Quantum Cosmology: Cosmology directly linked to the Planck Scale in General Relativity Theory and Newton Gravity

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Quantum Cosmology: Cosmology directly linked to the Planck Scale in General Relativity Theory and Newton Gravity

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Abstract

We will in this paper demonstrate that there is a link between cosmology and the Planck scale. It has in recent years been shown that the Planck length can be found totally independent on G, $\hbar$ and c and that a series of cosmological predictions can be predicted only from two constants, namely the Planck length and the speed of gravity. The speed of gravity can be easily found without knowledge of the speed of light. This gives a new perspective on cosmology and shows there is a link between the Planck scale and cosmology. This is fully consistent with a recent quantization of general relativity theory that links general relativity to the Compton frequency and the Planck scale. We look at both the Friedmann cosmology and the recently introduced cosmology based on the extremal solution of the Reissner-Nordström, Kerr and Kerr-Newman metric.

Keywords: Hubble constant, Hubble radius, universe equation, Freedman universe, extremal universe, Planck length, Compton length.

1 Bacground

Quantum cosmology has got increased attention among researchers, see for example [1–9]. However, it is still an unsolved question if there it is considered an unsolved problem if there is a link between cosmology and the Planck scale, just as it is considered an unsolved problem to come up with a quantum gravitational theory. Here, we will demonstrate that there is such a relation and that common cosmological units can be predicted and described through quantum units, in particular with relation to the Planck scale. What we will show is fully consistent with general relativity theory as general relativity recently has been quantized and linked to the Planck scale, see [10, 11]

Max Planck [12, 13] assumed there were three important universal constants, namely G, $\hbar$ and c and based on this and dimensional analysis derived what today is known as the Planck length: $l_p = \sqrt{\frac{G\hbar}{c^3}}$, the Planck time $t_p = \sqrt{\frac{G\hbar}{c^5}}$, the Planck mass $m_p = \sqrt{\frac{\hbar c}{G}}$, and the Planck temperature $T_p = \sqrt{\frac{\hbar c^5}{Gk_b^2}}$.

Already in 1984, Chahill [14, 15] suggests that the gravitational constant can be expressed from the Planck mass on the form

$$G = \frac{\hbar c}{m_p^2}, \tag{1}$$

which is simply the Planck mass formula solved with respect to G. But as pointed out by Cohen [16] in 1987, this seems to lead to a circular problem: one needs to know G to find the Planck units. So, it is of little or no use in expressing G in the form of Planck units if this is the case, a view held to this day. Also McCulloch [17] in 2016 points out the same formula as Cahill and Cohen for G, and that one needs to know G to find the Planck units. In 2016, Haug [18, 19] suggested that the gravitational constant is just a composite constant that can be expressed as

$$G = \frac{l_p^2 c^3}{\hbar}. \tag{2}$$

This is just solving the Planck length formula with respect to G, rather than the Planck mass formula, so it leads to the same circular problem as mentioned. However, in 2017, Haug showed for the first time how the Planck length could be found with no knowledge of G (see in particular the appendix), and later Haug [20–22] has shown how to find the Planck length and Planck time with no knowledge of both G or $\hbar$ in a practical feasible way.

In addition to this composite view of G, we will utilize that the rest-mas in kilogram for any mass, small or large, can be expressed by the formula

$$m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}, \tag{3}$$

where $\bar{\lambda}$ is the reduced Compton wavelength. This formula is nothing more than solving the Compton [23] wavelength formula with respect to m. This is also mathematically trivial, but to express the mass from the Compton wavelength rather than the Compton wavelength from the mass was perhaps first suggested in 2016, see [19, 24]. Masses larger than the Planck mass will have a reduced Compton wavelength shorter than the Planck length, something that seems impossible if one assumes as many physicists that the Planck length is the shortest possible length. However, masses larger than the Planck mass, we will claim, must be composite masses, and any composite mass consists of many elementary particles. Also, masses smaller than the Planck mass can be a composite mass, such as the proton. The reduced Compton wavelength in the elementary particles making up the composite mass can be aggregated with the formula

$$\bar{\lambda} = \frac{1}{\frac{1}{\bar{\lambda}_1} + \frac{1}{\bar{\lambda}_2} + \frac{1}{\bar{\lambda}_3} + \cdots \frac{1}{\bar{\lambda}_n}}, \tag{4}$$

where $\bar{\lambda}$ now is the aggregate of all the reduced Compton wavelength in the elementary particles making up the mass. This is fully consistent and can even be derived from the standard mass aggregation rule for non-bound components; that is, so far, we are excluding binding energy, which is given by:

$$\begin{aligned} m &= m_1 + m_2 + m_3 + \cdots + m_n \\ \frac{\hbar}{\bar{\lambda}}\frac{1}{c} &= \frac{\hbar}{\bar{\lambda}_1}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_2}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_3}\frac{1}{c} + \cdots + \frac{\hbar}{\bar{\lambda}_n}\frac{1}{c} \\ \bar{\lambda} &= \frac{1}{\frac{1}{\bar{\lambda}_1} + \frac{1}{\bar{\lambda}_2} + \frac{1}{\bar{\lambda}_3} + \cdots \frac{1}{\bar{\lambda}_n}}. \end{aligned} \tag{5}$$

However, we will also work with masses consisting of bound elements, and then the plain mass addition above will slightly overestimate the mass aggregate due to ignoring nuclear binding energy; see, for example, Walker, Halliday and Resnick [25]. Plain hydrogen has no nuclear binding energy, as it only consists of a proton and an electron. For known nucleons above plain hydrogen, the nuclear binding energy ranges from approximately 2.23 MeV for hydrogen-2 to 8.79 MeV for nickel-62, that is, from 0.24% to 0.94% of the total observed mass. Not taking into account binding energy can, therefore, result in up to a 0.94% overestimation of the mass based on simply counting protons and neutrons, and up to a 0.94% underestimation of the reduced Compton wavelength. In addition, there are additional binding energies, such as molecular bond energy, but these are extremely small again compared to the nuclear binding energy.

