Quantum mechanical and classic measurement result quantities are equal (even though their numerical values are not)

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Abstract: *A classic (i.e., physical) measurement result is a quantity (e.g., 6.9 seconds, 13 metres, 0.17 grams, etc.). A quantity is the product of two terms: a numerical value and a unit. The unit when calibrated to a standard has a +/- precision. In quantum mechanics (QM) a wave function produces a distribution of eigenvalues of unity eigenvectors. Each product of one eigenvalue of eigenvectors also represents one measurement result quantity. In a QM measurement process this distribution of quantities appears to collapse into one classic measurement result quantity (one numerical value and a unit). In classic measurement result quantities, (without noise or distortion) the numerical value of an observable appears fixed over repetitive measurements and the units have a precision relative to a standard. However in a QM measurement result quantity, the eigenvalues have Heisenberg's uncertainty and the eigenvectors are unity. This paper formally develops and verifies that both the QM and classic measurement result quantities are equal products and the classic unit precision is equal to the QM eigenvalue uncertainty.*

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INTRODUCTION

A BIPM [1] standardized measurement result is a quantity (defined in The International Vocabulary of Metrology (VIM) [2]) which is the product of two terms: a numerical value and a unit. For simplicity this paper only addresses the effects of quantization on a measurement result quantity without the effects of noise (external to the measurement system) or distortion (internal to the measurement system). Table 1 describes one example and three measurement forms.

	quantity	
measurements	numerical value	unit
1. time	13	seconds
2. classic (measurement instrument never calibrated)	numerical value	mean interval
Calibration changes each measurement instrument interval into a unit (or factor thereof) with a +/- precision.		
3. classic (calibrated measurement instrument)	numerical value (repetitive results are equal)	unit +/- precision
4. QM wave function	eigenvalue +/- uncertainty	eigenvector (unity)

Table. Measurement result quantities

L. Euler identified that an observable's measurement results are relative [3]. A physical observable is relative to a measurement instrument and the measurement instrument is relative to a physical standard, which is defined. The measurement instrument's property itself, as well as the numerical value and precision (see definition Section 4.0) of each measurement instrument's interval (MII), is relative to the standard. See Appendix A for an example.

In the Table the classic (3.) and QM wave function (4.) quantities, are shown to be equal by developing a measurement function that applies to both. Then the EPR paper titled: "Can quantum-mechanical description of physical reality be considered complete?" [4] can be answered in the affirmative when the relationship (i.e., calibration) of a measurement result quantity to a non-local physical standard is treated (see Fig. 1).

1. A Quantity

This paper applies and expands quantity calculus [5] to represent a measurement function. Currently quantity calculus defines a quantity (lower case) as the product of a numerical value (n) and unit (u). The relationship between the u in a quantity and each MII is defined by calibration and considered only empirical in the current theory of a measurement [6].

The current theory of measurement assumes that all u are equal, u calibration to a reference is empirical, and after calibration the mean MII = u. These three assumptions are self-perpetuating. That is, each assumption affirms the other two. This is why these three

assumptions have continued. Recognizing that Heisenberg uncertainty, which appears in both theory and experiment, opposes these three assumptions, this paper removes these three assumptions by applying a more rigorous (summation) measurement function:

$$\text{measurement result } \textit{Quantity } Q = \sum_{n=1}^{n} u_n \quad (1)$$

In (1) each u_n is the smallest interval of an additive scale without any u_n calibration, including during the design or construction of experimental measurement instruments.

A quantity ($n \times u$) is expedient for experimental measurements. A Quantity, see (1), generates a proper superset of a quantity and includes the result $n \times u$. (1) is proposed as the basic mathematical function that also describes all experimental measurement processes.

2. A relative measurement system

As first proposed in Relative Measurement Theory (RMT) [7], a relative measurement system (Fig. 1) consists of the sub-systems: local, calibration, and non-local. This relative measurement system is without any noise or distortion, and presents the descriptions (two or more additive scales), functions (equation (4)), and processes (measurement instruments) of a relative measurement system in theory and experiment.

