Research Article

The Guessing Game and Its Implications for Sport Psychology Research — A Tale of Lotteries, Penalties, Mixed Strategies, and Nash Equilibria

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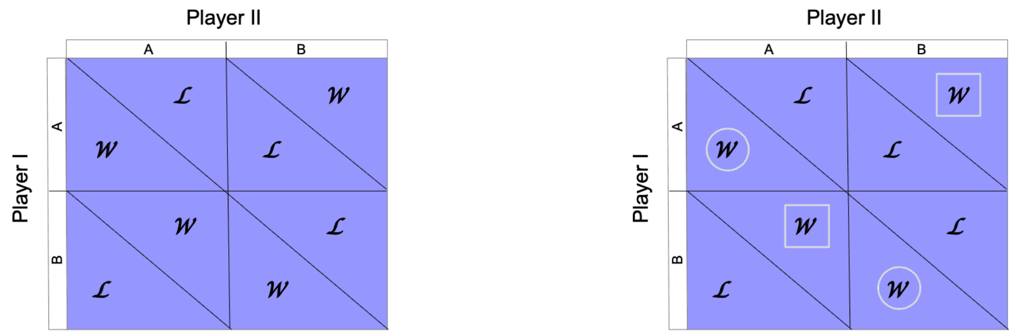
This paper introduces the Guessing game, a game characterized by a single unique Nash equilibrium (NE) in mixed strategies. Furthermore, this game is shown to be important in most sports activities. Finally, central parts of sport psychology research are shown to neglect the existence of the Guessing game. The paper hence concludes by questioning the value of such research.

1. Introduction

The two-player Guessing game may be defined as follows: Two players choose simultaneously strategies A or B. If both players choose the same strategy $\{A, A\}$ or $\{B, B\}$, Player I wins. However, if they choose different strategies $\{A, B\}$ or $\{B, A\}$, Player II wins. Clearly, the winning strategy in this game is for both players to try to guess what the other player does. For Player I, a correct guess implies choosing the same strategy as Player II, while Player II should choose the alternative strategy to his guess.

 doi.org/10.32388/CJEKHU



(a) Pay-off matrix for The Guessing game

(b) Best replies for the Guessing game

Figure 1. The Guessing game.

The resulting Pay-off matrix of this game is shown to the left (a) in figure 1. Assuming player rationality and $\mathcal{W} \succ \mathcal{L}$,[1] best replies, grey circles and squares to the right (b) in figure 1, are derived.

As none of the circles and squares locate in any sub square (AA, AB, BA, BB), the game contains no Nash equilibria in pure strategies. Consequentially, see [1], it must contain one (and only one) Nash equilibrium in mixed strategies. Hence, players of this game, obeying NE-strategies, should randomize. The NE-probabilities used for this randomization can be calculated straightforwardly. In order to find the solution, it is necessary to convert the utilities of winning and losing into explicit values, say $u(\mathcal{W}) = w$ and $u(\mathcal{L}) = l$. Given this conversion, it is easy[2] to show that:

$$\{p^*, q^*\} = \left\{\frac{1}{2}, \frac{1}{2}\right\} \tag{1}$$

where p^* is the probability that Player I plays A, and q^* is the probability that Player II plays A. The fact that we here get a coin toss as the equilibrium strategies for the players $\left(p^* = q^* = \frac{1}{2}\right)$ should not be surprising. Recall that our game definition in figure 1 involves no differences (for instance, quality) between the players. Hence, there is not to be expected that randomization should involve probabilities different from $\frac{1}{2}$. When we, in later sections of this paper, investigate more “realistic” guessing games, for instance in sports, we should not expect the same.

2. Guessing games in sport

Think about a male tennis player about to serve. Largely, he has two options, serving along the line or serving wide. Obviously, if the receiver can guess which choice the server makes before he serves, it

doi.org/10.32388/CJEKHU

would be beneficial for him. On the other hand, if the server can fool the receiver into guessing wrong, it would be beneficial for her. Clearly, a situation where both players would benefit from being unpredictable. This is basically the same situation as our previously defined guessing game. In reality, things are a bit more complex as quality differences may come into play. The server may, for instance, be a particularly good server, and the receiver may be particularly bad. This will certainly affect the equilibrium mixed strategy probabilities, moving away from the 50/50 solution. Still, both players will benefit from unpredictability, and a unique NE in mixed strategies should be expected.

Serving is not exclusive to tennis. Other sports like Badminton, Volleyball, Table tennis, and Beach Volleyball start with a serve. Consequentially, similar outcomes are to be expected in these sports[3].

