

# Conservation of Baryon and Lepton Number is an Effect of Electric and Magnetic Charges

Rainer W. Kühne

Department of Physics, Technische Universität Dortmund, Germany

e-mail: kuehne70@gmx.de

ORCID: 0000-0001-7449-761X

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## Abstract

The conservation of baryon number and lepton number has not yet been explained. Here I present a new nomenclature where I redefine isospin and hypercharge. By doing so I explain baryon and lepton number conservation as an effect of the electric-magnetic duality and the  $U(1) \times U(1)$  gauge symmetry of quantum electrodynamics. By using this method I predict the quantum numbers of an octet of magnetic monopoles. Another surprising result is that both leptons and quarks have nonzero magnetic isospin, a new quantum number.

### Keywords:

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## 1 Textbook Nomenclature

According to the conventional nomenclature presented in textbooks, up and down quark have strong isospin, whereas all the other elementary fermions have zero strong isospin.

All the left-handed quarks and leptons have weak isospin, whereas all the right-handed quarks and leptons have zero weak isospin. There are no right-handed neutrinos.

## 2 Criticism of Textbook Nomenclature

There exist three left-handed fermion octets.

(i) The first octet consists of three up quarks, three down quarks, electron, and electron-neutrino.

(ii) The second octet consists of three charm quarks, three strange quarks, muon, and muon-neutrino.

(iii) The third octet consists of three top quarks, three bottom quarks, tauon, and tau-neutrino.

The first octet includes three strong isospin doublets, whereas the other octets include no strong isospin doublets. The three octets are not described equivalently.

All the left-handed fermion octets include four weak isospin doublets. The three right-handed fermion septets include no weak isospin doublets. Left-handed and right-handed fermions are not described equivalently.

### 3 New Nomenclature

In order to describe both left-handed and right-handed fermions and the three octets equivalently, I will suggest a new definition of isospin, so that all the leptons and quarks have isospin partners and that all the octets consist of four isospin doublets.

(i) The isospin partner of the up quark is the down quark.

(ii) The isospin partner of the charm quark is the strange quark.

(iii) The isospin partner of the top quark is the bottom quark.

(iv) The isospin partner of the charged lepton is its corresponding neutrino.

A consequence of this redefinition is that right-handed neutrinos must exist.

Each quark exists in three chromoelectric colors. So each of the three fermion octets consists of four isospin doublets.

If  $Q$  denotes electric charge in units of the positron charge  $e$ ,  $I_3$  denotes the third component of isospin, and  $Y$  denotes hypercharge, then the Gell-Mann-Nishijima equation reads

$$Q = I_3 + Y/2 \quad (1)$$

If  $B$  denotes baryon number and  $L$  denotes lepton number, then:

(i) For quarks we get  $Y = B$ . The famous equation  $Y = B + S$  for strange quarks is valid only for strong isospin, but not here.

(ii) For leptons we get  $Y = -L$ .

(iii) For fermions we get in general

$$Y = B - L \quad (2)$$

The quantum numbers of the elementary particles are then:

(i) Up, charm, and top quark have  $Q = +2/3$ ,  $I_3 = +1/2$ , and  $Y = +1/3$ .

(ii) Down, strange, and bottom quark have  $Q = -1/3$ ,  $I_3 = -1/2$ , and  $Y = +1/3$ .

(iii) Electron-neutrino, muon-neutrino, and tau-neutrino have  $Q = 0$ ,  $I_3 = +1/2$ , and  $Y = -1$ .

(iv) Electron, muon, and tauon have  $Q = -1$ ,  $I_3 = -1/2$ , and  $Y = -1$ .

(v) Each of the three families of leptons has its own lepton number. Because of neutrino-oscillations, only their sum  $L$  can be conserved.

(vi) Each of the three families of quarks has its own baryon number. Because of Kobayashi-Maskawa mixing [1], only their sum  $B$  can be conserved.

According to this redefinition, isospin is conserved as long as both electric charge and hypercharge are conserved.