In Fig. 1. the upper dashed outline (local) is a local measure system including the observable (quantum mechanics term) or measurand (metrology term) and the measure instrument with an additive measure reference scale, following (1). The lower dashed outline (calibration) includes the calibrate instrument with additive equal states which correlates the local and non-local sub-systems.

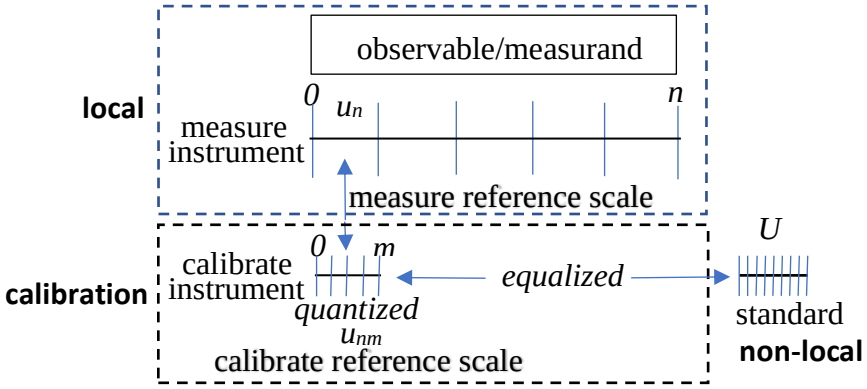

Figure 1. Relative measurement system.

The u_n intervals, where the mean interval is *1/n*, quantify the measure reference scale; the *m* equal states, each defined to be *1/m*, quantify the calibrate reference scale. $\pm 1/m$ is the precision of u_{nm} relative to *U*. The observable/measurand has an exact numerical value of $n \times m$ of *1/m* states.

Fig. 1 shows the successively smaller intervals of each horizontal scale, between the small vertical lines, that are required for a functional measurement system. That is, the measure reference scale mean intervals, *1/n* > calibrate reference scale states, *1/m* > *U* reference scale intervals. Each interval or state must be ^{3} the resolution which is the smallest identifiable change of each physical entity. Notice that *n* and *m* are integers (representing counts) when *1/n* and *1/m* represent the smallest intervals or states of their respective reference scales.

The calibrate reference scale quantizes and equalizes each u_n which determines the precision of each u_n to U. When each u_n is quantized and equalized to U, all u_n are *calibrated*:

$$\text{calibrated } u_n = u_n \pm 1/m = u_{nm} \qquad (2)$$

Although the numerical value of U does not appear in (2), the numerical value of U is not arbitrary. The numerical value of U determines the precision of each $u_{nm} = U \pm u_n$ in *1/m* states when there is noise or distortion (i.e., all experimental measurements). Changing u_n in (1) to u_{nm} from (2), produces:□

$$\text{measurement result Quantities } = \sum_{n=1}^{n} \left(u_n \pm 1/m \right) \qquad (3)$$

The *±1/m precision*, which changes each u_n individually, when summed over many measurement results establishes a Gaussian measurement result distribution. In statistically rare cases the distribution created by each *±1/m* is wide (see the examples: 5.1, 5.2 and 5.3 below). For a measurement result to include this statistical effect, a quantity (product) must be changed to a Quantity (summation). When a quantity (product) is used, this statistical effect is not treated and measurement discrepancies appear.

Currently the quantization of u_{nm} (i.e., *±1/m*) in QM is termed uncertainty and in metrology may be termed accuracy, precision or resolution. W. Heisenberg formally presented uncertainty in QM [8]. RMT included a verification that the *±1/m* quantization at the limit of precision and resolution (a Planck) is Heisenberg's uncertainty.

The differences that have emerged between classic and QM measurement results are based upon the different definitions of a unit. Understanding these differences requires a close look at the definitions.

3. Units have multiple definitions

Current accepted:

- In metrology, *u* is commonly the mean of the MII which is calibrated with a precision to a standard or a factor of a standard.
- In statistics, a *u* may represent a mean or a standard.
- In QM (a normed vector space), an eigenvector is a unity property numerically equal to one which may be assumed to be *U*.
- In representational measurement theory *u* is defined to be *U*.

