A memory dependent analysis on permeation of non-Gaussian laser pulse through human skin

Soumen Shaw and Santanu Banerjee
Department of Mathematics
Dinabandhu Andrews College, Kolkata-700084, India.

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Abstract

The present article deals with a thermodynamic analysis of laser treatment on human skin. Replacing Pennes model of thermodynamics, a well defined memory dependent hyperbolic type thermoelastic heat conduction model is adopted. In order to analyse the thermal shock experienced by the skin layers, a bi-layer skin containing two main layers: epidermis and dermis, is considered. An instantaneous point heat source and a non-Gaussian laser pulse are applied on the skin surface to analyse the thermal memory response into the skin-layers. Two different types of laser beams are considered to validate the thermoelastic observations with the optical results.

Key words: Non-Gaussian laser; thermodynamics; skin-layers; thermal shock; penetration depth.

1 Introduction

In light of recent advancements in biomedical science and engineering, laser treatment in various clinical therapies and surgeries plays an important role. Laser technology is being used in procedures of photodynamic therapies, photo-rejuvenations and mostly in thermal surgeries. One of the challenging task in these thermal treatments is to provide a suitable amount of heat energy in the exact portion of the diseased tissue and that too without affecting the neighboring tissues.

Laser beam play a vital role in treatment of certain types of malignant tumours and cancers also. The high intensified energies of lasers prove useful to shrink or even destroy tumour and pre-cancerous cell growths(especially in superficial tissues). Since this whole treatment requires the phenomenon of heat energy transfer upon an elastic environment (like human tissues), a mathematical modeling of thermoelastic equations coupled with bio-mathematical variables becomes essential. In 1948, H. H. Pennes [15] introduced a coupled bio-thermoelastic model to study the temperature profile in a portion of human forearm. This study provides a qualitative analysis of the arterial blood flow and its neighboring tissue temperatures, owing to the geometrical aspects. Even though convection plays its part in heat transfer due to arterial blood flow, Pennes neglected the thermal convection due to blood flow in his model. In 1974, introducing the Darcy velocity term in the theory of bio-thermoelasticity, Wulff [27] modified the Pennes model [15] accurately. Later on, several authors have attempted to improve the bio-thermal equations [3, 12, 24, 25]. Owing to the simplicity of the Pennes model, it serves more useful in terms of mathematical computations in bio-thermal problems subject to some modifications and considerations.

The classical theory of thermoelasticity was based upon the Fourier's law of heat conduction, in which the energy equation was parabolic in nature. Infinite speed of the thermal signals was inherent in that

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theory. To overcome the unrealistic featers of the thermal wave, in 20th century several modification were indroduced. The advancements of thermoelasticity and different aspects of heat transfer mechanism involving finite speed propagation of thermal signals, can be found in the nonograph of of Ignaczak and Ostoja-Starzewski (see Ref. [8] and the references therein).

In the present article our aim is to investigate a non-Gaussian laser pulse response into the human skin-layers thermodynamically. Based on the work of Pennes [15], adopting a generalized memory dependent heat conduction model, a new set of governing equations are developed. Employing eigen-function expansion method, the field functions are derived in laplace transform domain and then an efficient inverse-laplace transformation technique is applied by the method of discretization [7]. The numerical computational results are compared with experimental observations regarding the penetration depth of photons. For strong absorption and strong scattering, two different wavelengths of laser pulse are taken into consideration.

2 Governing Equations

A sudden thermal excitation (a point heat source viz. a laser pulse) on skin tissue can effect on comparatively a local area (neighbourhood) surrounding the point of application. We shall confine our study within the area of effected region. Here, we assume that the portion of skin tissue to be uniform with linear, homogeneous and isotropic thermoelastic properties in the present study. Based on the Pennes theory of heat flow and the recent developments on the relationship between arterial blood and tissue temperatures, the governing equations of bio-thermoelastic heat conduction model can expressed as follows [15, 22]:

1. Law of heat conduction
$$q_i = -k\theta_{,i} \tag{1}$$

2. Energy equation in Pennes model [15]
$$q_{i,i} = -\rho T_0 \dot{S} + \rho_b c_b w_b \left(T_b - T\right) + Q_m \tag{2}$$

3. Equation of motion
$$\sigma_{ij,j} = \rho \ddot{u}_i \tag{3}$$

4. Second law of thermodynamics
$$\rho S = \gamma e_{kk} + \frac{\rho c_e}{T_0}\theta \tag{4}$$

5. Constitutive relation (stress, strain and temperature relation)
$$\sigma_{ij} = 2\mu e_{ij} + \left(\lambda e_{kk} - \gamma\theta\right)\delta_{ij} \tag{5}$$

6. Linear kinematic relation
$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{6}$$

Here q_i denotes the components of heat flux vector, u_i is the component of displacement vector, σ_{ij} is the component of stress tensor, k is the thermal conductivity, ρ represents the mass density of skin tissue, c_e is the specific heat of skin tissue at reference temperature, γ denotes the thermal moduli given by $\gamma = \left(3\lambda + 2\mu\right)\alpha_t$; λ and μ being Lame's constants and α_t being the coefficient of linear thermal expansion of the skin tissue.

T denotes the absolute temperature of the body, T_0 is the reference temperature, $\theta = T - T_0$ with an assumption $\mid \frac{\theta}{T_0} \mid << 1$. The term $\rho_b c_b w_b \left(T_b - T\right)$ in Eqn.(2) describes the heat conduction between blood and tissue with Q_m being the metabolic heat generation due to blood flow in the veins. ρ_b, w_b, c_b

and θ_b denotes the blood mass density, blood perfusion rate, specific heat of blood and blood temperature in the skin tissue respectively. Besides, the super-dots refer to the ordinary time-derivative and comma followed by sub-indexes denote the corresponding partial differentiation with respect to space variables. It is assumed $T_b = T_0$ in the present study.