However, we can also treat energy as mass equivalent as quite often is done in physics since we have $m = \frac{E}{c^2}$, so any type of binding energy in a nucleus or a clump of matter can be treated as mass equivalent in this formula with a corresponding Compton wavelength (and reduced Compton wavelength). So we actually can generalize equation 5 to also take into account binding energy and other types of energy, as we can write

$$m = m_1 + m_2 + m_3 + \cdots + m_n + \frac{E_1}{c^2} + \frac{E_2}{c^2} + \cdots + \frac{E_n}{c^2} \tag{6}$$

, where E_1 to E_n are binding energies (these are typically "negative" as binding energy is typically released from the mass when elements bind) as well as other types of potentially relevant energy converted into mathematical equivalent rest-mass. This must hold true for even pure electromagnetic energy, as we must have

$$\begin{aligned} E = hf &= h\frac{c}{\lambda_\gamma} \\ \frac{E}{c^2} &= \frac{h\frac{c}{\lambda_\gamma}}{c^2} \\ m_\gamma &= \frac{h}{\lambda_\gamma}\frac{1}{c} \\ m_\gamma &= \frac{\frac{h}{2\pi}}{\frac{\lambda_\gamma}{2\pi}}\frac{1}{c} \\ m_\gamma &= \frac{\hbar}{\bar{\lambda}_\gamma}\frac{1}{c}, \end{aligned} \tag{7}$$

where λ_γ is the photon wavelength. That is, the equivalent mass from pure energy is identical to a mass that has a reduced Compton wavelength identical to the reduced photon wavelength (see [26] for an in-depth discussion of the importance of the Compton wavelength). The reduced Compton wavelength corresponds to the Photon wavelength divided by 2π; notice we then also used the reduced Planck constant. But we could alternatively write $m_\gamma = \frac{h}{\lambda_\gamma}\frac{1}{c}$ which gives exactly the same value as it simply consists of multiplying with 2π in the nominator and denominator. That is, we have

$$m_\gamma = \frac{\hbar}{\bar{\lambda}_\gamma}\frac{1}{c} = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}, \tag{8}$$

when $\bar{\lambda}_\gamma = \bar{\lambda}$. One cannot do Compton scattering on a photon (at least as far as we know), but one cannot observe the mass of a photon in the standard way, either. Still, the mass of a photon has been used in a series of calculations in a series of papers by assuming $E/c^2 = m$. We are, therefore, talking about the mass equivalence of energy and the Compton equivalent wavelength of a photon. The Compton equivalent wavelength of a photon must, as we see above, be identical to the reduced wavelength of the photon if $m = E/c^2$ must hold true.

Some will perhaps be very critical here and correctly claim the photon wavelength and the Compton wavelength are two totally different things and that this breaks with a basic understanding of concepts. The photon wavelength is, in general, related to the frequency of light, while the Compton wavelength is related to a wavelength one finds from the Compton scattering of such things as electrons. Still, treating energy as mass this way is fully consistent with both $m = E/c^2$ as we can get the correct energy and frequency back by simply multiplying the equivalent photon mass with c^2. The relation $m = E/c^2$ is very well known, and it is well known that $E = h\frac{c}{\lambda_\gamma}$., but where do the Planck constant and the wavelength of light go when we treat it as an equivalent theoretical mass, one normally stop at saying $\frac{h\frac{c}{\lambda_\gamma}}{c^2} = m$, so we will claim the deeper picture is not surprisingly $\frac{hf}{c^2} = \frac{h\frac{c}{\lambda_\gamma}}{c^2} = m = \frac{\hbar}{\lambda_\gamma}\frac{1}{c}$. That is, the wavelength of light then can be treated as an equivalent Compton wavelength. This, in our view, should not be considered more controversial than treating massless photons as equivalent mass, as quite often is done in physics.

Be aware, however, that we could not have done the same when using the de Broglie [27, 28] wavelength. For example, the de Broglie wavelength $\lambda_B = \frac{h}{mv\gamma}$ does not even have a mathematically defined wavelength for a rest-mass, see [26]. Further, when the mass is almost at rest, the de Broglie wavelength goes toward infinity, something that has led to a series of different interpretations of the de Broglie wavelength.