### 4 Baryon and Lepton Number

Quantum electrodynamics (QED) associates the  $U(1)_Q$  gauge symmetry with electric charge  $Q$ . Quantum flavordynamics (QFD) [2, 3] describes a  $SU(2)_I \times U(1)_Y$  gauge symmetry, where  $SU(2)_I$  is associated with isospin and  $U(1)_Y$  with hypercharge. A rotation by the Weinberg angle  $\Theta_W$  transforms the third part  $W_3^\mu$  of the gauge field associated with the  $SU(2)_I$  group and the gauge field  $B^\mu$  associated with the  $U(1)_Y$  group into the gauge field  $A^\mu$  associated with the photon of QED and the gauge field  $Z^\mu$  associated with the Z boson,

$$A^\mu = B^\mu \cos \Theta_W + W_3^\mu \sin \Theta_W \quad (3)$$

$$Z^\mu = -B^\mu \sin \Theta_W + W_3^\mu \cos \Theta_W \quad (4)$$

For the quantum numbers this rotation is described by the Gell-Mann-Nishijima equation.

The  $U(1)_Y$  gauge symmetry describes hypercharge  $Y = B - L$ , so  $B - L$  should be conserved. However, both baryon number  $B$  and lepton number  $L$  are conserved independently. So one would expect that  $B$  is associated with a  $U(1)$  gauge symmetry and that  $L$  is associated with another  $U(1)$  gauge symmetry. Hence, there should exist a  $U(1) \times U(1)$  gauge symmetry.

### 5 Electric-Magnetic Duality

Quantum electromagnetodynamics (QEMD) [4, 5] was suggested in order to describe electricity and magnetism equivalently. This theory includes both electric charge  $Q$  and magnetic charge  $q$ . Quanta which have nonzero magnetic charge are called Dirac

magnetic monopoles [6]. Quanta which have both nonzero electric charge and nonzero magnetic charge are called Schwinger dyons [7]. The gauge bosons of QEMD are the photon (now called Einstein electric photon [8]) and the new Salam magnetic photon [9]. The gauge group is  $U(1)_Q \times U(1)_q$ .

## 6 Generalized Electric-Magnetic Duality

The introduction of magnetic charges and the magnetic photon makes it necessary to generalize the standard theory of particle physics.

QFD will be generalized by a new theory which is described by a  $SU(2)_I \times U(1)_Y \times SU(2)_M \times U(1)_X$  gauge symmetry. The eight gauge bosons are the electric photon, the two W bosons, the Z boson, and the new magnetic photon, the two new isomagnetic W bosons, and the new isomagnetic Z boson.

A rotation by the isomagnetic Weinberg angle  $\Theta_M$  transforms the third part  $w_3^\mu$  of the gauge field associated with the  $SU(2)_M$  group and the gauge field  $b^\mu$  associated with the  $U(1)_X$  group into the gauge field  $a^\mu$  associated with the magnetic photon and the gauge field  $z^\mu$  associated with the isomagnetic Z boson,

$$a^\mu = b^\mu \cos \Theta_M + w_3^\mu \sin \Theta_M \quad (5)$$

$$z^\mu = -b^\mu \sin \Theta_M + w_3^\mu \cos \Theta_M \quad (6)$$

For the quantum numbers this rotation is described by the magnetic Gell-Mann-Nishijima equation

$$q = M_3 + X/2 \quad (7)$$

where  $q$  denotes magnetic charge in units of the unit magnetic charge  $m$ ,  $M_3$  denotes the third component of magnetic isospin, and  $X$  the herewith new introduced hypocharge.

I argued in earlier papers [4, 5] that quantum chromodynamics (QCD) [10, 11] will be generalized by a new theory which is described by a  $SU(3) \times SU(3)$  gauge symmetry. The associated charges are chromoelectric color and the new chromomagnetic color. The associated gauge bosons are the eight chromoelectric gluons and the eight new chromomagnetic gluons.

## 7 Hypocharge

In section 4 I suggested a  $U(1) \times U(1)$  gauge symmetry which is associated with baryon number  $B$  and lepton number  $L$ . Because of  $Y = B - L$  it is reasonable to associate this gauge symmetry with hypercharge  $Y$  and hypocharge  $X$ , thus  $U(1)_Y \times U(1)_X$ . Because of the two Gell-Mann-Nishijima equations and the two Weinberg angles  $\Theta_W$  and  $\Theta_M$  this symmetry can be transformed into the  $U(1)_Q \times U(1)_q$  gauge symmetry of QEMD.

