The correlation of classic and experimental measurement results with quantum measurement theory

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Abstract: In classic measurement theory each physical measurement result has a precision which becomes infinitesimal as noise and distortion approach zero, i. e., exact repetitive measurement results (± a Planck) are theoretically possible. This classic measurement theory is not well correlated with experimental measurement results. When noise and distortion are minimized, repetitive experimental measurement results display a Gaussian distribution. This paper addresses experimental measurements by first developing a formal measurement function and related definitions, which identify that a classic measurement result is not possible in theory as well as in experiments. This new measurement function is then correlated with quantum measurement theory. This correlation explains existing quantum measurement perplexities.

Keywords: *calibration, uncertainty, precision, quantization, metrology, entanglement, Gaussian.*

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1. Experimental measurement results

L. Euler identified that all measurement results are based upon mutual relations with a known quantity[1]. The paradigm applied in current measurement theory [2] is based upon an observable–measurement–observer paradigm called representational. L. Euler's paradigm is measurand[1]–measurement–standard paradigm, called relative. Relative Measurement Theory (RMT) [3], is based upon Euler's paradigm. The current paper develops the practice of RMT, which explains how RMT resolves the measurement perplexities that appear in quantum mechanics (QM).

A measurand's measurement result is currently defined as a quantity [4] which is the product of a numerical value (*n*) and a *unit*[2](*u*). In this paper *n* is an integer and then *u* is the smallest interval of a *relative scale*. Two mutual relations between a measurand and a standard are established by a:

- Relative scale - sets of *u*.
- *Reference point* - e.g., *0 u*.

In representational measurement theory both these mutual relations are considered empirical, i.e., part of an experiment's set-up. They will be shown to be significant in measurement theory as well.

A measurement system that generates a measurement result includes a relative scale which is projected on a measurand. Each interval (*u*) of this relative scale must be correlated to a standard unit (defined) for independent measurement results. This is the definition of calibration applied in this paper. When calibration to a standard unit is included, the first two mutual relations occur plus two additional (see Fig. 1) mutual relations between a measurand unit and a standard unit:

- Quantization of *u* - numerical value of *u*.
- Equalization of *u* - *precision* of *u*.

A precise measurement result quantity (i.e., precision smaller than ±*u*) of a measurand, in theory or experiment, can only come from a measurand quantized into yet smaller states than one *u*. Thus, a *calibration state* must be smaller than a *u* interval. Therefore, quantization due to calibration is required in any precise measurement theory as well as in any precise experimental measurement.

A physical measurement establishes these four mutual relations. The first two mutual relations are required in any measurement theory or experiment that produces comparable measurement results. The third mutual relation (quantization) is required in any precise measurement theory or experiment. The fourth mutual relation (equalization) is required for

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[1] Measurand (that which is measured) is a metrology term which is more appropriate than the QM term observable (that which is observed or seen).

[2] The first instance of a word defined in Annex A is italicized.

independent measurement results. This paper explains the perplexities that occur when these four mutual relations are not recognized, by first developing a new formal measurement function.

2. An experimental measurement result Quantity

For measurement result quantity comparisons, two relative scales with a common *u* are necessary. The common *u* occurs by: assumption, correlation of both relative scales, or by the calibration of each relative scale to a standard unit. This calibration process is considered solely empirical in the current theory of a measurement in physics. When calibration is solely empirical, precision (see definition in Annex A) is also empirical, which assumes classic measurement results, where the precision of *u* determined by calibration goes to zero in a perfect measurement system (where noise and distortion are assumed to be zero). However, classic measurement results are not possible in theory or experiment.

RMT verified [5] that all *u* are not theoretically or empirically equal, especially at quantum scales (consider Heisenberg's uncertainty). When all *u* are not equal in theory, quantization of *u* must be included in a measurement theory.

$$\text{quantized Quantity } Q = \sum_{n=1}^{n} u_n \qquad (1)$$

In (1), u_n represents each of the smallest intervals of an additive relative scale. However, each u_n is not defined as equal, as *u* is, for reasons that will be developed. A quantity ($n \times u$) is expedient for experimental measurements. A Quantity, see (1), is a proper superset of a quantity, i.e., it includes the result $n \times u$. Equation (1) is proposed as the first step towards a formal measurement function that applies to all measurements in theory and experiment.