We will here mainly work with the mass at the cosmological scale, where hydrogen and helium are estimated to make up roughly 74% and 24% of all baryonic matter in the observable universe. Hydrogen only consists of an electron and one proton and has no nuclear binding energy but will still have smaller binding energies in the form of gravitational binding energy and molecular binding energies. There is also hydrogen-2, also known as deuterium, in the universe; this consists of one proton plus one neutron and an electron, but estimates indicate that there are only about 26 atoms of deuterium per million hydrogen atoms, so this will, if anything, give an insignificant correction in relation to the nuclear binding energy. Helium-4, on the other hand, makes up almost 24 % of the baryonic mass in the universe and has a relatively high nuclear binding energy of a total 28295.7 KeV, which is almost 0.8% of the total mass of Hydrogen-4. Still, this is only 24% of the baryonic mass in the universe, so not considering the nuclear binding energy would still give a 0.2% error in the total baryonic mass and also in the Compton wavelength that is linearly proportional to the mass size if ignoring binding energy. An error of 0.2% is a lot when working with some experiments in nuclear physics, but when working at the cosmological scales as such things as estimating the mass of the universe or the Hubble constant, these have observation errors much larger than this, typically in the order of percent points. Again, in the main elements making up the universe's baryonic mass, there is quite low binding energy. Actually, the baryonic density is, in standard cosmology, estimated to be between 1 to 15 percent of the critical density; see, for example, Craig, Schramm and Turner [29]. That potentially 95% of the relevant density is due to dark energy could make the binding energy for the total equivalent mass in the universe less relevant; anyway, it should be accounted for in our model, as at the cosmological scale, we will not distinguish between what comes from energy and what from mass. Personally, we are skeptical of the dark energy hypothesis, but that is outside the scope of this paper. The main point is we do not need to differentiate between how much energy is relative to mass in the following analysis.

It is experimentally confirmed that mass has an effect on energy, for example, by bending light (deflection). It is in general consensus that energy, too, in the form of electromagnetic waves, has gravitational effects similar to its equivalent mass; this has likely not yet been exclusively experimentally confirmed, but the interest for it has increased as we are getting closer to be able to do experiments that can actually measure gravitational effects from electromagnetic waves, see [30–35]. If the hypothesis is right that the critical density of the universe mainly consists of energy, then our quantum cosmological model makes it indirectly possible to test if energy has its own gravitational field.

We will use the Compton wavelength and the Planck length to derive new quantum relations for such things as the Hubble scale and the mass in the universe. As will be seen from the formulas we derive, there is a linear relation between, for example, the Hubble constant and the reduced Compton wavelength of the mass in the universe. We will claim that even nuclear binding energy and gravitational binding energy likely will be embedded in our numbers after calibration to such as observed cosmological redshift. This is because both the cosmological models we will be looking into do not distinguish between energy and mass at the level we will be working at here. So, energy will be treated as equivalent mass as we have $E = mc^2$, so we are indirectly taking into account the mass defect. For example, when one works with the mass of the observable universe in the Friedmann model, then the model does not say how much of this is energy or mass, but treats energy and mass as equivalent; that is, we can choose if we want to operate with output units in mass or energy. For other purposes, there is a large body of literature trying to establish how much of the total energy in the universe is mass, how much is energy, and how much is dark energy etc., but that is outside the scope of this paper as the main purpose here is to establish the link between the Planck scale and the cosmological scale, and the methodology presented here also seems to work in practice.

With practice, we simply mean it seems like we are able to find the Planck length and the Compton wavelength of the observable universe (without knowledge of G or h), and based on these plus the speed of light (gravity), we are able to predict such thing as the Hubble constant, the Hubble time, the mass (energy) of the observable universe etc.

If we know the mass of an object in kilogram and the Planck constant, we can find the reduced Compton wavelength from the Compton wavelength formula

$$\bar{\lambda} = \frac{\hbar}{mc}. \tag{9}$$

Also, this formula can be used for any mass size. However, we want to rely on as few constants as possible, and we do not necessarily know the kilogram mass of large objects, so is there a way to find the Compton wavelength of any object without knowing the Planck constant or the object's mass in kilograms? We can start out with the Compton scattering of an electron. That is shooting photons at an electron, and in that case, the Compton wavelength is then given by

$$\begin{aligned}
\lambda_2 - \lambda_1 &= \frac{h}{mc}(1-\cos\theta) \\
\lambda_2 - \lambda_1 &= \frac{h}{\frac{h}{\lambda_e}\frac{1}{c}c}(1-\cos\theta) \\
\lambda_e &= \frac{\lambda_2 - \lambda_1}{1-\cos\theta},
\end{aligned} \tag{10}$$

where λ_1 and λ_2 are the wavelengths of the photon before and after collision with the electron. Further, θ is the angle between the photon before and after collision. In other words, we can find the Compton wavelength of the electron without knowing its mass or the Planck constant. Next, to find the Compton wavelength of the proton, we can utilize that the ratio of the Compton wavelength between the proton and the electron is identical to the cyclotron frequency ratio.

$$\frac{f_e}{f_P} = \frac{\frac{qB}{2\pi m_e}}{\frac{qB}{2\pi m_P}} = \frac{\bar{\lambda}_P}{\bar{\lambda}_e} \approx \frac{1}{1836.15} \tag{11}$$

So, one can just divide the electron Compton wavelength by 1836.15 to get the proton Compton wavelength. The interest in the proton Compton wavelength goes back to at least 1958 in a paper by Levitt [36]; in recent years, there has been increased interest in the Proton Compton wavelength [37]. After one has found the Compton wavelength of a proton, one can find the Compton wavelength in larger macroscopic masses by counting the number of atoms in such a mass. This is at least possible for even hand-size macroscopic uniform masses. To count atoms in such macroscopic masses is not easy but fully possible; see, for example, [38–42]. For the first macroscopic mass we handle, we must also adjust for the nuclear binding energy if we want high precession.