From the pioneering work of Pennes [15], theories of bio-heat transfer are getting enormous importance in the field of clinical physiology. In the last decade several developments are reported and excellent analytical expressions are demonstrated by employing different thermoelastic models on thermal responses of living tissue. Hitherto, adopting generalized thermoelastic models, the existing studies mainly taken care of the thermoelastic response of skin tissue with various material properties of the biological tissue. Most of cases, the material properties are taken as a constant. Recently, few attempt with variable thermal conductivity and specific heat are reported in the literature (for details see Refs. [13, 26] and references therein). Employing fractional order heat conduction model of generalized thermoelasticity, the Pennes model of arterial blood and tissue temperatures has been successfully replaced by Ezzat et. al. [5], Jiang and Haitao [11].

In 21st-century, the memory phenomena is successfully employed in various disciplines (from material modelling to medical science). The model takes the form:

$$ {}_0D_t^{\alpha}\epsilon(t) = C\sigma(t), \tag{7}$$

where ${}_0D_t^{\alpha}$ is the fractional derivative which depends on the strain history from 0 to t, and C is the positive constant.

Fractional order derivative is a generalization of the classical integer-order derivative and integral. If α is an integer n, then ${}_0D_t^{\alpha}\epsilon(t) = D^n\epsilon(t) = d^n\epsilon(t)/dt^n$. It has a long history, way back in 1695. It was originated from a famous letter regarding the meaning of 1/2 order derivative from Guillaume de l'Hôpital to Gottfried Wilhelm Leibniz in 1695 [14, 16].

There are several definitions of fractional derivative [4, 14, 16]. Caputo derivative is one of the most practicable definitions to describe the physical nature [2, 18], defined as

$$ {}_0D_t^{\alpha}f(t) = \frac{1}{\Gamma(m-\alpha)}\int_0^t \frac{f^m(\tau)}{(t-\tau)^{\alpha+1-m}}d\tau \tag{8}$$

where m is an integer satisfying $m-1 \leq \alpha < m$ and Γ denotes the Euler's Gamma function.

It was a promising tool for describing memory phenomena. The kernel function of fractional derivative is called memory function, but it does not reflect any physical process. Ambiguous physical meaning has been a big obstacle that keeps fractional derivative lagging far behind the integer-order calculus. In 1974, the science fraternity put an open problem as " what are the physical interpretations of fractional calculus". Till now, there is no simple answer to this open problem.

In 2011, Surpassing the fractional derivative, the Concept of the memory-dependent derivative has been introduced by Wang and Li [23]. For any continuous function $f(x,t)$, the memory dependent derivative(MDD) over a slipping interval $[t-a,t]$ is defined as

$$ D_a^{(m)}f(x,t) = \frac{1}{a}\int_{t-a}^{t} K(t-\xi)\frac{\partial^m f(x,\xi)}{\partial\xi^m}\partial\xi \tag{9}$$

where the time delay $'a(>0)'$ denotes the memory scale and the kernel function $K(t-\xi)$ $\big(0 < K(t-\xi) \leq 1, \quad \xi \in [t-a,t]\big)$ must be a differentiable function with respect to its arguments. The kernel function and the memory scales to be chosen in such a way that these are compatible with the physical problem, so this type of derivatives provides more possibilities to capture the material response.

In 2014, memory-dependent derivative has been applied in thermodynamics successfully [28]. In this process, non-locality effect comes into play in the theory of thermodynamics (For details see Ref. [20, 21] and the cross-references therein).

In general the kernel function is taken in the following form:

$$K(t,p) = \left(\frac{p-t}{a} + 1\right)^b \tag{10}$$

For $b = 0, 1, 2$ the kernel is called constant, linear or parabolic respectively.

Here in the present article, memory dependent derivative is employed in the heat conduction law and try to explain the memory responses over the temperature distribution as well as stress evolved into the skin layers.

Now, we replace the Fourier's law of heat conduction i.e. Eqn. (1) by the following Memory dependent generalized heat conduction law [28]

$$q_i + \tau D_\omega q_i = -k\theta_i, \tag{11}$$

in which τ denotes the thermal relaxation time, D_ω refer to the memory dependent time-derivative with ω being the thermal time delay.

From Eqs. (2), (4) and (11) one can obtain the generalized memory dependent bio-heat conduction equation

$$k\theta_{,ii} = (1 + \tau D_\omega)\left(\rho c_e \dot{\theta} + \gamma T_0 \dot{u}_{i,i} + \rho_b c_b w_b (T - T_b) - Q_m\right) \tag{12}$$

and using Eqs. (5) and (6), the equation of motion (Eqn. 3) can be expressed in terms of displacement vector as

$$\rho \ddot{u}_i = \mu u_{j,ij} + (\lambda + \mu) u_{i,jj} - \gamma \theta_{,i} \tag{13}$$

The above equations (12) and (13) provide the governing equations for memory dependent bio-thermoelasticity theory.

3 Problem formulation

Human skin, in human anatomy, the covering, or integument, of the body's surface that both provides protection and receives sensory stimuli from the external environment. The skin consists of three layers of tissue: the epidermis, an outermost layer that contains the primary protective structure, the stratum corneum; the dermis, a fibrous layer that supports and strengthens the epidermis; and the subcutis, a subcutaneous layer of fat beneath the dermis that supplies nutrients to the other two layers and that cushions and insulates the body (see Fig. 1).

Following the thermodynamical theory of Pennes [15] and to construct a mathematical model, we shall make some anatomical assumptions in the section of skin tissue under consideration. Firstly we shall restrict the effect of our problem till the dermis layer, i.e. we shall consider the portion of skin tissue with two main layers: the epidermis and the dermis. Secondly, the blood temperature shall be considered as the reference temperature, i.e. $T_0 = T_b$. Thirdly, owing to a high-intensity heat projection on the skin surface, the metabolic heat generation Q_m due to blood flow in Eqn.(2) shall be neglected. Lastly, since a high intensity heat beam is projected perpendicularly upon the outer boundary of the epidermis layer, the effect of the heated spot shall be much smaller when compared to the thickness of the bi-layered skin tissue [1].

Thus owing to the above assumptions, we shall consider all the field functions depends upon the location $\vec{x}$ and time t. Here the coordinate system is chosen that the x-axis is perpendicular to the skin surface, and the skin surface is represented by yz-plane. Hence, all thermoelastic field variables in the present problem will be depend on two independent variables x and t. Consequently, the displacement vector will have only one component $u(x,t)$ (say). The depth of the epidermis layer from the surface of the skin is taken to be h and the width of the dermis layer is taken to be l (as depicted in Fig. 2).