3. A relative measurement system.

Fig. 1 diagrams a calibrated relative measurement system which is currently applied only in experimental measurements. The relative measurement paradigm applies Fig. 1 in all measurements. Fig. 1 may also have transducers (not shown) which convert a measurand's property to a property represented by a relative scale.

The first two mutual relations in Fig. 1 are:

- The local relative scale (defined by the *U* property) determines that the set of *u* is *n*.
- *0* is the reference point common to the measurand and the local relative scale.

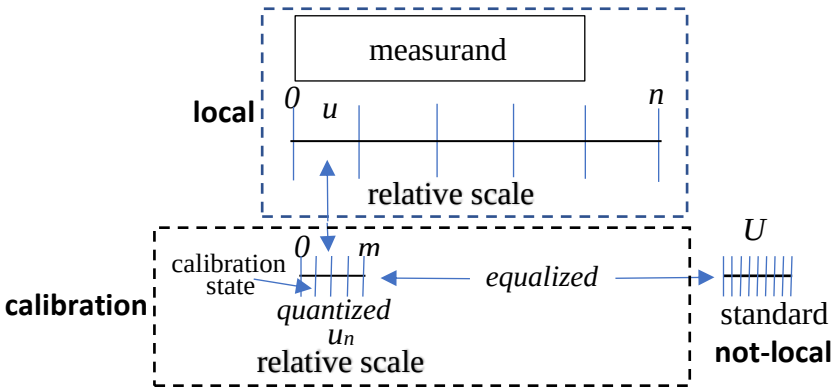

Figure 1. Relative measurement system.

A relative measurement system without calibration to a standard *U*) would still need to establish the first two mutual relations, A diagram of an uncalibrated relative measurement system would have two measurands with a common local relative scale (see example Section 6.2).

Fig. 1 shows four calibration states each defined as *1/m*, which quantify each of the five *u* intervals on the local relative scale. The measurand is defined to be $4u_n$ even though repetitive measurement results of this measurand will show a Gaussian distribution, whose *deviation* is quantified into calibration states, centered on $4u_n$.

The deviation of a measurement result quantity has three possible causes (operator errors are not considered): quantization (i.e., *u* or smaller), noise (external to the measurement system), distortion (internal to the measurement system). This paper focuses on the deviation due to quantization (i.e., calibration states), as this has been previously overlooked.

Fig. 1 illustrates the successively smaller quantized intervals (between the small vertical lines), of each horizontal scale that are required for a functional measurement system. That is, the local relative scale mean intervals are *1/n*. The *1/n* intervals are greater than the calibration relative scale defined calibration states (*1/m*). The *1/m* states are greater than the *U* reference scale resolution, which is the smallest identifiable change of *U*. Integers *n* and *m* represent counts when *1/n* and *1/m* represent the smallest intervals or states of their relative scales.

The third mutual relation, quantization, divides each *u* into *m* equal states. Then each u_n has a separate numerical value.

$$\text{quantized } u = u_n \qquad (2)$$

The fourth mutual relation, equalization, equalizes each u_n to *U* and completes a calibration process. As example:

$$\text{calibrated } u_n = U \pm 1/m \qquad (3)$$

Notice that the relationship between calibrated u_n and *U* is statistical (not fixed) and defines the precision ($\pm 1/m$) of each u_n to *U*. Then a precise measurement function must be a statistical sum of u_n. Including the precision of u_n in (1) produces:

$$\text{measurement result Quantities } = \sum_{n=1}^{n} u_n \pm 1/m \qquad (4)$$

The $\pm 1/m$ precision, which changes each u_n individually, establishes a Gaussian measurement result distribution when summed over many repetitive measurement results. In statistically rare cases the distribution established by each $\pm 1/m$ becomes increasingly dispersed (see the examples in Section 4). For a measurement function to represent the Gaussian distribution created by the statistical sums, a Quantity (summation) must be used. When a quantity (product) is used, the statistical sums are not treated and perplexing differences in measurement results appear.

4. Empirical measurement examples

The measurement paradigm developed in the first three sections is significantly different from the current measurement paradigm in physics today. The following three examples are offered in support of this new paradigm.