In this paper:

- u_n identifies each of the smallest MII of a measure reference scale before calibration to *U*. u_n represents an uncalibrated MII (mean $u_n = 1/n$) which has a local property, local size and undetermined precision. The property and size of each u_n are local, i.e., correlated only to other u_n on the same scale.
- u_{nm} is a numerical value of a MII calibrated to *U* and expressed in *1/m*. Each u_{nm} has a non-local property, non-local size and relative precision. The *1/m* intervals are the smallest states of the calibrate reference scale and relative measurement system.
- *U standard*, *U* (capitalized), is a non-local standard with a defined physical property and a defined numerical value. *U* represents one of the seven different BIPM base properties, their units or derivations. *U* may be defined without ± precision (i.e., exact). A *U*'s property and unit are required as a reference and *U*'s

numerical value is required to determine the precision of independent measurement instruments.

4. Additional definitions used in this paper

The u_n, u_{nm} and U definitions proposed above necessitate revising other definitions from the International Vocabulary of Metrology (VIM). The definitions below are related, where possible, to VIM definitions.

Quantity may be a product (q) or a sum (Q) as developed by equations (1) - (3). In VIM a quantity is a product, because instrument calibration is assumed to establish sufficient u_{nm} precision when $n>>m$.

Reference scale is the usually additive local property of the measure or calibrate reference instrument. Of course, a non-additive function between each u_n might be treated. A reference scale must be local to the observable (i.e., properties are locally comparable) to establish a measure. Both the size and the relative precision of the intervals relative to U are not defined. A reference scale has a reference or zero point. VIM applies the term *reference measurement standard* to both scales and standards.

A *measurement instrument* (calibrated in theory and experiment) has a reference scale and may have a transducer which converts an observable's property to a property of this reference scale.

A *measure instrument* (uncalibrated in theory and experiment) has a reference scale and may have a transducer which converts an observable's property to a property of this reference scale. A measure instrument determines the n of an observable/measurand.

A *calibrate instrument* (in theory and experiment) also has a quantized scale which, when equalized to a U, defines the property of the measure and calibrate instrument reference scales. The calibrate instrument determines u_{nm} precision.

Calibration in this paper applies to both physics theory and metrology and is defined to be the quantization and equalization (see Fig. 1) of each measurement instrument interval (or factor thereof) to a non-local unit standard. Calibration as defined in VIM is termed *instrument calibration* in this paper, and usually generates a mean u_n. Instrument calibration may include adjusting the n numerical value as adjusting the many u_n of an instrument is often not practical.

Precision is the deviation of the unit u_{nm} of a measurement result Quantity relative to a U standard. Precision is caused by the quantization due to calibration. This definition is more rigorous than the VIM definition.

Accuracy is the deviation of the numerical value of a measurement result Quantity relative to the mean numerical value (independent of the precision). In the VIM empirical definition, measurement accuracy includes the precision. This paper presents relative measurement theory, where *dispersion* is the sum of the precision and accuracy of a Quantity.

5. Empirical examples

Recognizing the subtle, but extensive, definitional changes proposed, some examples may be helpful to recognize how these definitional changes apply to specific measurements.

5.1 Length measurement instrument

A physical metre stick is *100* centimetres, or *100* u_n. Consider *n* (e.g., 0.*50* metre) is a numerical value of u_n and each u_n is calibrated to become u_{nm}. The first calibration is assumed to be a process where each u_n is calibrated to a standard and added to the next u_n. When first calibrated to *U* (a standard metre) each $u_{nm} = (U/100) \pm 1/m$ precision where, say each *1/m* is 1×10^{-6} (a micrometre) and *n* accuracy = $n \pm 1/m$. Without noise or distortion the *n* and *m* Quantity dispersion is determined by the random application of $\pm 1/m$ to each u_n producing a Gaussian distribution. In the rarest two cases, when *n* of the u_{nm}, all with a precision of $+1/m$, are summed and in another measurement of *n* of the u_{nm}, all with a precision of $-1/m$, are summed, the greatest Quantity dispersion occurs $\sim 2(0.50)10^{-6} = 10^{-6}$ metres. Notice that this dispersion increases as *n* increases. This dispersion is graphed in Fig. 2 below and formally developed in Section 6.