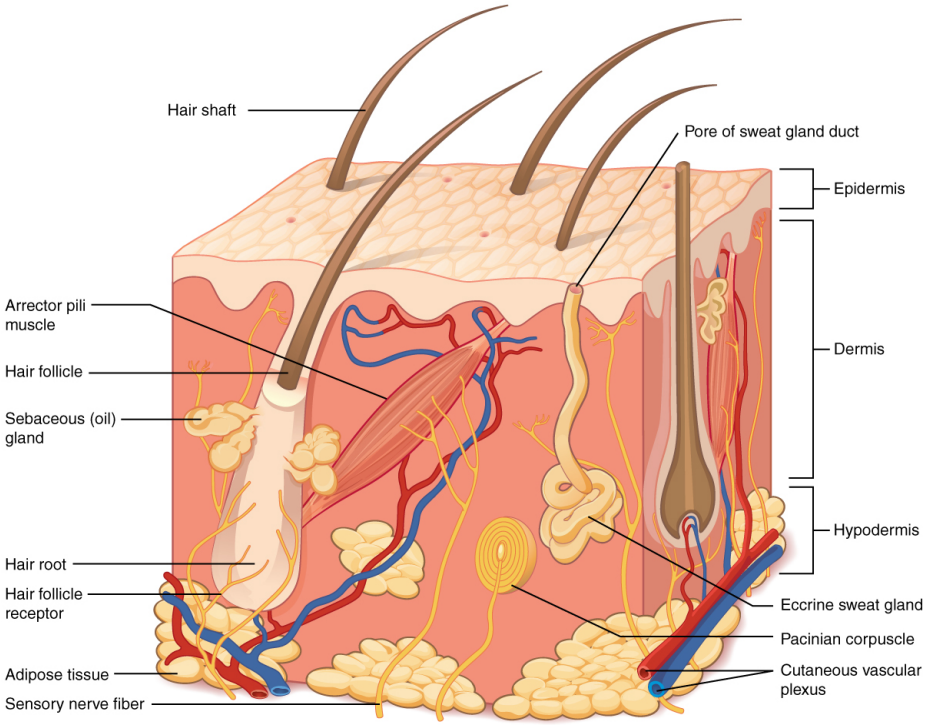

Figure 1: Layers of Skin. The skin is composed of two main layers: the epidermis, made of closely packed epithelial cells, and the dermis, made of dense, irregular connective tissue that houses blood vessels, hair follicles, sweat glands, and other structures.

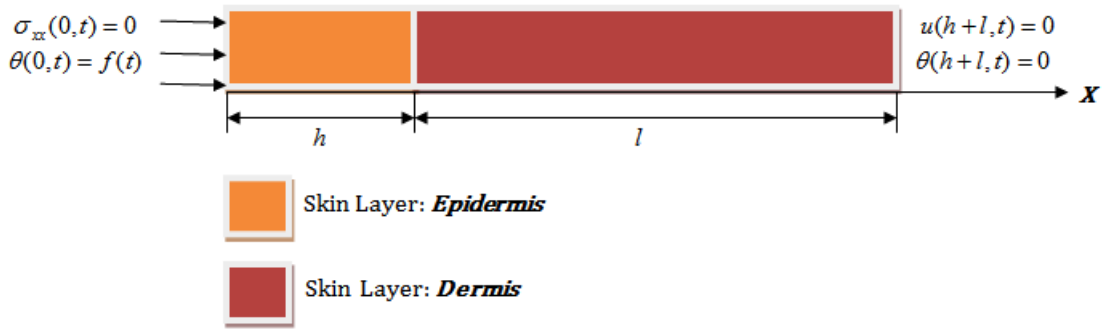

Figure 2: A schematic illustration of the bi-layered skin problem.

Now, according to aforementioned mathematical as well as anatomical assumptions, Eqs.(12), (13) and (5) yield

$$k\frac{\partial^2\theta}{\partial x^2}=(1+\tau D_\omega)\left(\rho c_e\frac{\partial\theta}{\partial t}+\gamma T_0\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right)+\rho_b c_b w_b\theta\right) \tag{14}$$

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}-\gamma\frac{\partial\theta}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2} \tag{15}$$

$$\sigma_{xx}=(\lambda+2\mu)\frac{\partial u}{\partial x}-\gamma\theta \tag{16}$$

We employ the following non-dimensional transformations for our convenience:

$$\left(x', h', l'\right) = \frac{1}{L}\left(x, h, l\right), \quad \left(\tau', t'\right) = \frac{\nu}{L}\left(\tau, t\right), \quad q_i' = \frac{Lq_i}{k\left(T_\infty - T_0\right)},$$
$$u' = \frac{\left(\lambda + 2\mu\right)u}{\gamma L\left(T_\infty - T_0\right)}, \quad \theta' = \frac{\theta}{T_\infty - T_0}, \quad \sigma_{xx}' = \frac{\sigma_{xx}}{\gamma\left(T_\infty - T_0\right)}$$

where L is having the dimension of a unit length, $\nu^2 = \frac{\lambda+2\mu}{\rho}$ and T_∞ is the temperature of thermal shock upon the outer boundary of epidermis.

Suppressing the primes, Eqs.(14)-(16) can be recast into the following non-dimensional form:

$$\frac{\partial^2\theta}{\partial x^2} = \left(1 + \tau D_\omega\right)\left(\alpha_1\frac{\partial\theta}{\partial t} + \alpha_2\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right) + \alpha_3\theta\right) \tag{17}$$

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial\theta}{\partial x} = \frac{\partial^2 u}{\partial t^2} \tag{18}$$

$$\sigma_{xx} = \frac{\partial u}{\partial x} - \theta \tag{19}$$

where

$$\alpha_1 = \frac{\rho c_e \nu L}{k} \quad , \quad \alpha_2 = \frac{\gamma^2 T_0 L\nu}{k\left(\lambda + 2\mu\right)} \quad , \quad \alpha_3 = \frac{L^2\rho_b c_b w_b}{k}. \tag{20}$$

4 Initial-boundary and inter-facial conditions of the skin layers

The medium is initially assumed to be in an inactive state. Thus the initial conditions of the problem at time $t = 0$ are given by,

$$u(x,0) = \dot{u}(x,0) = \theta(x,0) = \dot{\theta}(x,0) = \sigma_{xx}(x,0) = 0 \tag{21}$$

The laser is applied for very small duration, we can neglect any radial scattering or convection of thermomechanical properties. Owing to the short time period and the finiteness of thermoelastic wave velocities, we shall consider zero boundary conditions at the end of the dermis layer. Thus the thermal and mechanical boundary conditions as shown in Fig 2. can be expressed as:

$$\begin{aligned} \theta(0,t) &= f(t) \\ \sigma_{xx}(0,t) &= 0 \\ \theta(h+l,t) = u(h+l,t) &= 0 \end{aligned} \tag{22}$$

where $f(t)$ represents the temperature produced by the applied thermal shock.