4.1 Additive relative scale

An example of an additive relative scale is a thermometer which measures the property of thermodynamic temperature. This example demonstrates how additive imperfect intervals statistically increase the deviation of measurement results, producing a Gaussian measurement result distribution.

The measurement instrument consists of a hollow glass tube with a reservoir filled with mercury at one end, which fits inside another hollow glass tube that slides over the first. The two glass tubes are held together and placed in an adjustable temperature chamber which has a resolution of 0.1^0 (degree). Then the outside glass tube is marked at the level of mercury which appears and each $1.0^0\ u_n$ above this mark. $n + 1$ marks (e.g., $n = 100$ in the Celsius system) or 101 marks (a relative scale) are made to quantize the outside glass tube. Each of the $100\ u_n$ is correlated with the chamber to $1/0.1 = 10 = u_{nm} \pm 0.1^0$ precision.

After 101 marks are made, the instrument is removed from the chamber and an ice water bath is applied to the tube with mercury. The outside glass tube is now slid over the inside glass tube until the top of the inside mercury column lines up with the first mark on the outside glass tube. Now one mark on the outside glass tube is referenced to the temperature of ice water (0^0C) which is the reference point on this relative scale.

Consider the temperature of a glass of water (measurand) in contact with the reservoir of this measurement instrument. If the temperature of the water is 80^0, the *81st* mark on the outside glass tube represents $80^0 \pm 0.1^0$ nominal precision or $\pm 8^0$ worst case precision. The $\pm 0.1^0$ nominal precision occurs when the $\pm 0.1^0\ u_{nm}$ precision of each $80\ u_n$ is uniformly distributed and cancels. The $\pm 8^0$ precision occurs when each of the $80\ u_n$ has the same $+0.1^0$ or -0.1^0 precision, which sums.

In the proper design of experimental measurement systems, quantization effects are made smaller than noise or distortion and are ignored. But in this theoretical example of quantization effects, when each mark's precision is specified to be $\pm 0.1^0$, $\pm 8^0$ is very rarely possible. The range of the precision from $\pm 0.1^0$ to $\pm 8^0$ establishes a Gaussian distribution of measurement results (see Section 4.3 below). The effects of this Gaussian distribution on a measurement result are ignored when equal u are assumed. The statistical nature of these effects is significant in quantum scale measurements.

4.2 Length measurement instrument

A physical metre stick (a relative scale) is divided into *100* centimetres (smallest *u*). Consider a measurand whose numerical value is *n= 70*. In the more rigorous measurement theory proposed here, each u_n is treated individually and then added to the next u_n (*70* times).

When first calibrated to a factor of *U* (a standard metre) each $u_n = (U/100) \pm 1/m$ precision where, as example, each *1/m* is $1 \cdot 10^{-6}$ metres (i.e., there are 10^6 calibration states of this metre stick) and the *accuracy* of $n = n \pm 1/m$ is small enough to be ignored. In the proposed theory the Quantity *deviation* is established by the random application of $\pm 1/m$ to each u_n producing a Gaussian distribution. In the rarest two cases, when *n* of the u_n, all with a precision of $+1/m$, are summed and in another measurement of *n* of the u_n, all with a precision of $-1/m$, are summed, the maximum, and very rare, Quantity deviation appears $2(70)10^{-6} = 1.4 \cdot 10^{-4}$ metres, which is still sufficient precision ($\pm 0.7 \cdot 10^{-4}$) for a metre stick. When the number of calibration states of a relative scale is much greater than the *n* of the relative scale, the effect of measurement result deviation is often and realistically ignored. However, when the number of calibration states (*m*) and *n* are both small, e.g., quantum measurements, this effect becomes significant.

4.3 Gaussian measurement distributions

Fig. 2 presents the characteristic Gaussian shape of a large distribution of repetitive experimental measurement results. This shape has been verified in many different forms of measurement results where noise and distortion have been minimized [6]. A Gaussian distribution of repetitive measurement results, caused by the summing of the $\pm 1/m$ precision, strongly supports the changes to measurement theory proposed in this paper.

