Possible Hexaquark Explanation for the 4406.9 MeV/c2 Resonance in Electron-Positron Collisions at Center-of-Mass Energies from 4.23 to 4.70 GeV

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Abstract

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The BESIII Collaboration observed two resonance structures in the $e^+ e^- \rightarrow \pi+ \pi- \psi(3823)$ reactions at center-of-mass energies from 4.23 to 4.70 GeV. The first resonance (R_1) has a mass of 4406.9 MeV/c^2 and is based on a $\pi^+ \pi^- \Psi(3823)$ structure. A second resonance (R_2) is based on a $\pi^+ \pi^- \gamma\ \eta_{c1}$ structure with a mass of 4647.9 MeV/c^2. The spin and parity of these states was not provided in Ref. 1.

A first-order hexaquark mass formula is applied to evaluate the possible mass and J^π value of the candidate R_1 hexaquark. The R_2 state, with a photon included in the final state, is not consistent with the first-order hexaquark model, and is not evaluated. The first-order hexaquark model yields a R_1 resonance mass within 1% of the experimental value. The model also suggests that the R_1 has a 2^- spin assignment.

KEYWORDS: First Order Mass Formula, Hexaquark, Weak Coupling Model, $\pi^+ \pi^- \Psi(3823)$ structure, and R_1 Resonance at 4406.9 MeV/c^2.

1.0 Introduction

In Ref. 1, the energy dependent production cross section was measured for the $e^+ e^- \rightarrow \pi^+ \pi^- \Psi(3823)$ reaction at center-of-mass energies from 4.23 to 4.70 GeV, and resonance structures were observed in the cross section with significances exceeding 5σ. The results of the fit to the $e^+ e^- \rightarrow \pi^+ \pi^- \Psi(3823)$ distribution, yield two resonances[1]. The first resonance (R_1) has a mass of 4406.9 MeV/c^2 and is based on a $\pi^+ \pi^- \Psi(3823)$ structure. A second (R_2) is based on a $\pi^+ \pi^- \gamma\ \eta_{c1}$ structure with a mass of 4647.9 MeV/c^2. The spin and parity of these states was not provided in Ref. 1.

As noted in Ref. 1, these particles are not readily accommodated in the spectrum of conventional charmonium states, and are considered to be potential candidates for QCD exotic hadrons. In this paper, the general first-order weakly coupled meson-meson-meson hexaquark mass formulas of Refs. 2 - 5 are applied to evaluate the possible mass and J^π value of the candidate R_1. The R_1 mass relationship is specifically based on a weakly coupled $\pi^+ \pi^- \Psi(3823)$ structure that follows the model of Ref. 4. A weakly coupled $\pi^+ \pi^- \gamma \eta_{c1}$ structure is not used to model R_2. The model of Ref. 4 can describe the $\pi^+ \pi^- \eta_{c1}$ structure, but the γ component in the final state is not included in the model. Therefore, the proposed weak coupling mode is not expected to reasonably describe the R_2 mass. Accordingly, this paper only addresses the R_1 resonance.

2.0 Model and Formulation

Zel'dovich and Sakharov[6,7] proposed a semiempirical mass formula that provides a prediction of meson and baryon masses in terms of effective quark masses. Within this formulation, quark wave functions are assumed to reside in their lowest 1S state. These meson mass formulas are used as the basis for deriving a first-order hexaquark mass formula. In particular, the model proposed in this paper assumes the hexaquark is partitioned into three meson clusters with the interaction between the clusters providing a minimal contribution to its mass.

The meson mass (M_m) formula of Refs. 6 and 7 is:

$$\mathbf{M_m = \delta_m + m_1 + m_2 + b_m [m_o^2 / (m_1 m_2)] \sigma_1 \cdot \sigma_2} \quad \mathbf{(1)}$$

where m_1 (m_2) are the mass of the first (second) quark comprising the meson, m_0 is the average mass of a first generation quark[8,9], and the σ_i (i = 1 and 2) are the spin vectors for the quarks incorporated into the meson. The parameters δ_m and b_m are 40 MeV/c^2 and 615 MeV/c^2, respectively[7].

The last term in Eq. 1 represents the spin-spin interaction of the quarks and $\boldsymbol{\sigma_1 \cdot \sigma_2}$ is the scalar product of the quark spin vectors. $\boldsymbol{\sigma_1 \cdot \sigma_2}$ has the value -3/4 and +1/4 for pseudoscalar and vector mesons, respectively[7].

In formulating the hexaquark mass formula, effective quark masses provided by Griffiths[8] are utilized. These effective masses for d, u, s, c, b, and t quarks are 340, 336, 486, 1550, 4730, and 177000 MeV/c^2, respectively. The effective masses are utilized in Eq. 1.

These six quarks are arranged in three generations: [d(-1/3), u(+2/3)], [s(-1/3), c(+2/3)], and [b(-1/3), t(+2/3)][9]. The three generations are specified by the square brackets and the quark charges [in elementary charge units (e)] are given within parentheses.

The hexaquark mass (M) is modeled in terms of three weakly interacting meson clusters M_m(i)

$$\mathbf{M = M_m(1) + M_m(2) + M_m(3) + \Phi_{12} + \Phi_{23}} \quad \mathbf{(2)}$$

where i = 1, 2, and 3, and Φ represents the interaction between the clusters.(e.g, Φ_{ij} is the interaction between meson cluster i and cluster j). Other interaction structures (e.g., $\Phi_{12} + \Phi_{13} + \Phi_{23}$) are possible, but the form of Eq. 2 is representative given the assumed zero angular momentum value between the weakly coupled clusters. Moreover, the meson masses are assumed to be much larger than the Φ_{ij} values.