Now the task is to find out how  $X$  depends on  $B$  and  $L$ .

Let us start with the ansatz

$$X = aB + bL \quad (8)$$

where  $a$  and  $b$  are hitherto unknown real numbers. Quarks have  $q = 0$ ,  $B = 1/3$ , and  $L = 0$ . So they have  $X \neq 0$  and therefore  $M_3 \neq 0$ . Leptons have  $q = 0$ ,  $B = 0$ , and  $L = 1$ . So they have  $X \neq 0$  and therefore  $M_3 \neq 0$ . The gauge group associated with  $M_3$  is  $SU(2)$ . So it is reasonable that both leptons and quarks have  $M_3 = \pm 1/2$  and therefore  $X = \pm 1$ . This can be satisfied only if  $a = \pm 3$  and  $b = \pm 1$ .

The total electric charge, isospin, chromoelectric color, and hypercharge of each of the three conventional fermion octets is zero.

For symmetry reasons it is reasonable to assume that fermionic magnetic monopoles exist in octets and that the total magnetic charge, magnetic isospin, chromomagnetic color, and hypocharge of each of the magnetic fermion octets is zero.

Let us use the following nomenclature. Hanselons are elementary magnetic fermions with nonzero chromomagnetic color. Gretelons are elementary magnetic fermions with zero chromomagnetic color.

In this case each magnetic fermion octet consists of three hanselons, one gretelon, and their respective isomagnetic partner, thus six hanselons and two gretelons.

The conditions

(i)  $a = \pm 3$  and  $b = \pm 1$

(ii)  $Q = 0$  and  $Y = B - L \neq 0$  and therefore  $I_3 = \pm 1/2$  and therefore  $Y = \pm 1$  for magnetic monopoles

(iii) zero total hypocharge of the octet

can be satisfied if each hanselon has  $L = 1$  and  $B = 0$  and each gretelon has  $B = 1$  and  $L = 0$ , hence  $a = 3$ ,  $b = -1$ , and

$$X = 3B - L \quad (9)$$

By using  $Y = B - L$  we get

$$B = (X - Y)/2 \quad (10)$$

$$L = (X - 3Y)/2 \quad (11)$$

## 8 Predicted Quantum Numbers of Particles

The conventional and new quanta have the following quantum numbers.

(i) Up, charm, and top quark have  $Q = +2/3$ ,  $I_3 = +1/2$ ,  $B = +1/3$ ,  $L = 0$ ,  $Y = +1/3$ ,  $X = +1$ ,  $M_3 = -1/2$ ,  $q = 0$ , chromoelectric color, no chromomagnetic color.

(ii) Down, strange, and bottom quark have  $Q = -1/3$ ,  $I_3 = -1/2$ ,  $B = +1/3$ ,  $L = 0$ ,  $Y = +1/3$ ,  $X = +1$ ,  $M_3 = -1/2$ ,  $q = 0$ , chromoelectric color, no chromomagnetic color.

(iii) Electron, muon, and tauon have  $Q = -1$ ,  $I_3 = -1/2$ ,  $B = 0$ ,  $L = +1$ ,  $Y = -1$ ,  $X = -1$ ,  $M_3 = +1/2$ ,  $q = 0$ , no chromoelectric color, no chromomagnetic color.

(iv) Electron-neutrino, muon-neutrino, and tau-neutrino have  $Q = 0$ ,  $I_3 = +1/2$ ,  $B = 0$ ,  $L = +1$ ,  $Y = -1$ ,  $X = -1$ ,  $M_3 = +1/2$ ,  $q = 0$ , no chromoelectric color, no chromomagnetic color.

(v) Electric photon and magnetic photon have  $Q = q = I_3 = M_3 = B = L = X = Y = 0$ , no chromoelectric color, no chromomagnetic color. Their rest mass must be zero in order to satisfy the Dirac quantization condition.

(vi) Chromoelectric gluons have chromoelectric color and are otherwise neutral.

(vii) Chromomagnetic gluons have chromomagnetic color and are otherwise neutral.

