the VIM empirical definition, measurement accuracy may also include precision.

5. Empirical measurements

Considering the subtle, but extensive, classic measurement theory and definitional changes proposed, some examples may be helpful.

5.1 Additive reference scale

An example of an additive reference scale is a thermometer which measures the property of thermodynamic temperature. This example demonstrates how additive imperfect intervals statistically increase the dispersion of measurement results, producing a Gaussian measurement result distribution.

The measurement instrument consists of a hollow glass tube with a reservoir filled with mercury at one end, which fits inside another hollow glass tube that slides over the first. The two glass tubes are held together and placed in an adjustable temperature chamber which has a resolution of 0.1^0 (degree). Then the outside glass tube is marked at the level of mercury which appears and each 1.0^0 u_n above this mark. $n + 1$ marks (e.g., $n = 100$ in the Celsius

system) or *101* marks are made to quantize the outside glass tube. Each of the *100* u_n is correlated with the chamber to $1/0.1 = 10 = u_{nm} \pm 0.1^0$ precision.

After 101 marks are made, the instrument is removed from the chamber and an ice water bath is applied to the tube with mercury. The outside glass tube is now slid over the inside glass tube until the top of the inside mercury column lines up with the first mark on the outside glass tube. Now one mark on the outside glass tube is referenced to the temperature of ice water (0^0C) which is one point on this reference scale for thermodynamic temperature.

Consider the temperature of a glass of water in contact with the reservoir of this measurement instrument (observable/measurand). If the temperature of the water is 80^0, the *81st* mark on the outside glass tube represents $80^0 \pm 0.1^0$ nominal precision or $\pm 8^0$ worst case precision dispersion. The $\pm 0.1^0$ nominal precision occurs when the $\pm 0.1^0$ u_{nm} precision of each *80* u_n is uniformly distributed.

The $\pm 8^0$ precision dispersion occurs when each of the *80* u_n has the same $+0.1^0$ or -0.1^0 precision, which sums. Of course, such a $\pm 8^0$ distribution is not really possible in practice and for this reason is often ignored in current practice. But in a measurement theory, when each mark's precision is specified to be $\pm 0.1^0$, $\pm 8^0$ is possible in theory. The range of the precision dispersion from $\pm 0.1^0$ to $\pm 8^0$ establishes a Gaussian distribution of measurement results (see Section 5.3). The statistical effects of this Gaussian distribution on a measurement result is ignored when equal *u* are assumed.

5.2 Length measurement instrument

A physical metre stick is divided into *100* centimetres (smallest *u*), or *100* u_n. Consider an observable/measurand whose quantity, $80u_n$ consists of $n = 80$ and each u_n is calibrated to become u_{nm}. In the measurement theory proposed each u_n is calibrated to a standard and added to the next u_n (*80* times). When first calibrated to a factor of *U* (a standard metre) each $u_{nm} = (U/100) \pm 1/m$ precision where, e.g., each $1/m$ is $1 \cdot 10^{-6}$ (i.e., there are 10^6 calibration states of this metre stick) and the *n* accuracy = $n \pm 1/m$ is ignored. In the proposed theory the Quantity dispersion is established by the random application of $\pm 1/m$ to each u_n producing a Gaussian distribution. In the rarest two cases, when *n* of the u_{nm}, all with a precision of $+1/m$, are summed and in another measurement of *n* of the u_{nm}, all with a precision of $-1/m$, are summed, the greatest Quantity dispersion appears $2(80)10^{-6} = 1.6 \cdot 10^{-4}$ metres, which is sufficient precision ($\pm 0.8 \cdot 10^{-4}$) for a metre stick. When the number of calibration states of a measurement instrument is much greater than *n*, the effect of measurement result dispersion is realistically ignored for statistical reasons, which are explained in Section 6.

5.3 Gaussian normal measurement distributions

Fig. 2 presents the characteristic Gaussian shape of a large distribution of repetitive experimental measurement results. This shape has been verified in many different forms of measurement results where noise and distortion have been minimized [9]. The Gaussian shape of measurement results in Fig. 2 is established by the quantization due to calibration states. This strongly supports the proposal to treat calibration in both physics theory and experiment.