One can measure the gravitational effect from the mass where one has counted the number of atoms. Next, one can find the Compton wavelength, for example, in the Earth, since the ratio of the Compton wavelength is identical to

$$\frac{g_1 R_1^2}{g_2 R_2^2} = \frac{\bar{\lambda}_2}{\bar{\lambda}_1}, \tag{12}$$

g_1 and g_2 are here the gravitational acceleration for mass one and mass two. We [43] have recently demonstrated that one can extract the Compton wavelength of the universe mass in a similar way from cosmological redshift.

To solve any of the Planck unit formulas with respect to G or the Compton wavelength formula with respect to m is trivial. Putting several trivial things together can sometimes lead to breakthroughs. When G is multiplied with m and written on this form, we will notice that the Planck constant always cancels out:

$$GM = \frac{l_p^2 c^3}{\hbar}\frac{\hbar}{\bar{\lambda}}\frac{1}{c} = c^2\frac{l_p^2}{\bar{\lambda}}. \tag{13}$$

That is, to know GM, we do not need to know $\hbar$, but only c and l_p, but to know G and m on their composite forms individually, we need to know $\hbar$, c and l_p. We have demonstrated in the papers just mentioned that the Planck length and the Compton wavelength[1] can be found with no knowledge of G or $\hbar$. Table 1 shows a series of gravity phenomena, and we see all observable gravity phenomena contain GM and not GMm. In two-body problems where the gravitational effects from also the second mass m are significant, the gravity parameter is $\mu = GM + Gm$ and not GMm. This means the Planck constant is never needed in gravity predictions, even at the quantum level; also, the gravity constant is not needed either. As we can see from the results most to the right in the table, all we need is knowledge of the constants l_p and c and, in addition, a variable deciding the size of the gravitational object, and this is the Compton wavelength of the object. In addition, we naturally need to know the distance to the gravity object where we want to make predictions or test our model against observations.

That is, we will claim one only needs two constants to make gravity predictions, that is, the Planck length and the speed of light, in addition to variables, as can be seen from Table 1 in gravitational phenomena normally linked to general relativity theory the only constant we need to know is the Planck length.

[1] Or the reduced Compton wavelength.

Mass	$M = \frac{\hbar}{\lambda_M}\frac{1}{c}$ (kg)
Non observable (contains GMm)	
Gravitational constant	$G, \left(G = \frac{l_p^2 c^3}{\hbar}\right)$
Gravity force	$F = G\frac{Mm}{R^2}$ $(kg \cdot m \cdot s^{-2})$
Observable predictions: (contains only GM)	
Gravity acceleration	$g = \frac{GM}{R^2} = \frac{c^2}{R^2} l_p \frac{l_p}{\lambda_M}$
Orbital velocity	$v_o = \sqrt{\frac{GM}{R}} = c\sqrt{\frac{l_p}{R}\frac{l_p}{\lambda_M}}$
Orbital time	$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi\sqrt{R^3}}{c\sqrt{l_p\frac{l_p}{\lambda_M}}}$
Periodicity pendulum[a] (clock)	$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi R\sqrt{\frac{L}{GM}} = \frac{2\pi R}{c}\frac{\sqrt{L}}{\sqrt{l_p\frac{l_p}{\lambda_M}}}$
Frequency Newton spring	$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi R}\sqrt{\frac{GM}{x}} = \frac{c}{2\pi R}\sqrt{\frac{l_p}{x}\frac{l_p}{\lambda_M}}$
Velocity ball Newton cradle[b]	$v_{out} = \sqrt{2\frac{GM}{R^2}H} = \frac{c}{R}\sqrt{2Hl_p\frac{l_p}{\lambda_M}}$
Observable predictions (from GR): (contain only GM)	
Advance of perihelion	$\sigma = \frac{6\pi GM}{a(1-e^2)c^2} = \frac{6\pi l_p}{a(1-e^2)}\frac{l_p}{\lambda_M}$
Gravitational redshift	$z = \frac{\sqrt{1-\frac{2GM}{R_1 c^2}}}{\sqrt{1-\frac{2GM}{R_2 c^2}}} - 1 = \frac{\sqrt{1-\frac{2l_p}{R_1}\frac{l_p}{\lambda_M}}}{\sqrt{1-\frac{2l_p}{R_2}\frac{l_p}{\lambda_M}}} - 1$
Time dilation	$T_R = T_f\sqrt{1-\sqrt{\frac{2GM}{R}}^2/c^2} = T_f\sqrt{1-\frac{2l_p}{R}\frac{l_p}{\lambda_M}}$
Deflection	$\delta = \frac{4GM}{c^2 R} = \frac{4l_p}{R}\frac{l_p}{\lambda_M}$
Microlensing	$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{(d_S-d_L)}{d_S d_L}} = 2\sqrt{l_p\frac{l_p}{\lambda_M}\frac{d_S-d_L}{(d_S d_L)}}$

Table 1: The table shows that any observable gravity phenomena contain GM and not GMm and further than when assuming G is a composite, then we end up that we can predict all observable gravity phenomena only from l_p and c.