5.2 Additive reference scale

An example of an additive reference scale is a thermometer which measures the property of thermodynamic temperature. This example demonstrates how additive intervals statistically increase the dispersion of measurement results, producing a Gaussian measurement result distribution.

The measurement instrument consists of a hollow glass tube with a reservoir filled with mercury at one end, which fits inside another hollow glass tube that slides over the first. The two glass tubes are held together and placed in an adjustable temperature oven which has a resolution of 0.1^0 (degree). Then the outside glass tube is marked at the level of mercury which appears for the first degree state and each 1.0^0 u_n above the zero mark. $n + 1$ marks (e.g., $n = 100$ in the Celsius system) or *101* marks are made to quantize the outside glass tube. Each of the *100* u_n is correlated by the oven to $1/0.1 = 10 = u_{nm} \pm 0.1^0$ precision.

After 101 marks are made, the instrument is removed from the oven and an ice water bath is applied to the tube with mercury. The outside glass tube is now slid over the inside glass tube until the top of the inside mercury column lines up with the first mark on the outside glass tube. Now one mark on the outside glass tube is referenced to the temperature of ice water (0^0C) which is one reference point for thermodynamic temperature.

Consider the temperature of a glass of water in contact with the reservoir of the referenced measurement instrument. If the temperature of the water is 70^0, the *71st* mark on the outside glass tube represents $70^0 \pm 0.1^0$ nominal precision or $\pm 7^0$ worst case precision dispersion. The $\pm 0.1^0$ nominal precision occurs when the $\pm 0.1^0$ u_{nm} precision of all *70* u_n is uniformly distributed. This distribution of measurement results is termed a Gaussian distribution (see Section 5.3).

The $\pm 7^0$ precision dispersion occurs (very, very rarely) when each of the *70* u_n has the same $+0.1^0$ or -0.1^0 precision, which then sums. This statistical effect is ignored when defined

equal units are applied (e.g., in an eigenvector representation of a measure result). Notice there is also a $70^0 \pm 0.1^0$ numerical value of n accuracy in this example.

5.3 Gaussian normal measurement distributions

Fig. 2 presents the characteristic Gaussian shape of a large distribution of repetitive experimental measurement results. This shape has been verified in many different forms of measurement results where noise and distortion have been minimized [9]. The Gaussian shape of the u_{nm} precision dispersion of a distribution in Fig. 2 is created by the quantization of calibration (as discussed in Section 5.2). This provides another verification of the requirement to treat quantization by calibration in both physics theory and experiment.

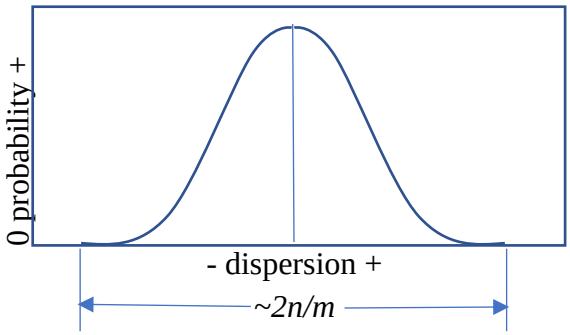

Figure 2. Gaussian distribution of measurement results.

5.4 Comparison of two measurement instrument results

A comparison of two measurement results (q_a and q_b) from two measurement instruments (a and b) is a ratio of their numerical values (n_a and n_b) and the numerical values of their mean u (i.e., u_a and u_b), shown as: $n_a u_a / n_b u_b$. Currently in physics theory $u_b = u_a$ is assumed because calibration is assumed. Given Heisenberg's uncertainty in theory and unit precision in experiment this assumption is not valid in all cases.

In this paper, since calibration is applied in theory, the calibration of a and b to U refines u_a into u_{am} and u_b into u_{bm}, which gives the ratio as $n_a u_{am} / n_b u_{bm}$. Then u_{am} and u_{bm} cancel because they are equalized by calibration.