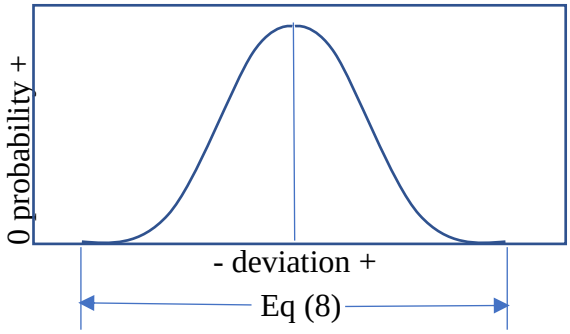

Figure 2. Gaussian distribution of measurement results.

5. The effects of quantization on accuracy, precision and deviation

Including an example of the quantization of accuracy in (4):

$$\text{measurement result Quantities} = \sum_{n=1}^{n \pm 1/m} u_n \pm (1/m) \qquad (5)$$

Equation (5) is a measurement function that applies (without noise or distortion) to all formal and experimental measurement systems. From (5):

$$u_n \text{ precision} = \pm\left(1/m\right) \qquad (6)$$

This precision, ± *1/m* relative to *U* in each of the *n* u_n in (5), varies randomly (in theory) and statistically sums into the worst case precision:

$$\text{the worst case precision of (5)} = \pm\sum_{n=1}^{n}\left(\pm(1/m)\right) \qquad (7)$$

Equation (7) identifies how repetitive measurement result Quantities of the same measurand can appear to be different (non-commuting), when *n* and *m* are small (a common quantum measurement), and not when *n* or *m* are large relative to each other (a common experimental measurement, see Section 4.3). Then from (7) the maximum deviation of calibration states (including the accuracy of *n* which, in this example, is ± *1/m*), is 2*n/m* precision + *2/m* accuracy:

$$\text{the maximum Quantity deviation of (5)} = 2n/m + 2/m \qquad (8)$$

Equation (7) identifies that each calibrated measurement result Quantity has a deviation which is determined by both the *n* and *m* of a Quantity. In (7) when *n* or *m* is large relative to the other, the ± *1/m* calibration states likely have a very small effect on precision, due to the central limit theorem's effect on the sum of *n* (u_n ± *1/m)* when *n* is large and/or due to the central limit theorem's effect on the sum of ± *1/m* calibration states in each u_{nm} when *m* is large.

Conversely, when *n* and *m* are both small, two repetitive measurement result Quantities of the same measurand will often be different. This appears as non-commuting numerical values in QM measurement results.

6. Mutual relations in quantum measurement theory

The logical development presented in the first five sections is based on the relative measurement paradigm. When the units are seen as arbitrary and the calibration of them to *U* as empirical, the requirement for mutual relations between units in theory is lost and a representational measurement paradigm appears. Based upon a representational paradigm, the QM experiments and thought experiments discussed below are perplexing. When a relative paradigm is applied the perplexities are explained.

6.1 Remote entanglement

There have been many attempts to understand remote entanglement. J.S. Bell's formalization (since verified experimentally [7]), which perhaps consolidates the earlier attempts, is addressed below. J. S. Bell, in his paper [8]: "...there must be a mechanism whereby the setting of one measurement device can influence the reading of another instrument, however remote." That mechanism is calibration.

N. D. Mermin [9] described logically and analyzed statistically the neutron spin experiments [10] that verify the remote entanglement that Bell formalized in QM. Remote entanglement is described by Mermin without QM formalism, which indicates it occurs in all measurement results. He identifies that the measurement result quantities of two entangled particles have a logical relationship that is not possible without an unknown interaction between

the two measuring instruments. Not recognizing that two measurement instruments must be correlated for a relative measurement result to occur, Mermin identifies the unknown interaction - reference point correlation - without realizing it.

In Mermin's device, the two measurement instruments each have a 3 position selector which selects the angle (one of three 120^0 intervals) of the spin property chosen. Mermin's two measurement instruments require a common reference point among the three 120^0 intervals since the common property (spin) and common relative scale (three 120^0 intervals, each a u_n) are determined by experimental set-up.

There are 9 possible combinations of the two position selectors: Three (Mermin's case a) when each selector is in the same position (33.3%) and six (Mermin's case b) when the positions of the two selectors are different (66.7%). When the two remote selectors are in different positions (the two measurement instruments are uncorrelated), each *n* (0^0 or 180^0) of the two particles spin vectors appears randomly over a large number of runs. Only in Mermin's case a is each *n* of the two particles correlated, because the two measurement instruments have been correlated independent of the measurements. When this necessary reference point correlation occurs in experiments, but does not appear in physics measurement theory, it is perplexing.