3.0 First-Order Mass Formula

The spin and parity of a hexaquark within the first-order mass formula, summarized in Section 2.0, is determined by coupling the three meson clusters

$J^{\pi} = J^{\pi}(1) \times L(12) \times J^{\pi}(2) \times L(23) \times J^{\pi}(3)$ (3)

where the first-order mass formula assumes a minimally interacting L(12) = L(23) = 0 configuration[2-5] between the clusters. Considering that L(12) = L(23) = 0, Eq. 3 simplifies the weakly coupled three meson system to the form

$J^{\pi} = J^{\pi}(1) \times J^{\pi}(2) \times J^{\pi}(3)$ (4)

Eq. 4 provides a primitive J^{π} assignment using the three meson clusters comprising R_1. The π^+ and π^- have a 0^- assignment, and the ψ(3823) is a 2^- state. Using Eq. 4, the R_1 has the following assignment

$J^{\pi}(R_1) = 0^- \times 0^- \times 2^- = 2^-$ (5)

The R_1 mass is obtained using the methodology of Refs. 2-5

$M(\pi^+ + \pi^- + \psi(3823)) = M(\pi^+) + M(\pi^-) + M(\psi(3823))$ (6)

where the Φ_{12} and Φ_{23} noted in Eq. (2) are sufficiently small relative to the meson masses to be ignored[2-5]. Accordingly, Eq. 6 represents the R_1 structure as a quasimolecular six quark systems whose basic character is a weakly bound meson-meson-meson systems where the mesons reside in their ground states.

Since the ψ(3823) is not the ground state, Eq. 6 is modified following the approach of Refs. 2-5

$M(\pi^+ + \pi^- + \psi(3823)) = M(\pi^+) + M(\pi^-) + M(\psi(1S)) +$

$\Delta(M(\psi(3823)) - M(\psi(1S)))$ (7)

where $\Delta(M(\psi(3823)) - M_{sm}(\psi(1S)))$ is the mass difference between the two states.

4.0 Results and Discussion

The angular momentum coupling from Eq. 5 and the first-order hexaquark mass formula of Eqs. 2 and 7 are used to construct the $\pi^+ \pi^- \Psi(3823)$ state. As noted previously, the spin and parity assignment for the R_1 is determined to be 2^-. No spin assignment was provided in Ref. 1.

Eqs. 2 and 7 lead to a predicted R_1 mass of 4383.6 MeV/c^2. This result is about 1% smaller than the experimental value[1] 4406.9 MeV/c^2. Although these results are encouraging, they are based on a first-order mass formula with a number of uncertainties including the assumed quark masses[8] and the magnitude of the meson-meson cluster interaction. However, the model does provide an initial description of the R_1 state that is reasonable agreement with the experimental data[1].

5.0 Conclusions

The recently proposed R_1 structure[1] is investigated using a first-order hexaquark mass formula. This mass relationship is based on weakly coupled $\pi^+ \pi^- \psi(3823)$ meson clusters. The assumed $\pi^+ \pi^- \Psi(3823)$ configuration leads to a mass result that is within about 1% of the measured R_1 value[1]. A J^π value of 2^- is also predicted. These results suggest the R_1 structure[1] is consistent with a weakly bound $\pi^+ \pi^- \psi(3823)$ meson cluster system.

References

1) BESIII Collaboration, Observation of Resonance Structures in $e^+ e^- \rightarrow \pi^+ \pi^- \psi(3823)$ and Mass Measurement of $\psi(3823)$, Phys. Rev. Lett. **129**, 102003 (2022).

2) J. J. Bevelacqua, Description of Selected Hexaquark States in Terms of a First-Order

Mass Formula, Physics Essays **31** (1), 104 (2018).

3) J. J. Bevelacqua, Description of $\Omega\Omega$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ Dibaryon States in Terms

of a First-Order Hexaquark Mass Formula, Qeios **27N2QF**, 1 (2022).

https://doi.org/10.32388/27N2QF.

4) J. J. Bevelacqua, Possible Hexaquark Explanation for the State X(2600) in the

$\pi^+ + \pi^- + \eta'$ System Observed in the Process $J/\Psi \rightarrow \gamma\, \pi^+ \pi^- \eta'$, Qeios **S7UNV7**, 1 (2023).

https://doi.org/10.32388/S7UNV7.

5) J. J. Bevelacqua, Unflavored Excitations of Nucleon-Nucleon States in Terms of a First-Order Mass Formula, Qeios **LJUROY**, 1 (2023). https://doi.org/10.32388/LJUROY.

6) Ya. B. Zel'dovich and A. D. Sakharov, *Kvarkovaia struktura i massy sil'novzaimodeistvuyushchikh chastits*, Yad. Fiz. **4**, 395 (1966).

7) A. D. Sakharov, Mass formula for mesons and baryons, Sov. Phys. JETP **51**, 1059 (1980).

8) D. Griffiths, *Introduction to Elementary Particles*, 2^{nd} ed., (Wiley-VCH, Weinheim, 2008).

9) Particle Data Group, Review of Particle Physics, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).