[a] The formula is a very good approximation when the angle of the pendulum is small, as it is in most pendulum clocks. It is not accurate for large angles, but is again exact for an angle of 360; that is to say, for a full circle, see [44].

[b] Where H is the height of the ball drop.

Pay attention to the fact that in all formulas, we essentially have the term $\frac{l_p}{\lambda_M}$, which is the Planck length divided by the reduced Compton wavelength. This represents the reduced Compton frequency per Planck time for the mass in question and is what gives us quantization in gravity. As we will see, this also applies to cosmology. Both gravity and cosmology can be referred to as Planck quantized. For a Planck mass, the term $\frac{l_p}{\lambda} = \frac{l_p}{l_p} = 1$. Observable reduced Compton frequencies come in integers, so they are quantized. For masses smaller than the Planck mass, the frequency will be less than one, and no frequency less than one can be observed. However, it can be interpreted as a probability of being in a Planck mass state within a given Planck time observational window. The Planck constant is nowhere to be found, and as a result, some may question whether this can be considered quantum gravity. Additionally, the Planck constant also seems unnecessary in many aspects of quantum mechanics; see [26] for more details. The Planck constant is only needed when we want to express something in kilograms or joules. The kilogram is related to an arbitrary clump of matter that we have chosen to call a kilogram.

2 Cosmology and the Planck scale

Gravity theory is also closely linked to cosmology. One of the most studied cosmological models is the Friedmann [45] model, which comes from Einstein's [46]. General relativity theory. However, there has been considerable effort in trying to link cosmology to the Planck scale, without much success. In this paper, we will demonstrate how the Friedmann equations can be expressed in Planck unit form. We will also do the same with a new cosmological model recently derived from general relativity theory, based on extremal solutions.

One of the central Friedmann equations is given by:

$$H_0^2 = \frac{8\pi G\rho + \Lambda c^2}{3}, \tag{14}$$

where Λ is the cosmological constant, and ρ is the critical mass density in the Friedmann universe. Furthermore, H_0 represents the Hubble constant. By setting the cosmological constant to zero and solving for mass, we obtain:

$$M_c = \frac{c^3}{2GH_0}. \tag{15}$$

It is worth noting that to include the cosmological constant in the Friedmann equation, one must insert it ad-hoc into Einstein's field equation, as Einstein [47] himself did for the first time in 1917 and referred to it as his extended field equation.

Recently, in our work [48], we have shown that from the extremal solutions of the Reissner-Nordström [49, 50], the Kerr [51], and the Kerr-Newman [52] metric, a new cosmological equation emerges:

$$H_0^2 = \frac{8\pi G\rho - \Lambda c^2}{3}, \tag{16}$$

where $\Lambda = 3\left(\frac{H_0}{c}\right)^2 = \frac{3}{R_H^2}$. The extremal universe model is derived from the extremal solutions of Einstein's 1916 field equation. In the 1916 field equation, there was no explicit inclusion of a cosmological constant, yet it arises naturally from simple derivations of the extremal solutions of the Reissner-Nordström, Kerr, and Kerr-Newman metrics.

In 1917, when Einstein first introduced the cosmological constant, he suggested it to be $\frac{2}{r^2}$, where r represented some kind of horizon, and the Hubble constant or cosmological redshift had not yet been suggested or observed at that time. It is significant that Λ in our new cosmological model directly emerges from the derivation of the extremal solution of the 1916 field equation. In contrast, in Einstein's 1917 proposal and in the Friedmann model, as well as the Λ-CDM model, the cosmological constant is somewhat ad hoc inserted.

Solving equation 16 for mass yields:

$$M_u = \frac{c^3}{GH_0}, \tag{17}$$

which is twice the mass of the Friedmann critical universe: $M_u = 2M_c$. This also implies $H_0 = \frac{GM_u}{c^3}$.

Table 1 shows the Friedmann model and also the extremal universe cosmological model written on what we can call the Planck form or quantum form. Pay attention to the fact that the Friedmann model is slightly more complicated than the extremal model. What is reported in the table is only the Friedmann model for a critical universe, that is, when the cosmological constant is set to zero. In other words, only for a flat universe.

Be aware this is much more than just re-writing the gravity constant from the Planck units. We can relatively easily find the Planck length and the Planck time without any knowledge of G; we can also find the Compton wavelength of the total mass (and energy) in the observable universe with no knowledge of G, see [43]. Further, we can replace the universal constants G, $\hbar$ and c with only c and the Planck length l_p, both for observable gravity phenomena as shown in Table 1, and for cosmology predictions in Tables 2 and 3. Our new way to also predict cosmological phenomena using only two constants, namely the Planck length and the speed of light, adds support to that it looks like this theory/view is on to something, and it is fully consistent with the recent quantization of general relativity theory.