For simplicity, we shall neglect all thermoelastic resistances at the interface of the epidermis and dermis layer. Hence, the conditions at the junction of the layers shall be considered to be continuous with respect to the thermal and elastic variables. Consequently,

$$\begin{aligned} q_x^E(h,t) &= q_x^D(h,t) \\ \theta^E(h,t) &= \theta^D(h,t) \\ u^E(h,t) &= u^D(h,t) \\ \sigma_{xx}^E(h,t) &= \sigma_{xx}^D(h,t) \end{aligned} \tag{23}$$

From here onwards the superscripts "E" and "D" shall denote the thermoelastic parameters in the epidermis and the dermis layer, respectively.

5 Non-Gaussian laser pulse

The temporal profile of a non-Gaussian laser pulse is defined as (Fig. 3),

$$L_P(t) = \frac{L_0}{t_p^2} t e^{-\frac{t}{t_p}} \tag{24}$$

where L_0 is the laser intensity that is defined as the total energy carried by a laser pulse per unit area of the laser beam, and t_p is the characteristic time of the laser pulse which may also be referred as the time duration of the laser pulse.

The function has the property

$$\int_0^\infty L_P(t)\, dt = 1 \tag{25}$$

and

$$\max_{\forall t} [L_P(t)] = L_P(t_p) \tag{26}$$

The following figure provides the temporal profile of the laser power L_P/L_0, by considering $tp = 2p$ s.

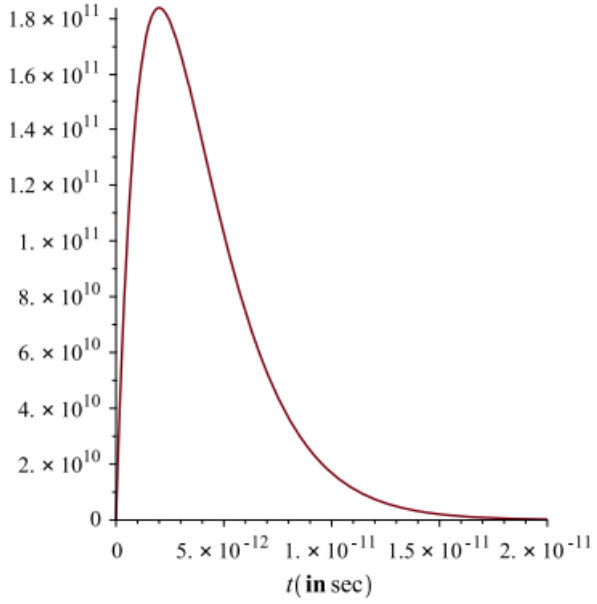

Figure 3: The temporal profile of the laser power L_P/L_0

6 Method of solution

In order to extent our analysis, here we apply Laplace transformation with respect to the time variable t defined as

$$\widetilde{F}(x,s) = L\left[F(x,t); t \to s\right] = \int_0^\infty exp\left(-st\right) F(x,t) dt$$

Consequently, Eqs.(17) - (19) yield,

$$\frac{d^2\widetilde{\theta}}{dx^2} = s_1\left(\alpha_1 s + \alpha_3\right)\widetilde{\theta} + s_1 s \alpha_2 \frac{d\widetilde{u}}{dx} \tag{27}$$

$$\frac{d^2\widetilde{u}}{dx^2} = \frac{d\widetilde{\theta}}{dx} + s^2\widetilde{u} \tag{28}$$

$$\widetilde{\sigma}_{xx} = \frac{d\widetilde{u}}{dx} - \widetilde{\theta} \tag{29}$$

where,

$$s_1 = 1 + \tau\xi$$

and (for details see Ref. [6])

$$\xi = \frac{1}{\omega}\left[\left(1 - \frac{2b}{\omega s} + \frac{2a^2}{\omega^2 s^2}\right) - exp\left(-\omega s\right)\left(1 - 2b^2 + a^2 + \frac{2\left(a^2 - b\right)}{\omega s} + \frac{2a^2}{\omega^2 s^2}\right)\right]$$

Now, Eq.(27) and Eq.(28) can be recast in a vector matrix differential equation from as follows:

$$\frac{d\widetilde{V}(x,s)}{dx} = \widetilde{A}(s)\widetilde{V}(x,s) \tag{30}$$

where,

$$\widetilde{V} = \begin{bmatrix} \widetilde{\theta} & \widetilde{u} & \dfrac{d\widetilde{\theta}}{dx} & \dfrac{d\widetilde{u}}{dx} \end{bmatrix}^T$$

and

$$\widetilde{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{31} & 0 & 0 & a_{34} \\ 0 & a_{42} & 1 & 0 \end{bmatrix}$$

in which

$$a_{31} = s_1\left(\alpha_1 s + \alpha_3\right), \quad a_{34} = s_1 s\alpha_2, \quad a_{42} = s^2 \tag{31}$$

6.1 Solution for epidermis layer

The epidermis layer has the boundary of $0 \leq x \leq h$. Remembering that the blood perfusion is in the dermis layer only (the epidermis consists of keratin with no blood supply), we can discard the term α_3 (i.e. $\alpha_3 = 0$) in Eq.(20). Thus the characteristic equation of matrix $\widetilde{A}$ is obtained as

$$\zeta^4 - t_1^E\zeta^2 + t_2^E = 0 \tag{32}$$

where

$$t_1^E = ss_1\left(\alpha_1^E + \alpha_2^E\right) + s^2 \quad , \quad t_2^E = s_1 s^3\alpha_1^E \tag{33}$$