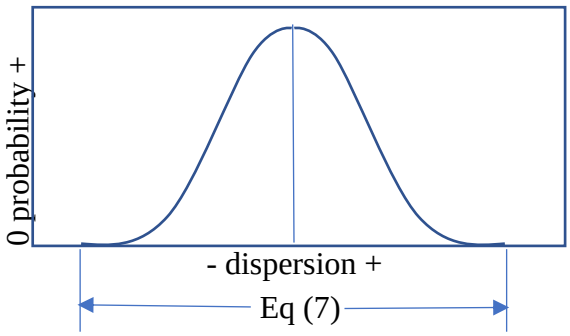

Figure 2. Gaussian distribution of measurement results.

6. The effect of u_{nm} on precision and dispersion

Including the quantization of accuracy in (3):

$$\text{measurement result Quantities} = \sum_{n=1}^{n\pm 1/m} u_{nm} \qquad (4)$$

Equation (4) is a measurement function that applies (without noise or distortion) to all formal results and experimental processes. From (2):

$$u_{nm} \text{ precision} = \pm\left(1/m\right) \qquad (5)$$

This precision, $\pm\, 1/m$ in each of the n u_n, varies randomly (in theory) and sums (4) into a measurement result. Then the sum of the u_{nm} precision is:

$$\text{the worst case precision of (4)} = \pm n\left(1/m\right) \qquad (6)$$

Equation (6) identifies how repetitive measurement result Quantities of the same observable appear different, when $n \sim m$ (a quantum measurement), and not when $1/n >> 1/m$ (an experimental measurement, when $1/m$ is often treated as infinitesimal). Then from (6) the maximum dispersion of calibration states (including the accuracy of n which, in this example, is $\pm 1/m$), is $2n/m$ precision dispersion + $2/m$ accuracy dispersion:

$$\text{the maximum Quantity dispersion of (4)} = 2n/m + 2/m \qquad (7)$$

Equation (7) defines the base of the Gaussian distribution of measurement results shown in Fig. 2. Equation (4) identifies that each calibrated measurement result Quantity has a dispersion which is determined by both the n and m of a Quantity. In (4) when m is large relative to n, the $\pm 1/m$ calibration states have a small effect on precision. Because, each $\pm 1/m$ (calibration state) cancels, or close to cancels in almost all measurement results (as example see 5.1), due to the central limit theorem's effect on the distribution of u_{nm} calibration states.

Conversely, when $n \sim m$ and m is small (neutron spin measurements $n = m = 2$) [10], the sum of each $\pm 1/m$ precision will likely not cancel. In the neutron spin experimental measurements, it is likely (50% probability in the neutron spin measurements, assuming accuracy does not have an effect) two repetitive measurement result Quantities of an unchanged observable will be different. This creates the illusion of non-commuting quantities.

7. Explaining the perplexities that appear in quantum measurement theory

The previously development (4) identifies that the calibration states are not similar to u. The defined time independent calibration states may be correlated with the defined time independent stationary states in quantum mechanics (QM) measurement theory [11].

Currently the precision of u_{nm} (i.e., $\pm 1/m$) in QM appears as quantum uncertainty and in metrology may appear as accuracy, precision or resolution. W. Heisenberg formally presented uncertainty in QM [12]. RMT provided a verification that $1/m$ (a calibration state) at the limit of precision and resolution (a Planck) is quantum uncertainty.

7.1 Heisenberg's uncertainty

Heisenberg's thought experiment [13] explores why uncertainty appears in QM, based upon his assumption that uncertainty does not appear in classic measurements. Heisenberg's assumption about classic measurements is not valid. The worst case precision (6) identifies that all measurement results (theory or experiment) will not commute (e.g., two repetitive measurements of one quantity of a Fourier dual (x_1p_1 & p_2x_2) are not equal, ($x_1 \neq x_2$ & $p_1 \neq p_2$) in one half of the repetitive measurement results when $n = m = 2$. Not commuting appears because each repetitive measurement result of x and p, a sum (n) of the sum of the m calibration states of either quantity, varies statistically. These statistical dispersions occur whether or not x and p are Fourier duals. The quantum uncertainty Heisenberg and others described occurs in all quantized measurement results.

7.2 Double slit experiments

The double slit experiments as explained by Feynman [14] offer an example of how calibration defines a property. Feynman concludes, "...when we look at the electrons the distribution of them on the screen is different than when we do not look."