Without the calibration of u_a and u_b the ratio $n_a u_a / n_b u_b$ has a different numerical value than the ratio $n_a u_{am} / n_b u_{bm}$. This difference is then assumed to be a difference in the numerical value ratio of n_a/n_b (because all u are assumed equal) and is further assumed to be caused by a faster than light mechanism [10].

In current physics theory the calibration of both u_a and u_b is often considered an initial condition, e.g., in QM all eigenvectors are unity. In effect, the Heisenberg uncertainty of an eigenvector is moved to the eigenvalue. Calibration is necessary in theory to equalize the numerical values of u_{am} and u_{bm}, because a factor change in the numerical value of both u_{am} and u_{bm} should not change the ratio n_a/n_b. In this manner a numerical value of centimetres (e.g., *1/100* of a metre) is compared with a numerical value of a metre in 5.1, above [11].

J. S. Bell, in his paper [12]: "...there must be a mechanism whereby the setting of one measurement device can influence the reading of another instrument, however remote." There

have been many attempts to understand this discontinuous behavior. J.S. Bell's formalization, which perhaps consolidates the earlier attempts, is addressed below.

N. D. Mermin [13] described logically the discontinuity that Bell formalized in QM. The states of two particles relative to two remote detectors (the measurement instruments) appear to be more numerous (eight states) than a pair of particles provides (four states). That this effect is described by Mermin without QM formalism indicates it is basic to all measurements.

Applying quantity calculus, a particle has 2 numerical values (e.g., spin up or spin down). Then a pair of particles has 4 numerical values. In Mermin's description the two measurement instruments each have a 3 position selector which selects the *u* (spin direction unit) detected. There are 9 possible states of the two selectors: 3 when each selector is in the same position (calibrated) and 6 when the positions of the two selectors are different (uncalibrated).

Only when the two measurement instruments are calibrated to each other (Mermin's case a) is the measurement of a particle value in units of spin direction possible. When the two remote selectors are uncalibrated (Mermin's case b), the 4 *n* values of the two particles appear randomly just as quantity calculus indicates. Calibrated/uncalibrated (2 sets of states) is the function that multiplies the 4 numerical values to the 8 states Mermin identifies.

Restating the above, the *a* and *b* measurement instruments are independent but the measurement results q_a and q_b, whose units (u_{am} and u_{bm}) are equalized by calibration, appear to "...influence the reading of another instrument..." (i.e., they are calibrated) across any distance. That is, the measurement result ratio $n_a \times u_a / n_b \times u_b$ will be different (especially at quantum scales) when u_a and u_b are not calibrated in theory to each other. This quantity calculus development of the comparison of two measurement instrument results, identifies that calibration is the mechanism Bell describes. This is evidence of the significance of calibration to the quantum mechanical description of physical reality.

6. The effect of u_{nm} on precision and dispersion

A local comparison which determines the precision of two or more measurement results requires a common local property and a common local u_n which are provided by a local measure reference scale. A non-local comparison requires the precision of u_n to be determined by calibration to the numerical value of *U* which also defines the property being compared.

From (3) *n* is modified to include its accuracy, producing:

$$\text{measurement result Quantities} = \sum_{n=1}^{n\pm 1/m} \left(u_n \pm 1/m\right) \qquad (4)$$

Equation (4) is a measurement function that applies to formal results and experimental processes. From (2):

$$u_{nm} \text{ precision } = \pm\left(1/m\right) \qquad (5)$$

This precision, ± *1/m* in each of the *n* u_{nm}, varies randomly and sums (see (4)) into a measurement result. Then the sum of the *n* accuracy and u_{nm} precision is:

$$\text{the worst case accuracy and precision of (4)} \;= \pm\left(n \pm 1/m\right)\left(1/m\right) \qquad (6)$$

Equation (6) also identifies how different measurement result Quantities of the same observable appear when $n \sim m$ (quantum) and not when $n >> m$ (classic). Then from (6) the total number of $1/m$ states: $(n/m + 1/m) + (n/m - 1/m)$ equals:

$$\text{the maximum Quantity dispersion of (4)} \;= (2n/m) \pm (1/m) \qquad (7)$$