Beach Volleyball is particularly interesting, as it contains other guessing games. For instance, the situation where one team attacks at the net. Then, the attacker typically can choose between two strategies: a pokey or a smash. A pokey is an action that has the potential of neutralizing a block move from the defender. Instead of using a hard smash, which can be neutralized by the block, a pokey typically involves a very soft touch (with crooked fingers) playing the ball over or at a sharp angle, passing the defender. Again, a situation resembling the guessing game. If the attacker can predict a block, a pokey is the right answer. On the other hand, if the defender can predict a pokey, standing still and not jumping to block is the best strategy. So, again, the guessing game emerges. NE's in mixed strategies are to be expected.

Baseball also contains guessing games. A recent contribution [2] investigates the game between the batter and the pitcher in Baseball. This situation involving decisions made in infinitesimal time indicates a simultaneous game, and the author identifies the mixed strategy Nash equilibrium as well as testing it.

Football is perhaps the sport containing the most guessing games. Think about one attacker trying to dribble a defender. There are several ways of successfully dribbling an opponent, but two obvious ones are: playing the ball on the side or trying a "tunnel'[4]'. Again, if the defender can guess what the attacker chooses to do, it is clearly easier to stop him, and vice versa. Think about a head duel. Two players try to reach a high ball using their heads. Either you can jump and go for the ball or alternatively push your opponent by using your body. Again, a situation where unpredictability is the key. There are a lot of other sub-games in football resembling guess game situations. Still, I guess most would agree that the most important one, with the power to decide matches, is the penalty kick. The fact that penalty kicks (very often) may involve NE's in mixed strategies should more or less be intuitively obvious. The kicker's ability to guess the keeper's action or the keeper’s ability to guess the kicker's action provides decisive benefits in achieving their targets. This is surely also confirmed by research, see for instance [3][4][5][6][7][8] and [9].

Most sports should contain guessing games – and for a reason – uncertainty of outcome. This concept, introduced by Rottenberg [10], simply states that sport without tension is less interesting. After all, who would spend time on a sport contest if the outcome was known beforehand? The guessing game, discussed above, is important in this context. After all, the existence of unique NE’s in mixed strategies introduces outcome uncertainty in sports. Players’ execution of randomized strategies produces outcome uncertainty by definition.

3. Research “forgetting” the guessing game

In [11], the authors discuss the principal problems involved in approximating games by stochastic programs. The basic conclusion is that games involving mixed strategy NE’s should not be approximated by optimization problems. After all, neither deterministic nor stochastic programs can produce solutions involving randomization. Hence, optimization must be applied with extreme care if the aim is to analyse game situations, especially if the game situation involves mixed strategy NE’s.

Some other research looks into this problem in a more practical setting. In [6] or [12], the story of the rise and fall of the Norwegian national football team is told. In the early nineteen nineties, Norway was placed second on the FIFA ranking. This was achieved by applying a new and different playing style. The coach in those days, Egil “Drillo” Olsen, was more interested in how the players behaved without the ball than with the ball. His system was based on some empirical observations, presented in his master thesis linking scored goals to the number of passes played before the goal was scored. It turned out that a small number of passes was favourable compared to a large number (the optimization). This observation led to a playing system trying to use a minimal amount of passes before scoring a goal. Either long balls from the keeper or defence typically headed by the wing-player down to a midfielder running directly into the opponents' defence, or even better, breaking the opponent early when he was trying through many passes to build an attack. And, it worked, in the beginning. The problem related to the story so far is that the system did not expect reactions from the opponent. In this case, a neutralizing strategy is simple: just copy the Norwegian playing style, and if you on average have better players, which most opponents of Norway did have (both in those days and

doi.org/10.32388/CJEKHU

today), you will normally beat Norway. Of course, this was what happened. Norway started to lose national matches and is today placed where they belong, around number 50 on the FIFA rank.

A story, somewhat different, a new point score system, investigates the same type of fallacy, see [13].

A certain area of sports psychology is interesting in this setting: the sport psychological research related to penalty kicks. A recent book, "PRESSURE" by Geir Jordet [14], is especially interesting in this setting. In this book, Jordet talks a lot about actual historic shoot-outs. By all means, very interesting. Personally, I fell for the sad story of Roberto Baggio, the famous Italian midfielder/striker in the nineties, who published his autobiography entitled, "A Hole in the Sky". This title alludes to his penalty in the World Cup final against Brazil in 1994, when he missed by playing the ball into the sky. Apart from nice stories about penalties, Jordet discusses his own research on penalties in this book. His research on the matter can essentially be summed up as follows.

1. Observe penalties and define two disjoint groups separated by a difference in player behaviour.
2. Count up success rates (on penalties) in the two groups.
3. Check if success rates are significantly different, and if so, *recommend* the behaviour with the highest success rate.