The roots of Eq.(32) are assumed to be of the form: $\zeta = \pm m_1, \pm m_2$
where

$$m_1^2 + m_2^2 = t_1^E \quad , \quad m_1^2 m_2^2 = t_2^E$$

The eigenvector $\widetilde{X}_i$ corresponding to the respective eigenvalue ζ_i of the matrix $\widetilde{A}$ is given by,

$$\widetilde{X}_i = \widetilde{X}\big|_{\zeta_i = m_i}, \ (i = 1, 2, 3, 4) \tag{34}$$

in which

$$\widetilde{X}\big|_{\zeta_i} = \begin{bmatrix} a_{42}^E - \zeta_i^2 \\ -\zeta_i \\ \zeta_i\left(a_{42}^E - \zeta_i^2\right) \\ -\zeta_i^2 \end{bmatrix} \tag{35}$$

Hence, the solution of Eq.(30) can be expressed as follows:

$$\widetilde{V}(x,s) = \sum_{i=1}^{4} \widetilde{C}_i \widetilde{X}_i exp(\zeta_i x) \tag{36}$$

where $x > 0$ and,

$$\widetilde{C}_i = \begin{bmatrix} c_{i1} & c_{i2} & c_{i3} & c_{i4} \end{bmatrix} \tag{37}$$

Therefore, from Eqs. (28) and (29) we obtain the solutions for the field functions in the epidermis layer as follows:

$$\widetilde{u}^E(x,s) = \sum_{i=0}^{4} u_i^E exp(\zeta_i x) \quad , \quad u_i^E = \zeta_i c_{2i} \tag{38}$$

$$\widetilde{\theta}^E(x,s) = \sum_{i=0}^{4} \theta_i^E exp(\zeta_i x) \quad , \quad \theta_i^E = d_i^E u_i^E \tag{39}$$

$$\widetilde{\sigma}_{xx}^E(x,s) = \sum_{i=0}^{4} \sigma_i^E exp(\zeta_i x) \quad , \quad \sigma_i^E = b_i^E u_i^E \tag{40}$$

where

$$d_i^E = \frac{s^2 - m_i^2}{m_i} \quad , \quad b_i^E = -\frac{s^2}{m_i} \tag{41}$$

6.2 Solution in dermis layer

For dermis layer $x \in [h, h+l]$. In this portion of the skin layer, blood perfusion is actively present. The characteristic equation of matrix $\widetilde{A}$ is thus obtained as

$$\eta^4 - t_1^D \eta^2 + t_2^D = 0 \tag{42}$$

where

$$t_1^D = s s_1 \left(\alpha_1^D s + \alpha_2^D\right) + s_1 \alpha_3^D + s^2 \quad , \quad t_2^D = s_1 s^2 \left(\alpha_1^D s + \alpha_3^D\right) \tag{43}$$

Roots of the Eq.(42) to be of the form : $\eta = \pm n_1, \pm n_2$ where

$$n_1^2 + n_2^2 = t_1^D \quad , \quad n_1^2 n_2^2 = t_2^D$$

The eigen vectors $\widetilde{Y}_i$ corresponding to the respective eigen values η_i of the matrix $\widetilde{A}$ are given by,

$$\widetilde{Y}_i = \widetilde{Y}\big|_{\eta_i = n_i}, \ (i = 1, 2, 3, 4) \tag{44}$$

where

$$\widetilde{Y}\big|_{\eta_i = n_i} = \begin{bmatrix} a_{42}^D - \eta_i^2 \\ -\eta_i \\ \eta_i \left(a_{42}^D - \eta_i^2\right) \\ -\eta_i^2 \end{bmatrix} \tag{45}$$

The solution of Eq.(30) is expressed as:

$$\widetilde{V}(x,s) = \sum_{i=1}^{4} \widetilde{P}_i \widetilde{Y}_i exp(\eta_i x) \tag{46}$$

where $x > 0$ and,

$$\widetilde{P}_i = \begin{bmatrix} p_{i1} & p_{i2} & p_{i3} & p_{i4} \end{bmatrix} \tag{47}$$

Similar to epidermis layer, the field functions in the dermis layer as follows:

$$\widetilde{u}^D(x,s) = \sum_{i=0}^{4} u_i^D exp(\eta_i x) \quad , \quad u_i^D = \eta_i p_{2i} \tag{48}$$

$$\widetilde{\theta}^D(x,s) = \sum_{i=0}^{4} \theta_i^D exp(\eta_i x) \quad , \quad \theta_i^D = d_i^D u_i^D \tag{49}$$

$$\widetilde{\sigma}_{xx}^D(x,s) = \sum_{i=0}^{4} \sigma_i^D exp(\eta_i x) \quad , \quad \sigma_i^D = b_i^D u_i^D \tag{50}$$

where

$$d_i^D = \frac{s^2 - n_i^2}{n_i} \quad , \quad b_i^D = -\frac{s^2}{n_i} \tag{51}$$

Now, invoking the intial-boundary conditions (21) - (23) we can determine the expressions for c_{2i} as well as p_{2j} (i, j = 1,2,3,4).

7 Absorption and scattering of thermal shock irradiation

For the purpose of treatment of various skin disorders with light, it is important to have deep photon penetration into the skin. Consequently, it makes use of absorption of photons by chromophores present in viable cells of either the epidermis or the dermis.

Strong absorption:

For some (laser) wavelengths in the UV (e.g., The wavelength about 193 nm at the ArF excimer laser) and the IR, tissue absorption may be substantially larger than scattering. We consider thermal damage of surrounding tissue by thermal conduction from the thermal shock-irradiated site. The thermal relaxation time, τ, is defined as in the following equation on the assumption that the penetration depth, δ, for the laser wavelength is much smaller than the laser spot size [26]

$$\tau = \frac{\delta^2}{4\xi}, \tag{52}$$

where ξ is the thermal diffusivity of tissue. Thermal diffusivity of tissue corresponds approximately to that of water ($\xi = 1.3 \times 10^{-7} m^2/s$). First of all, we estimate the penetration depth of the stratum corneum for 193 nm wavelength. In this wavelength, where absorption is stronger than scattering, the penetration depth δ could be expressed as the following equation:

$$\delta = \frac{1}{\mu_a}, \tag{53}$$

where μ_a is the absorption coefficient. The measured absorption coefficient of the human stratum corneum at the 193 nm wavelength was reported as 6,690 cm^{-1} [10].