6.2 Heisenberg's uncertainty

In Heisenberg's analysis [11] a single particle's two Fourier dual properties (his notation: *p*, momentum and *q*, position) are measured multiple times identifying their precision p_1 and q_1. This precision produces a Gaussian distribution the same as u_n precision. The precision of Fourier dual properties (having inverse time units) will always vary inversely as time changes, which Heisenberg recognizes. Since two Fourier dual properties are measured on one particle, the required first three mutual relations are treated. Then the product of the precision of the two inversely related properties of one entity, $p_1 q_1 \approx h$ is approximately equal to a Planck (the minimum calibration state), which supports the measurement precision quantization proposed in this paper. A more formal analysis of the relation between uncertainty and precision is presented in the RMT paper.

Heisenberg's assumption that classic measurements in theory can be without uncertainty is shown to be invalid in Section 5, above.

6.3 Double slit experiments

Feynman's [12] explanation of the double slit experiments offers an example of how a relative scale (whose property is defined by *U*) defines a property of a measurand. Feynman concludes, "...when we look at the electrons the distribution of them on the screen is different than when we do not look."

In these experiments the relative scale is a set of slits with a sensing screen which projects both length and momentum properties onto the screen. It is well understood that particles have multiple properties. An operator, looking at the sensing screen, identifies a dot indicating the particle's property of momentum. The same operator, looking at the same sensing screen identifies a wave's property of length (wavelength of the frequency determined by the slits), but

the operator is applying one sensing screen presenting two different relative scales. The operator's selection of a wavelength or dot reading on the sensing screen is a relative scale selection (first mutual relation). When selecting a relative scale is not included in measurement theory, the operator's choice (by looking) is perplexing.

7. Relating relative measurement theory to other theories

In 1891, J. C. Maxwell [13] proposed that a measurement result quantity is:

$$\text{measurement result } \textit{quantity} \quad q = n \times u \tag{9}$$

In (9), n is a numerical value, and u is a unit ("taken as a standard of reference" [14]). From Maxwell's formal definition and quote, u is equal (without $\pm$ precision) to a U standard unit. Then (9) indicates that perfect precision is possible in theory, which makes all units equal in theory. and a standard arbitrary in the same theory, well before Heisenberg uncertainty identified that perfect precision is impossible.

Currently the precision of u_n appears in QM as quantum uncertainty; in metrology, this quantization effect is empirically made smaller than noise or distortion and is usually ignored. W. Heisenberg formally presented uncertainty in QM [15]. RMT provided a formal verification that a calibration state at the limit of precision and resolution (a Planck) is quantum uncertainty. Given this verification, the defined time-independent calibration states, which are states of each u_n, may be correlated with the defined time independent stationary states in QM [16].

In the 20th century, von Neumann's Process 1 QM measurement function [17] was developed. von Neumann's Process 1 formalizes a measurement as the sum of inner products of the stationary states (f_n) of a measurand and a relative scale. Process 1 includes a statistical projection operator, $P_{[\phi n]}$[18], which projects each now statistical state on each inner product. von Neumann does treat the first three mutual relations by comparing the stationary states of the measurand's measurement result to the measurand's prior measurement result. Then the comparison of eq. (5) to Process 1 is possible, recognizing that the statistical calibration states are correlated to the statistical states in Process 1.

A. Einstein based the special and general theory of relativity [19] on a relative measurement paradigm. As examples: in special relativity a rod = u. In general relativity ds in a continuous (Gaussian) four dimensional space is the equivalent of a calibration state in a one dimensional quantized space. Since this paradigm is not currently applied in QM, applying a relative measurement paradigm will assist the integration of these two formal forms of physics.

The Einstein, Podolsky, Rosen (EPR) paper titled: "Can quantum-mechanical description of physical reality be considered complete?" [20] can be answered in the affirmative only when the relative measurement paradigm is applied in QM theory. The EPR paper considers a classic measurement, which is not possible in experiment or theory, and does not recognize the need for a standard (which Bell termed a non-local structure[21]) for an independent measurement result to occur.