Equation (7) explains how the Gaussian shape of a normal distribution of measurement results shown in Fig. 2 occurs, and identifies that every calibrated measurement result Quantity has a dispersion which is determined by both the n and the u of a Quantity. In (4) when $n >> m$, the sum of each $\pm 1/m$ cancels, or close to cancels in a high probability of measurement results (see 5.2), due to the central limit theorem's effect on a normal (i.e., symmetric) distribution of measurement results.

Conversely, when n is small (neutron spin measurements $n = 2$) [14], the sum of each $\pm 1/m$ will not generally cancel. In such cases, two repetitive measurement result Quantities of the same observable will likely be different. This is considered in QM as repetitive measurement results that do not commute (i.e., $xp \neq px$). This violation of arithmetic appears because the precision of the units associated with the x and p quantities is not treated.

This paper has developed how, when the smallest quantization due to calibration is included, repetitive measurement results will generally not be equal. When the $\pm 1/m$ (u_{nm} precision) relative to U approaches the size of a measurement result Quantity, U is not valid to apply and u_{nm} is necessary. At this size of a Quantity the u_{nm} precision produces an identifiable change in a measurement result Quantity ratio.

7. Applying a quantity explains perplexing experiments

7.1 Heisenberg's quantum uncertainty

Heisenberg's thought experiment [15] analyzes p_1x_1, the product of the variation of the momentum ($p = Mv$) and the variation of the position (x), to understand quantum uncertainty. The mass M is assumed constant. Heisenberg notes the difficulty of treating position and velocity without contradiction, but does not recognize that this requires the use of units as his thought experiment considers only numerical values.

The frequency (f) of the light which examines p and x determines the minimum length variation, x_1 which is the wavelength unit, $\lambda = c/f$. This wavelength unit is the limit of a position's precision that can be measured. Notice that each λ is equivalent to $1/m$ developed previously, the smallest x position state. Heisenberg recognizes that the frequency of the light determines the λ and the period T, but, since calibration is empirical in current theory, does not identify that setting f is a calibration process which defines what this paper terms the u_n precision. This calibration process also creates an entropy change ($log\ m$) which Heisenberg refers to as a "quantum jump".

The minimum x_1 to measure v is one λ unit. The numerical value of one period to measure v is $T = 1$ in units of $1/f$. Given velocity $v = x/t$, then $v = \lambda/T$ without contradiction. Multiplying $p_1 = \lambda/T$ (ignoring the constant M) by $x_1 = \lambda$, the product is λ^2/T. Since the numerical value of $T = 1$, x_1 is λ in each term p and x.

Recognizing the Heisenberg uncertainty of the product of these two canonically conjugate eigenvalues is equal to the square of their unit of length per *T,* further verifies Relative Measurement Theory.

7.2 Double slit experiments

The double slit experiments [16] offer an example of how calibration defines a property. In the double slit experiments, both the momentum (particle) and frequency (wave) properties of each particle appear in one measurement experiment. The measurement instrument in these experiments includes a sensing plate which is both a momentum and a frequency transducer, while the slits are the measurement reference scale. It is well understood that all particles have multiple properties, i.e., momentum, frequency, spin, mass, etc.

An operator, reading the sensing plate, identifies the property of momentum, another reading of the same sensing plate identifies the property of frequency. The operator's choice of a dot (momentum) or pattern (frequency) reading on the sensing plate is a property selection process which usually occurs by calibration. When calibration is assumed to be solely empirical, the operator's selection of a dot or pattern representing a property (a calibration process) appears as particle duality rather than the selection of a property by calibration.

8. Relating relative measurement theory to other theories

In 1891, J. C. Maxwell [17] proposed that a measurement result quantity is:

$$\text{measurement result } \textit{quantity} \quad q = n \times u \tag{8}$$

In (8), *n* is a numerical value, and *u* is a unit ("taken as a standard of reference" [18]), which together form a mutual relation, capable of being compared. Equation (8) is the basis of quantity calculus. From Maxwell's usage and quote, *u* is a mean that is equal (without $\pm$ precision) to a *U* standard unit. Equation (8) and Maxwell's usage suggests that perfect precision is possible, well before Heisenberg's uncertainty determined that such precision is impossible.