Meanwhile, by using the measured absorption coefficient of the cornea at the 193-nm wavelength ($\alpha_{cor} = 2.7 \times 10^3 cm^{-1}$) [17] and the composition of the stratum corneum (water: 15%, protein: 70%, lipid: 15%) [19] and the cornea (water: 78%, protein: 20%), we could estimate the absorption coefficient of the stratum corneum, α_{sc}, as

$$\alpha_{sc} = 2.7 \times 10^3 \times \frac{70}{20} \quad cm^{-1} = 9450 \quad cm^{-1}. \tag{54}$$

Considering that the absorption coefficient of the stratum corneum at the 193 nm wavelength is 6,690 (reported value)– 9,450 (estimated value) cm^{-1}, the penetration depth δ becomes

$$\delta = \frac{1}{\alpha_{sc}} \approx 1.1 - 1.5 \quad \mu m. \tag{55}$$

On the basis of the above estimation, the thermal relaxation time τ of the stratum corneum at the 193-nm wavelength results in

$$\tau = 2.3 - 4.3 \quad \mu s. \tag{56}$$

The 10-ns pulse duration of the ArF laser is less than a hundredth of the estimated thermal relaxation time τ. The 0.5-s pulse interval is more than a hundredfold of τ. Therefore, the thermal damage by means of heat conduction may be limited in the case of the 193-nm laser ablation described above. Thickness of the thermal damage layer might be kept in the order of the penetration depth. There might be no thermal damage to the epidermis because the thermal damage layer might be thinner than the stratum corneum.

Strong scattering:

Between the wavelengths about 300 nm to 1000 nm nonpigmented tissues have scattering dominating over absorption. It is reported that the transmission of laser light through different skin types is much deeper than the scattering for the laser wavelengths range 532–1064 nm. Under these circumstances the transport equation can be approximated by a diffusion equation and the total fluence rate, $\phi(z)$, can be expressed as follows [29]:

$$\phi(z) = \phi_0 K e^{-\frac{z}{\delta}} \tag{57}$$

where ϕ_0 and $\phi(z)$ are represent the incident photon fluence and the photon fluence at depth z respectively. K denotes the photon buildup factor and δ is the photon penetration depth which is defined as

$$\delta = \frac{1}{\sqrt{3\mu_a \left(\mu_a + \mu_s \left(1 - g\right)\right)}} \quad , \tag{58}$$

in which μ_a and μ_s are the absorption and scattering coefficients; g is the anisotropy scattering factor of the skin.

The Experimental values for the absorption, scattering coefficients and the anisotropy scattering factor are given by

$\mu_a = 0.2mm^{-1}$, $\mu_s = 18.8mm^{-1}$ and $g = 0.8$ which is a set of values that approximates in vitro dermal tissue at 633 nm wavelength [9].

Hence, the calculated value of the photon penetration depth into the skin (for the wavelength 633 nm) is $\delta \approx 0.7 \quad mm$.

8 Numerical analysis

In this section, we try to validate the analytical expressions obtained in the previous sections and the reported experimental results. For numerical computational purpose, the material constants of the layered skin tissue and blood at the reference temperature are given as;

Epidermis:

$\lambda = 8.27 \times 10^8$ kg/m s^2, $\mu = 3.446 \times 10^7$ kg/m s^2, $\rho = 1190$ kg/m^3, $c_e = 3600$ J/kgK, $k = 0.235$ W/mK

Dermis:

$\lambda = 8.27 \times 10^7$ kg/m s^2, $\mu = 3.446 \times 10^6$ kg/m s^2, $\rho = 1116$ kg/m^3, $c_e = 3300$ J/kgK, $\alpha = 1 \times 10^{-4}$ /K, $T_0 = 310$ K

Blood:

$\rho_b = 1060$ kg/m^3, $c_b = 3770$ /kgK, $w_b = 0.005$ /s .

In addition, $L = 1 \times 10^{-3}$ m. The non-dimensional thickness of the epidermis layer is $h = 0.3$ and that of for dermis layer is $l = 1.5$.

For numerical validation purpose, we have considered two different types of thermal shocks applied on the skin surface: (i) **Instantaneous point Heat source:** $f(t) = L_0\delta(t)$ (where $\delta(t)$ represents the Dirac delta function), and (ii) **Non-Gasussian laser pulse:** as discussed in section-5.

For instantaneous point heat source, temperature distribution (Fig. 4), stress profile (Fig. 5) and displacements (Fig. 6) are illustrated over non-dimensional depth from the skin surface. It has been observed that due to thermal memory temperature distribution is not taking so 'flat' nature from the point of application. Consequently, the stress generated in epidermis layer can also be experienced at dermis layer also.

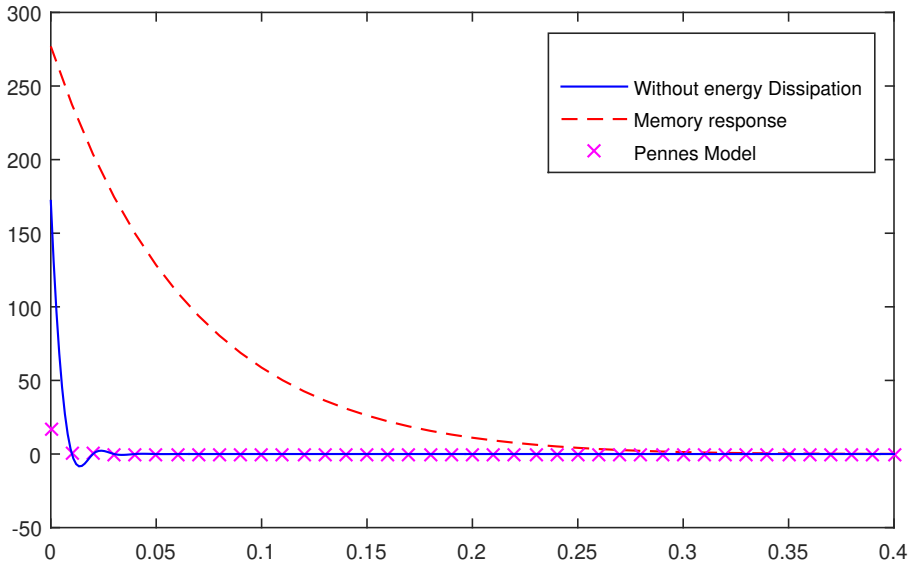

Figure 4: Distribution of temperature with location.