This means that, for the first time, we have models of the cosmos directly linked to the Planck scale. How should this be interpreted? In our view, it should be interpreted as quantum gravity already being embedded within Newtonian and Einsteinian gravity, not intentionally, but by understanding gravity at a deeper level. This is consistent with the recent re-formulation of Einstein's field equations and metrics to be on Planck scale form, see [10, 48]. Most observable gravitational phenomena, if not all, in our view, are indirect detections of the Planck scale. That's why we can now extract the Planck length from most gravity phenomena and even from cosmological redshift without any knowledge of G or $\hbar$. This stands in stark contrast to the consensus view, where gravity theory is considered distinct from the quantum scale. Not everyone agrees with our perspective, but we encourage other researchers to carefully investigate and study it before forming their opinions.

Once again, pay attention to the fact that we, in all formulas, basically have the term $\frac{l_p}{\bar{\lambda}_c}$, that is, the Planck length divided by the reduced Compton wavelength of the critical universe in the Friedmann model and $\frac{l_p}{\bar{\lambda}_u}$ is the Planck length divided by the reduced Compton wavelength of the mass (equivalent mass) in the extremal universe. Again, this is the reduced Compton frequency per Planck time, so it is the quantization of mass and energy and gives Planck quantization of gravity and cosmology.

Table 3 shows a series of more ways to express the different aspects of the cosmos with Planck units. When we use the Planck mass (in kilogram terms) instead of the Planck length or Planck time, we, in addition, need to know the Planck constant, but as we have demonstrated in Table 2, there is no need to use the Planck mass as we can predict these cosmological phenomena from the Planck length and or Planck time. So, we need to know the two constants l_p and c for cosmology predictions. Still, even the Planck mass can be found without knowledge of G.

It is worth noting that the extremal universe model can also be derived from the new exact solution of the Einsteins field equation recently derived by Haug and Spavieri [55].

	Friedmann critical universe	Extremal universe
Universe equation	$H_0^2 = \frac{8\pi G\rho}{3} = \left(\frac{c}{2l_p \frac{l_p}{\bar{\lambda}_c}}\right)^2$	$H_0^2 = \frac{8\pi G\rho - \Lambda c^2}{3} = \frac{4\pi G\rho}{3} = \left(\frac{c}{l_p \frac{l_p}{\bar{\lambda}_u}}\right)^2$
Cosmological constant		$\Lambda = 3\left(\frac{H_0}{c}\right)^2 = 3\left(\frac{\bar{\lambda}_u}{l_p^2}\right)^2$
Universe kilogram mass denisty	$\rho_c = \frac{3H_0^2}{4\pi G} = \frac{3\hbar\bar{\lambda}_c^2}{16\pi c l_p^6}$	$\rho_u = \frac{3H_0^2}{4\pi G} = \frac{3\hbar\bar{\lambda}_u^2}{4\pi c l_p^6}$
Universe energy Joule denisty	$\rho_c = \frac{3H_0^2 c^2}{4\pi G} = \frac{3\hbar\bar{\lambda}_c^2 c}{16\pi l_p^6}$	$\rho_u = \frac{3H_0^2 c^2}{4\pi G} = \frac{3\hbar\bar{\lambda}_u^2 c}{4\pi l_p^6}$
Universe mass kilogram	$M_c = \frac{\hbar}{\bar{\lambda}_c}\frac{1}{c}$	$M_c = \frac{\hbar}{\bar{\lambda}_u}\frac{1}{c}$
Hubble constant	$H_o = \frac{c}{2l_p \frac{l_p}{\bar{\lambda}_c}}$	$H_o = \frac{c}{l_p \frac{l_p}{\bar{\lambda}_u}}$
Hubble Radius	$R_H = \frac{c}{H_0} = 2l_p\frac{l_p}{\bar{\lambda}_c}$	$R_H = \frac{c}{H_0} = l_p\frac{l_p}{\bar{\lambda}_u}$
Hubble Circumference	$C_H = 2\pi\frac{c}{H_0} = 4\pi l_p\frac{l_p}{\bar{\lambda}_c}$	$C_H = 2\pi\frac{c}{H_0} = 2\pi l_p\frac{l_p}{\bar{\lambda}_u}$
Hubble volume	$V_H = \frac{4}{3}\pi R_H^3 = \frac{32}{3}\pi l_p^3\frac{l_p^3}{\bar{\lambda}_c^3}$	$V_H = \frac{4}{3}\pi R_H^3 = \frac{4}{3}\pi l_p^3\frac{l_p^3}{\bar{\lambda}_u^3}$
Age Universe	$T_H = \frac{R_H}{c} = \frac{1}{H_0} = 2l_p\frac{l_p}{\bar{\lambda}_c c} = 2t_p\frac{l_p}{\bar{\lambda}_c}$	$T_H = \frac{R_H}{c} = \frac{1}{H_0} = l_p\frac{l_p}{\bar{\lambda}_u c} = t_p\frac{l_p}{\bar{\lambda}_u}$
Hubble frequency	$f_H = \frac{1}{T_H} = \frac{1}{2t_p}\frac{\bar{\lambda}_c}{l_p}$	$f_H = \frac{1}{T_H} = \frac{1}{t_p}\frac{\bar{\lambda}_u}{l_p}$
Mass to radius ratio relation	$\frac{M_c}{R_H} = \frac{m_p}{2l_p}$	$\frac{M_u}{R_H} = \frac{m_p}{l_p}$
Compton wavelength universe mass	$\bar{\lambda}_c = \frac{\hbar}{cM_c} = 2l_p\frac{l_p}{R_H} = 2l_p^2\frac{Z}{d}$	$\bar{\lambda}_u = \frac{\hbar}{cM_u} = l_p\frac{l_p}{R_H} = l_p^2\frac{Z}{d}$
Cosmological red-shift	$Z \approx \frac{dH_0}{c} = \frac{d}{2l_p\frac{l_p}{\bar{\lambda}_c}}$	$Z \approx \frac{dH_0}{c} = \frac{d}{l_p\frac{l_p}{\bar{\lambda}_u}}$
Planck length from Cosmological red-shift	$l_p = \sqrt{\frac{d\bar{\lambda}_c}{2Z_c}}$	$l_p = \sqrt{\frac{d\bar{\lambda}_u}{Z_u}}$