The measurement instrument in these experiments includes a sensing screen which is both a position and momentum transducer, while the slits are the reference scale. An operator, looking at the sensing screen, identifies a dot indicating the particle's property of momentum. The same operator's looking at the same sensing screen identifies a wave's property of length (wavelength of the frequency), but these are two independent calibrated measurements. The operator's choice of a wavelength or dot reading on the sensing screen is a property selection process which occurs by calibration. Calibration, which defines the property being measured, is what Feynman refers to as a "look". When calibration is not considered part of a measurement process, the property measured, which is always relative, appears to be changed by looking, but is actually defined by calibration.

7.3 Remote entanglement of two measurement instruments' results

A physical comparison of two measurement result quantities (q_a and q_b) from two measurement instruments *(*a and b) is a ratio of their numerical values (n_a and n_b) and the numerical values of their mean u (i.e., u_a and u_b), shown as: $n_a \times u_a / n_b \times u_b$. Currently in physics theory experimental calibration is assumed, therefore $u_b = u_a$. Given the development in Section 6 and the quantum uncertainty identified in Section 7.1, $u_b = u_a$. is not always valid.

Applying calibration in theory, the calibration of a and b to U refines u_a into u_{am} and u_b into u_{bm}, which gives the ratio as $n_a \times u_{am} / n_b \times u_{bm}$. Then u_{am} and u_{bm} cancel because they are equalized by calibration. When the calibration of both u_a and u_b is considered an initial empirical condition, this does not recognize that the ratio $n_a \times u_a / n_b \times u_b$ has a different numerical value than the ratio $n_a \times u_{am} / n_b \times u_{bm}$. This difference is then assumed to be a difference in the ratio of n_a/n_b (because all u are assumed equal) which suggests a remote entanglement [15] between the measurements of n_a and n_b, which is very perplexing.

There have been many attempts to understand remote entanglement. J.S. Bell's formalization, (since verified experimentally [16]), which perhaps consolidates the earlier attempts, is addressed below. J. S. Bell, in his paper [17]: "...there must be a mechanism whereby the setting of one measurement device can influence the reading of another instrument, however remote." That mechanism is calibration as defined in this paper.

N. D. Mermin [18] described logically and analyzed statistically the neutron spin experiments that verified the remote entanglement that Bell formalized in QM. Remote entanglement is described by Mermin without QM formalism, indicating it is basic to all measurements. He identifies that the measurement result quantities of two entangled particles have a logical relationship that is not possible without an unknown interaction between the two measuring instruments. Not recognizing that a measurement result is a Quantity, Mermin describes calibration as that interaction, without realizing it.

In Mermin's description two measurement instruments each have a 3 position selector which determines the u_n detected (i.e., spin vector at 120^0 intervals) meaning there are 3 possible u_n. Then, there are 9 possible combinations of the two position selectors: Three (Mermin's case a) when each selector is in the same position and six (Mermin's case b) when the positions of the two selectors are different. When the two remote selectors are different (the two measurement instruments' u_n are uncalibrated) and the n (0^0 or 180^0) of the two particles appears randomly. Only in Mermin's case a (the two measurement instruments' u_n are calibrated) is the n of the two particles correlated.

In summary, the a and b measurement instruments are independent but the measurement results q_a and q_b, whose units (u_{am} and u_{bm}) are equalized by calibration, appear to "...influence the reading of another instrument...". That is, the two measurement instruments are calibrated. The measurement result quantity ratio $n_a \times u_a / n_b \times u_b$ will be different (especially at quantum scales) when u_a and u_b are not calibrated in theory to each other. This is strong evidence of the significance of calibration to any description of physical reality.

8. Relating relative measurement theory to other theories

In 1891, J. C. Maxwell [19] proposed that a measurement result quantity is:

$$\text{measurement result } \textit{quantity} \quad q = n \times u \tag{8}$$

In (8), n is a numerical value, and u is a unit ("taken as a standard of reference" [20]), which together form a mutual relation, capable of being compared [3]. Equation (8) is the basis of quantity calculus. From Maxwell's usage and quote, u is equal (without $\pm$ precision) to a U standard unit. Equation (8) and Maxwell's usage suggest that perfect precision is possible and a standard, though required, is arbitrary, well before quantum uncertainty identified that perfect precision is impossible.