(viii) W bosons have  $Q = \pm 1$  and  $I_3 = \pm 1$  and are otherwise neutral.

(ix) Isomagnetic W bosons have  $q = \pm 1$  and  $M_3 = \pm 1$  and are otherwise neutral.

(x) Z boson and isomagnetic Z boson are neutral.

(xi) Hanselons have chromomagnetic color, no chromoelectric color,  $B = 0$ ,  $L = +1$ ,  $X = -1$ ,  $Y = -1$ ,  $Q = 0$ ,  $I_3 = +1/2$ , ( $M_3 = +1/2$  and  $q = 0$  or  $M_3 = -1/2$  and  $q = -1$ ).

(xii) Gretelons have no chromomagnetic color, no chromoelectric color,  $B = +1$ ,  $L = 0$ ,  $X = +3$ ,  $Y = +1$ ,  $Q = 0$ ,  $I_3 = -1/2$ , ( $M_3 = +1/2$  and  $q = +2$  or  $M_3 = -1/2$  and  $q = +1$ ).

(xiii) The quantum numbers of the antiparticles corresponding to the particles (i)–(iv) and (xi)–(xii) have the opposite sign. Antileptons and antiquarks are the isomagnetic partners of leptons and quarks. Antihanselons and antigretelons are the isospin partners of hanselons and gretelons.

(xiv) Higgs boson and magnetic Higgs boson are neutral. Their spin is zero.

(xv) It is  $\sin^2 \Theta_W \simeq 0.23$ , so one can assume that  $\sin^2 \Theta_M$  is also of order unity. The relation between positron charge  $e$  and weak coupling constant  $g_W$  is  $e = g_W \sin \Theta_W$ . The relation between unit magnetic charge  $m$  and magnetic weak coupling constant  $g_M$  is  $m = g_M \sin \Theta_M$ . The Dirac quantization condition is  $em = 2\pi$  (where  $\hbar = c = \varepsilon_0 = 1$ ), hence

$$\begin{aligned} g_M &= \frac{m}{\sin \Theta_M} = \frac{2\pi}{e \sin \Theta_M} \\ &= \frac{2\pi}{g_W \sin \Theta_W \sin \Theta_M} > 1 \end{aligned} \quad (12)$$

All the quarks and leptons have nonzero magnetic isospin  $M_3$ . Since magnetic isospin has not yet been observed, this can only mean that the rest masses of the isomagnetic W and Z bosons are larger than 100 GeV.

(xvi) All the leptons, quarks, hanselons, and gretelons have spin 1/2. All the 12 conventional and all the 12 new gauge bosons have spin 1 and negative parity. Higgs boson and magnetic Higgs boson have spin 0 and positive parity.

(xvii) All the leptons, quarks, hanselons, and gretelons have nonzero isospin. It is therefore possible to create hanselon-antihanselon pairs and gretelon-antigretelon pairs via neutral currents. One possibility is electron-positron scattering

$$e^- e^+ \rightarrow Z^0 \rightarrow G\bar{G} \quad (13)$$

(xviii) With regard to isospin all the leptons, quarks, hantons, and gretelons are (iso-)dyons, because they have both isospin and magnetic isospin.

(xix) The magnetic Fermi constant has not yet been determined. It is probable that its value is between those of the Fermi constant and the gravitational constant.

(xx) The coupling constant associated with magnetic charge is larger than unity. The corresponding binding energy would lead to negative energy densities for small distances. This violation of the weak energy condition can be prevented if the magnetic coupling constant is a running coupling constant, becomes smaller for higher energies, and the rest masses of the magnetic monopoles are of the order of the Planck mass.

(xxi) A similar problem with magnetic isospin can be solved if the rest masses of the isomagnetic W and Z bosons are of the order of the Planck mass. This suggests that the magnetic Fermi constant is of the order of the gravitational constant.

(xxii) The magnetic analogue to the graviton is the tordion. The gauge group is the Poincare group [12]. The magnetic analogue to mass is spin [5, 13]. The spin of the graviton is 2, its rest mass is zero. The spin of the tordion is 3, its rest mass is of the order of the Planck mass [5, 13].

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