Table 2: Some other ways to express the cosmological equations rooted in the Planck scale. However, for example, making the cosmological observations linked to the Planck mass rather than the Planck length seems to just add complexity, and it leads to one need for one more constant, namely the Planck constant.

	Friedmann critical universe	Extremal universe
Universe equation	$H_0^2 = \frac{2c^2 l_p^2}{\bar{\lambda}_c R_H^3}$	$H_0^2 = \frac{c^2 l_p^2}{\bar{\lambda}_u R_H^3}$
Universe equation	$H_0^2 = \frac{\bar{\lambda}_c^2 c^2}{4l_p^4}$	$H_0^2 = \frac{\bar{\lambda}_u^2 c^2}{l_p^4}$
Hubble constant from t_p	$H_0 = \frac{\bar{\lambda}_c}{2t_p^2 c}$	$H_0 = \frac{\bar{\lambda}_u}{t_p^2 c}$
Hubble constant from m_p	$H_0 = \frac{\bar{\lambda}_c m_p^2 c^3}{2\hbar^2}$	$H_0 = \frac{\bar{\lambda}_u m_p^2 c^3}{\hbar^2}$
Hubble constant from l_p and t_p	$H_0 = \frac{\bar{\lambda}_c}{2t_p l_p}$	$H_0 = \frac{\bar{\lambda}_u}{t_p l_p}$
Hubble constant from m_p and t_p	$H_0 = \frac{\bar{\lambda}_c m_p c}{2t_p\hbar}$	$H_0 = \frac{\bar{\lambda}_u m_p c}{t_p\hbar}$
Radius universe from t_p	$R_H = \frac{c}{H_0} = \frac{2t_p^2 c^2}{\bar{\lambda}_u}$	$R_H = \frac{c}{H_0} = \frac{t_p^2 c^2}{\bar{\lambda}_u}$
Radius universe from m_p	$R_H = \frac{c}{H_0} = \frac{2\hbar^2}{\bar{\lambda}_u m_p^2 c^2}$	$R_H = \frac{c}{H_0} = \frac{\hbar^2}{\bar{\lambda}_u m_p^2 c^2}$
Radius universe from m_p and t_p	$R_H = \frac{c}{H_0} = \frac{2\hbar^2}{\bar{\lambda}_u m_p^2 c^2}$	$R_H = \frac{c}{H_0} = \frac{t_p\hbar}{\bar{\lambda}_u m_p}$

Table 3: Cosmology written with its relation to the Planck scale.

3 Relativistic Newton theory and conservation of space-time

In addition, we will highlight that the same cosmological model, which one can derive from both the extremal solution and the Haug-Spavieri metric, can surprisingly be obtained from the special relativistic modified Newtonian gravitational theory [56] and its corresponding quantum gravity theory [57]. We believe that it is important to pay more attention to the fact that the extremal solutions of the Reissner-Nordström, Kerr, and Kerr-Newman metrics, as well as the Haug-Spavieri metric in general relativity theory, correspond to many cosmological and black hole-related derivations that one can obtain from the simply relativistic modified Newtonian gravitational theory.

The Schwarzschild solution is calibrated to Newton's weak field limit in the last step before it can be used to predict anything. However, even in a strong gravitational field, it predicts multiple identical formulas as the Newtonian weak field limit, such as the same escape velocity and the same radius of a black hole-type object, namely $r_s = \frac{2GM}{c^2}$. We will here go against the consensus and claim that calibrating a model to a model that only holds in a weak field can easily cause the calibrated model to also hold truly only in a weak gravitational field.

On the other hand, the extremal solutions and the Haug-Spavieri metric correspond to the relativistic modified Newtonian theory in both weak and strong fields. However, there is one significant exception: in the relativistic Newtonian theory, one gets the bending of space and time but flat space-time.

4 It requeres much less information to find $l_p\frac{l_p}{\bar{\lambda}}$ than l_p and $\bar{\lambda}$ sepratley

In addition to the speed of light (gravity), in some cosmological predictions, we need to know the term $l_p\frac{l_p}{\bar{\lambda}}$. This last term is embedded in all cosmological predictions. We can find both the Planck length and the Compton wavelength of the universe, totally independent of knowledge of the Planck constant and the gravitational constant G. For the entire cosmos, we simply need to measure the cosmological redshift Z from light coming from an object at a distance d. Then we have:

$$l_p\frac{l_p}{\bar{\lambda}_u} = \frac{d}{Z}, \tag{18}$$

where $\bar{\lambda}_u$ is the reduced Compton wavelength of all the mass and energy in the observable universe based on the extremal solution of Einstein's field equations. Alternatively, we have:

$$l_p\frac{l_p}{\bar{\lambda}_c} = \frac{d}{2Z}. \tag{19}$$

Next, to find the reduced Compton wavelength of the observable critical universe or the extremal universe, we simply have to find the Planck length, as can be done independently of knowledge of G and $\hbar$, as demonstrated in [20, 22].