In the 20th century, QM offered a new measurement function, von Neumann's Process 1 [21]. Process 1 includes a statistical projection operator, $P_{[\phi n]}$[22], which represents each sum of *m* calibration states in this paper. von Neumann's Process 1 formalizes a measurement as the sum (*n*) of the inner products of the stationary states of the observable and the measurement instrument. Then the comparison of eq. (4) to Process 1 is straight forward when the calibration states are correlated to the stationary states in Process 1.

Fig. 1 relates representational measure theory [23] (local) with the calibration and non-local portions of a relative measurement system. Representational measure theory does not recognize that a quantity is relative; assumes measure result comparisons can occur without a reference scale or standard; treats a unit as arbitrary [24], which requires any calibration to be empirical [25]; and indicates that all measure result quantity dispersion is due to noise, distortion and errors in the measurement system [26].

Now, the Einstein, Podolsky, Rosen (EPR) paper titled: "Can quantum-mechanical description of physical reality be considered complete?" [27] can be answered in the affirmative when the correlation (i.e., calibration) of each *u* to a non-local physical standard is treated in theory. The basic criteria, explained in the EPR paper, that experimental measurement results (physical reality) can in theory be certain ("predict with certainty"), is not rigorous. The EPR paper considers classic and experimental measurement results to be equal. Experimental measurement results can only be as precise as a calibration process defines.

Since a Quantity consisting of a numerical value and a u_{nm} has not been applied in QM for almost 90 years, other perplexing effects have been noted. In Measurement Unification, 2021 [28] explanations are given of quantum teleportation experiments and Mach-Zehnder interferometer experiments. The Schrödinger's Cat thought experiment is explained in a short preprint [29]. Together with the explanations above, these explanations verify that u_n calibration unifies classic and experimental measurement processes and QM measurement functions.

9. Conclusion

A physical standard (*U*) is the basis from which all physical phenomena are defined. Without such standards any understanding of physical phenomena, e.g., QM, appears statistical.

Perhaps, when Maxwell indicated that the $u = U$, he assumed that a theoretical measurement result could be exact. It is not clear if he considered calibration an empirical process or that a standard was arbitrary. However, these incorrect assumptions did emerge. When calibration is only an empirical process, then a standard is not required in theory and a measurement property is assumed. When a standard is not required to define a property, the ratios of two measurement results are relative to each other rather than each relative to a standard. Then the various forms of quantum uncertainty appear as relations between Fourier duals, not as a precision relative to a standard of any quantized measurement result quantity in a state space.

Examined closely, all repetitive measurement quantities are distributions caused by quantization and controlled by calibration. This paper has developed how quantization creates

variation between states, units and standard units. These variations are not included in current physics measurement theory, which causes perplexities.

Appendix A

Defining the property of a measurement instrument

The first calibration of a measurement instrument to a physical unit standard defines the measurement instrument's property (length, mass, time, spin, etc.).

It is often assumed that a measurement instrument has a physical property (length, mass, spin, time, etc.) that it measures. This is a simplification. All physical properties are relative and must be defined by a common property (usually represented by a BIPM standard).

As example, consider an electrical multi-meter which can display the numerical values in units of the electrical properties: volts, ohms and amps on its meter. A rotary switch on the multi-meter selects which property to display. What occurs if the printing of the terms: volts, ohms and amps on the multi-meter's rotary switch is removed? That is, the visible prior history of the multi-meter's measured properties is removed.

In this case the operator can use a known reference voltage (e.g., the known numerical value in volts of a common battery) and apply it to the multi-meter to determine which switch setting causes the display of the numerical value of the battery voltage. The same approach may be used by applying a known reference resistance to determine the ohms switch position. Without any prior history, calibration to a known reference/standard defines the property as well as determines the precision of the measurement instrument.

The very first calibration of any measurement instrument to a standard, or perhaps an instrument or part that is applied in the design or manufacture of the measurement instrument, defines the property being measured.

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[27] A. Einstein, B. Podolsky, N. Rosen (EPR), Can quantum-mechanical description of physical reality be considered complete?, Physical Review, Vol 47, May 15, 1935..
[28] K. Krechmer, Measurement Unification, Measurement, Vol. 182, September 2021, https://www.sciencedirect.com/science/article/pii/S0263224121005960?via%3Dihub
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