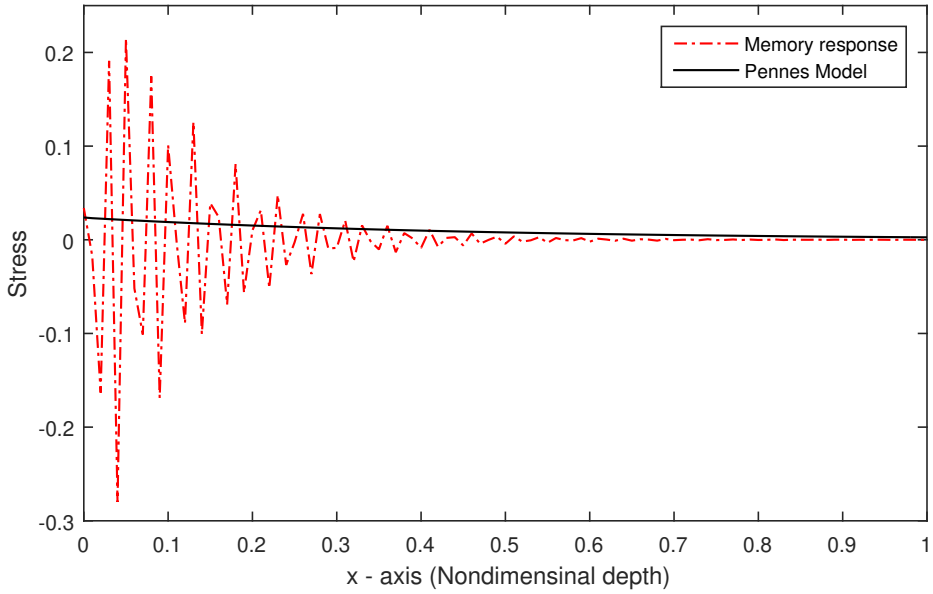

Figure 5: Distribution of stress with location.

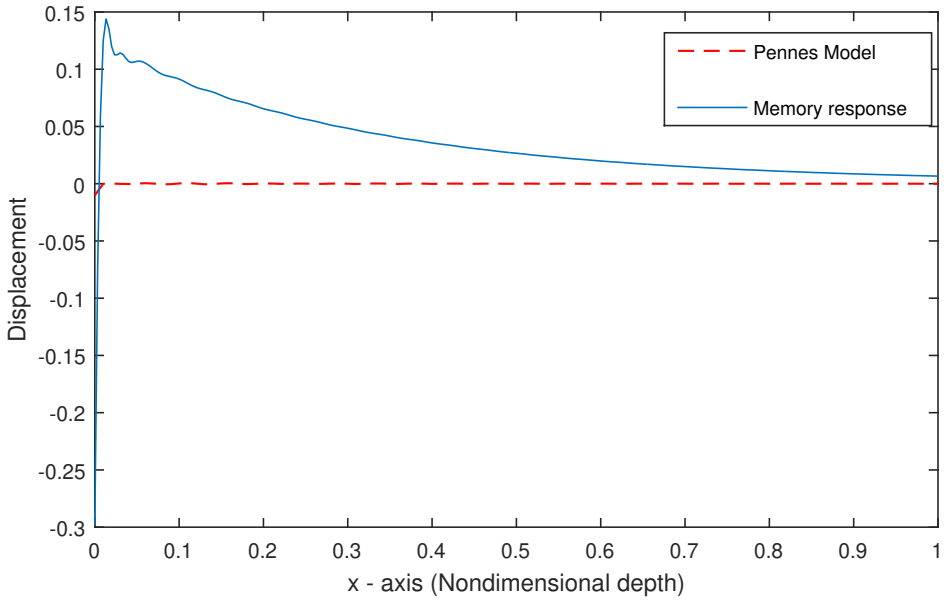

Figure 6: Variation of displacement with location.

In the case of a laser-pulse applied on the skin tissue, the photon penetration into the skin depends upon the wavelengths of the light and that yields the thermal stress into the skin layers. Here, the distribution of temperature (Fig. 7), stress profile (Fig. 8) and comparison of displacements (Fig. 9) are shown for two different wavelengths of laser-pulse (for computational purpose, we have taken $t_p = 1s$). At the wavelength 193 nm, experiments shows that photon can penetrate maximum 1.5 μm, that is its impact should be absorbed by the epidermis layer. Whereas, at the wavelength 633 nm, it has been proved that scattering would be the dominating factor over absorption, and the penetration depth is almost 0.7 mm.

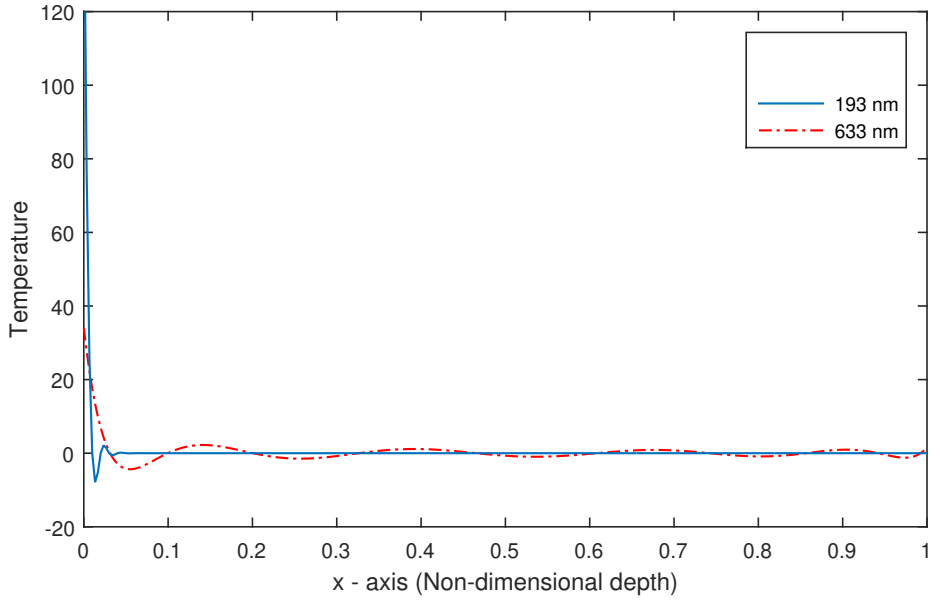

Figure 7: Distribution of temperature for two different laser-pulse.