Not only is it easier to find $l_p\frac{l_p}{\bar{\lambda}}$ than l_p and $\bar{\lambda}$ separately, but it also leads to much higher precision. To find l_p and $\bar{\lambda}$ separately is only needed when we want to study the quantization of gravity, not to predict macroscopic gravity phenomena where we only need to know $l_p\frac{l_p}{\bar{\lambda}}$.

5 Conclusion

We have presented how both the Friedmann model and a recent cosmological model based on the extremal solution of the Reissner-Nordström, Kerr, and Kerr-Newman metrics can be represented using the Planck length, the speed of light (gravity), and the Compton wavelength of the mass in question–in this case, the mass of the universe. Importantly, we can find the Planck length and the Compton wavelength of the universe with no prior knowledge of G or $\hbar$. This means that, for the first time, we have been able to link the smallest scale, the Planck scale, with the largest scales in the universe, namely cosmic scales.

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Appendix, some derivations

The Friedmann universe

Just to demonstrate some derivations of the results given in the tables. The mass in the Freedman universe is given by

$$M_c = \frac{c^3}{2GH_0}. \tag{20}$$

Further, the Hubble constant is then given by

$$H_0 = \frac{c^3}{2GM_c}. \tag{21}$$

Next we can replace G with $G = \frac{l_p^2 c^3}{\hbar}$ and the universe mass with $M_c = \frac{\hbar}{\bar{\lambda}_c}\frac{1}{c}$ which gives

$$H_0 = \frac{\bar{\lambda}_u c}{2l_p^2}, \tag{22}$$

where $\bar{\lambda}_u$ is the reduced Compton wavelength of the critical mass in the Freedman universe. This means the (reduced) Compton wavelength of the critical mass in the Friedmann universe is given by

$$\begin{aligned}
H_0^2 &= \frac{8\pi G\rho}{3} \\
\left(\frac{\bar{\lambda}_c c}{2l_p^2}\right)^2 &= \frac{2GM_c}{R_H^3} \\
\frac{\bar{\lambda}_u^2 c^2}{4l_p^4} &= 2\frac{l_p^2 c^3}{\hbar}\frac{\hbar}{c}\frac{1}{\bar{\lambda}_c}\frac{1}{R_H^3} \\
\frac{\bar{\lambda}_c^3}{4l_p^4} &= 2\frac{l_p^2}{R_H^3} \\
\bar{\lambda}_c^3 &= \frac{8l_p^6}{R_H^3} \\
\bar{\lambda}_c &= \left(\frac{8l_p^6}{R_H^3}\right)^{1/3} \\
\bar{\lambda}_c &= \frac{2l_p^2}{R_H},
\end{aligned} \tag{23}$$

where $\bar{\lambda}_c$ is the reduced Compton wavelength of the mass in the critical Friedmann universe.

Further since $R_H = \frac{c}{H_0}$ we can also write this as

$$\bar{\lambda}_c = \frac{2l_p^2 H_0}{c} = \frac{2l_p^2}{R_H}. \tag{24}$$

The extremal universe

The mass in the extremal universe is given by

$$M_u = \frac{c^3}{2GH_0}. \tag{25}$$

Next we can replace G with $G = \frac{l_p^2 c^3}{\hbar}$ and the universe mass with $M_u = \frac{\hbar}{\bar{\lambda}_c}\frac{1}{c}$ which gives

$$H_0 = \frac{\bar{\lambda}_u c}{l_p^2}, \tag{26}$$

where $\bar{\lambda}_u$ is the reduced Compton wavelength of the mass in the extremal universe. The mass in this extremal universe is twice that of the Friedmann universe. The reduced Compton wavelength of the mass in this universe we can express as

$$\begin{aligned}
H_0^2 &= \frac{8\pi G\rho}{3} \\
\left(\frac{\bar{\lambda}_u c}{l_p^2}\right)^2 &= \frac{GM_u}{R_H^3} \\
\frac{\bar{\lambda}_u^2 c^2}{l_p^2} &= \frac{l_p^2 c^3}{\hbar}\frac{\hbar}{c}\frac{1}{\bar{\lambda}_u}\frac{1}{R_H} \\
\frac{\bar{\lambda}_u^3}{l_p^4} &= l_p \frac{l_p}{\bar{\lambda}_u R_H^3} \\
\bar{\lambda}_u^3 &= \frac{l_p^6}{R_H^3} \\
\bar{\lambda}_u &= \left(\frac{l_p^6}{R_H^3}\right)^{1/3} \\
\bar{\lambda}_u &= \frac{l_p^2}{R_H},
\end{aligned} \tag{27}$$

and since $R_H = \frac{c}{H_0}$ we can also re-write this as

$$\bar{\lambda}_u = \frac{l_p^2 H_0}{c} = \frac{l_p^2}{R_H}. \tag{28}$$