The keyword here is perhaps the word *recommend* in bullet point 3. Is this a good recommendation? I am very reluctant about the quality of this type of advice. To stress my point, let me give an alternative example which should explain my reluctance.

Suppose I, with the aid of the above-defined research model, were to try to find out whether it is better to place penalties in the middle of the goal than to shoot to any side. Following the model above, I would pick some penalty data and partition it into two groups: middle shots (M) and wide shots (W). Suppose I count up and calculate the two success rates and observe that the choice of M produces 91% goals, while W only results in 77% goals. For the sake of completeness, let me also assume that 91% is significantly larger than 77% with these data, at a reasonable significance level. Would it then be wise to recommend placing all shots in the middle? Of course not. It may very well be quite successful in the beginning. However, after some penalties of this kind, the keeper will start thinking and change behavior. He will, instead of the normal action of going to any side, start standing still, and surely this will change the success rate in disfavor of the shooter. So using empirical data to make recommendations of this type is clearly quite hazardous. Actually, one could ask if it has any value whatsoever.

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If Jordet were the only researcher doing this kind of research, it would not be much of a problem. Unfortunately, this is definitely not the case. A simple search on Google Scholar produces 23,200 hits (in September 2024) for the search term "sport psychology + penalties." Surely, not all of these hits use the research model outlined above. But some investigation tells me that a significant amount of researchers engage in this line of research. One paper is especially relevant, as it is a kind of survey or meta-study. This paper [15] is very handy, as it ends with a checklist of recommendations with pointers to research giving the recommendations. First of all, the volume is quite high. Only this paper presents around 100 articles involving (roughly) 600 researchers (seemingly) spending their careers finding out how to take penalties.

Some of the actual recommendations are particularly interesting (for the penalty taker):

i. should be left-footed
ii. wear a red jersey
iii. if technically possible, kick the ball close to but still below the crossbar
iv. in penalty shoot-outs, celebrate a goal as theatrically as possible

Although iii) is good advice, but all so hard to do, all these pieces of advice are silly. I am sorry, but it is impossible to conclude differently. If there is any truth in the initial game theory in this paper, measuring success or failure in penalty kick performance without taking reactions from the opponent into account is at best a dangerous approximation.

4. Conclusion

The fact that research labelled penalty sports psychology suffers from the obvious fallacy of not realizing that a penalty shoot-out is a game raises the question of value. The fact that many persons are writing many papers based on a research model that simply is incorrect stresses this question of value even more. It is perhaps too harsh to judge this research as valueless, but it surely raises questions. Recommending (optimal) action when the optimal action is a random process can never be correct. Hopefully, this article may provide learning and drive this research in different and more valuable directions.

Surely, one could always argue that our Guessing game model is far too simple. For instance, the change in the penalty kick rule allowing the keeper to move (on the line) before the shot, introduced relatively recently, is clearly significant. It opens up for a far more complex (sequential) game. The

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kicker can choose to wait and observe the keeper's movement before placing the shot. This is referred to as keeper-dependent penalties in this branch of research, see for instance [14]. Consequently, our Guessing game model is further away from reality, and our argument related to mixed strategy NE's becomes less valid. Still, when the pressure of the occasion (or the importance of the match) increases, I would guess that the number of keeper-dependent penalties decreases. After all, this is the situation this research focuses on, the pressure of penalties in the big matches.

Statements and Declarations

Disclosure statement

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

This article contains no data.

Statement of ethics/IRB approval

Ethics approval was not required for this study, as it contains no experimental or observational data, neither from humans nor animals.

Footnotes

[1] The sign $\succ$ means preferred over.

[2] Defining $[p, 1-p]$ and $[q, 1-q]$ as the mixed strategies for Player I and Player II respectively, the expected pay-off for Player I is easily calculated as $E_I(p,q) = qpw + p(1-q)l + (1-p)ql + (1-p)(1-q)w$. It turns out that finding the mixed strategy NE is especially simple in this case, as solving the parametric linear programming problem $\max_p E_I(p,q)$ can be done by taking the partial derivative of this expression with respect to p and equating it to zero. This gives $\frac{\partial}{\partial p}[E_I] = 0 \Rightarrow q(2w-2l) = w - l \Rightarrow q^* = \frac{1}{2}$. By the symmetry of the pay-off matrix, p^* also equals $\frac{1}{2}$.

[3] Obviously, the situation of serving is in principle a sequential game. The receiver observes the serving direction and can react (sequentially) to the server's choice. Still, a successful return may need

the right choice of placement or guessing where the serve is going. For some more discussion of possible mixed strategy Nash equilibria in Tennis, refer to [16], [17], and [18].

[4] A “tunnel” is playing the ball between the defenders' feet.

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Declarations

Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.