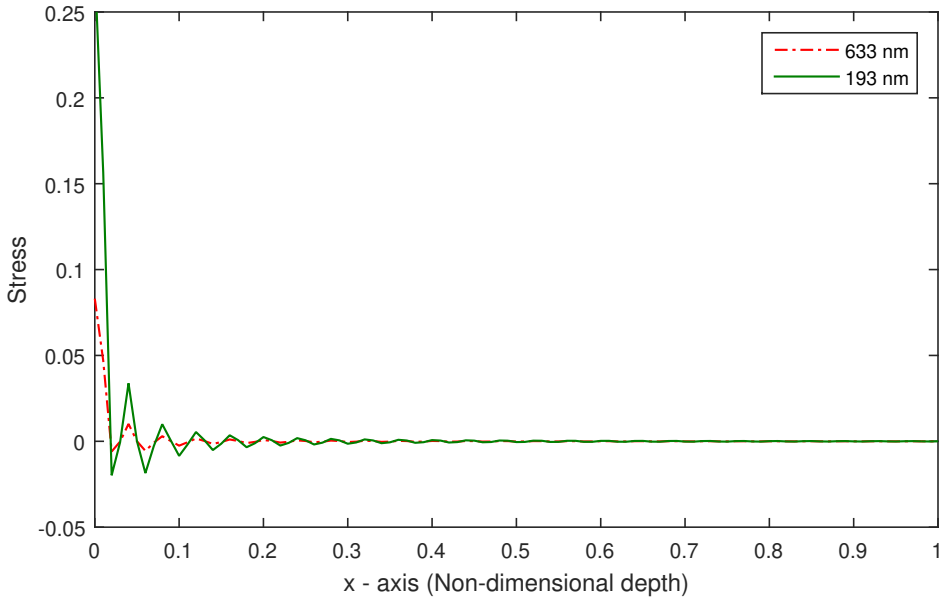

Figure 8: Stress profile for two different laser-pulse.

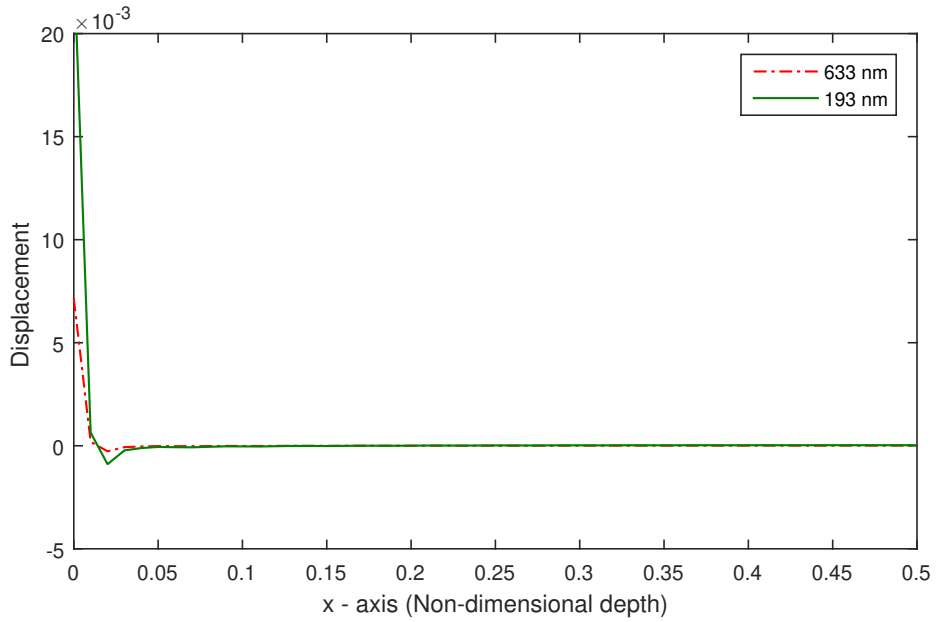

Figure 9: Comparison of displacements for two different laser-pulse.

In the present article, we observe that at the wavelength 193 nm, temperature changes occurred in the epidermis layer only. But, at 633 nm wavelength, temperature changes are observed beyond the 0.7 mm depth from the skin surface. Though the penetration depth of photon is 0.7 mm from the surface, temperature change occurs beyond the penetration depth of photon in non-Gaussian laser-pulse.

Contrary to the temperature profile, stress generated into the epidermis layer only. At shorter wavelength of laser-pulse skin layer experienced more thermal stress than longer wavelength.

There is a very small difference in displacement profile for two aforementioned wavelengths. Shorter wavelength able to create more displacements.

Thus, the thermodynamic result goes(validate) with the optical experimental observations to some extent.

9 Conclusion

In the present article, we have investigate thermodynamically the penetration depth of the non-Gaussian laser-pulse into the human skin. From the numerical computational results you may conclude that:

1. A good agreement is found with optical analysis. At longer wavelength scattering coefficient is much dominated factor over absorption coefficient.

2. For longer wavelength of laser pulse, the temperature changes experienced in the dermis layer beyond the photon's penetration depth.

3. On application of a point heat source, thermal memory in the heat conduction law yields a low attenuation in the temperature as well as stress distribution.

4. This investigation illustrates that on increasing the wavelength of light used in skin therapy, there is a corresponding increase in the temperature into the skin layers.

Conflict of interest:

There is no conflict of interest with any individual or any research group.

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