Fig. 1 relates representational measurement theory (local) with the calibration and not-local aspects of a relative measurement system. D. H. Krantz, et al [22], note that representational measurement theory does not recognize a quantity; assumes measurement result comparisons can occur without a relative scale or standard; treats units as equal [23], which requires any calibration to be empirical [24]; and indicates that all measurement result quantity deviation is due to noise, and distortion in the measurement system [25]. RMT presents a different view.

Since the relative measurement paradigm is not applied in QM theory, other perplexing effects have been noted. In Measurement Unification, 2021 [26], explanations are given of quantum teleportation experiments and Mach-Zehnder interferometer experiments. The Schrödinger's Cat thought experiment is explained in a short preprint [27]. These, together with the explanations in this paper, strongly support RMT.

8. Conclusion

Representational measurement theory is long standing dogma that will eventually change. A physical standard (U) is the basis from which all physical phenomena are defined. Without such standards any understanding of physical phenomena is less precise.

Examined closely, all repetitive measurement result Quantities in theory and experiment have a deviation relative to a standard. Then a comparable, precise, independent, measurement result Quantity is:

$$(n \pm \text{accuracy}) \sqcup (u_n \pm \text{precision}) = (n \sqcup u_n) \pm \text{deviation.} \qquad (10)$$

These are the new measurement result Quantities.

Perhaps, when Maxwell indicated that the $u = U$, he assumed that a theoretical measurement result could be exact, that calibration is empirical and u was arbitrary. This appears to have instigated the limited representational measurement theory which has continued and was not reconsidered, even after Heisenberg's uncertainty was recognized.

When calibration is only an empirical process, then u may be unitary in theory and a standard is not required. When a standard is not required to define a property, the ratio of two measurement results are relative to each other rather than each relative to U. Then the forms of quantum uncertainty appear as relations between Fourier duals, not as the precision of any quantized measurement result Quantity relative to a standard in a state space. When the relative measurement paradigm is applied, measurement results and Gaussian distributions in physics are no longer perplexing.

Annex A Definitions

1.0 Units have multiple definitions

Currently accepted in physics and mathematics:

- In representational measurement theory (all physics except metrology) *u* is defined to be *U*.
- In metrology (assumed only empirical), *u* is commonly the mean *u* which is calibrated to a standard unit or a factor of a standard unit.
- In statistics, a mean *u* or *U* may be a standard unit.
- In bra-ket notation (common QM notation), a state is a ket vector representing *u* or *U*, and is treated as unity [28].

In this paper:

- *u* identifies each of the smallest *u* of a local relative scale without calibration to *U*. Each uncalibrated *u* has a local property, local size and undetermined precision. Local means correlated only to other *u* on the same relative scale or projected measurand.
- u_n has a numerical value (*n/m*). Each u_n calibrated to *U* has a not-local property, not-local size and precision relative to *U* or a factor thereof.
- *U standard unit* (capitalized), is not-local with a defined physical property and a defined numerical value of reference scale resolution states. Only *U* may be defined as a true value, although it will have a resolution in a quantized state space. *U* represents one of the seven different BIPM base properties and units or their derivations [29].

2.0 Additional definitions used in this paper

Similar to the changes proposed for the unit definitions, the paradigm change proposed in this paper necessitates changing or adding other basic measurement definitions.

Accuracy is the ± change of the numerical value (*n*) of a measurement result Quantity relative to its mean numerical value.

Calibration state, *1/m*, is the smallest defined equal state of a relative measurement system. A calibration state quantifies u_n and is a stationary (time independent) state of u_n.

Deviation is the ± statistical sum of precision and accuracy. The ± deviation of one measurement result of a distribution of repetitive measurement results.

Precision is the ± change of one or more u_n relative to a *U* standard or factor thereof as determined by calibration.

Relative scale may be considered a one dimensional coordinate system or a theoretical measurement apparatus. A relative scale establishes the equivalence of a property, as well as the order and additivity of the *u* set. [30] Both the size and the precision of the relative scale

intervals relative to *U* are not defined. This paper only addresses additive relative scales. For comparison purposes, a relative scale may provide a *reference point*, e.g., *0* reference point.

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