In the 20th century, QM offered a new measurement function, von Neumann's Process 1 [19]. Process 1 includes a statistical operator, $P_{[\phi n]}$[20], which is equivalent to $\pm 1/m$ in this paper. Both von Neumann's Process 1 and Dirac's bra-ket notation (without a statistical operator) treat a measurement as an inner product of eigenvalues and eigenvectors. Then the comparison of eq. (4) to Process 1 or the use of bra-ket notation is straight forward when the equal *1/m* states are treated as eigenstates.

Fig. 1 (above) relates a representational measure theory [21] (local) with the calibration and non-local portions of a relative measurement system. This representational measure theory does not recognize that a quantity is relative; assumes measure result comparisons can occur without a reference scale or standard; treats a unit as arbitrary [22], which requires any calibration to be empirical [23]; and indicates that all measure result quantity dispersion is due to noise, distortion and errors in the measurement system [24].

The basic criteria, as explained in the EPR paper, that experimental measurement results (physical reality) can in theory be certain ("predict with certainty"), is not rigorous.

Experimental measurement results can only be as precise as a calibration process defines. This changes the EPR criteria for physical reality. All measurement results are distributions and appear in metrology with ± precision and in QM theory with uncertainty.

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Since a Quantity consisting of a numerical value and a u_{nm} has not been applied in QM for almost 90 years, many perplexing effects have been noted. Measurement Unification, 2021 [25] explains how in the Stern-Gerlach experiments that J. S. Bell considered, the effect (two new sets of states) of the calibration of each instrument to the other is not recognized. Other explanations are given of quantum teleportation experiments, Mach-Zehnder interferometer experiments, Mermin's device (also based upon the Stern-Gerlach experiments), and the Schrödinger's Cat thought experiment. These explanations in Measurement Unification verify how u_n calibration unifies experimental processes and QM measurement functions.

9. Conclusion

This paper has attempted to identify and resolve the existing assumptions about units by expanding quantity calculus. Perhaps Maxwell's quantity, which assumed that the mean $u_n = U$, misled theorists to treat calibration as only an empirical process. However, QM evolved away from Maxwell's single dimension quantity with *n* and *u* to the richer inner product of a eigenvalues and eigenvectors.

However, the precise relationships between *u*, *U* and eigenvectors, determined by calibration, were not considered in QM measurement theory, likely because of the differing definitions of *u*. And when calibration is not included in physics theory, discontinuities appear. The incompleteness identified in the EPR paper can be resolved by including the precision determined by calibration to a non-local unit standard in both QM mathematical functions and experimental measurement processes.

Appendix A

Defining the property that a measurement instrument measures

The first calibration of a measurement instrument to a physical unit standard defines the measurement instrument's property (length, mass, time, spin, etc.).

It is often assumed that a measurement instrument has a physical property (length, mass, spin, time, etc.) that it measures. This is a simplification. All physical properties are relative and must be correlated to a common property (usually represented by a BIPM standard) to be defined.

As example, consider an electrical multi-meter which can display the numerical values of electrical properties in units of volts, ohms and amps on its meter. A rotary switch on the multi-meter selects which unit property to display. What happens if the printing of the terms: volts, ohms and amps on the multi-meter's rotary switch is removed? That is, the visible prior history of the multi-meter's unit properties is removed.

In this case the operator can use a standard voltage (e.g., a known numerical value of volts of a battery - a standard) and apply it to the multi-meter to determine which switch setting causes the display of the numerical value of the battery voltage. The same approach applies for amps and ohms. Without the prior history, the calibration to a standard defines the property as well as determines the precision of the measurement instrument.

The very first calibration of any measurement instrument to a standard, or perhaps an instrument or part that is applied in the design or manufacture of the measurement instrument, defines the property being measured.

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