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Properties of elementary particles, dark matter, and dark energy

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Abstract

This paper points to, proposes explanations for, and extrapolates based on patterns that pertain to the following - properties of objects, elementary particle data, dark matter data, and dark energy phenomena. The paper suggests new elementary particles, a specification for dark matter, a description of dark energy, and insight regarding galaxy formation. Data pertaining to dark matter (especially ratios of dark matter effects to ordinary matter effects) and to dark energy phenomena (including aspects that associate with tensions - between data and modeling - that pertain to large-scale phenomena) might tend to confirm the suggestions. The proposed explanations associate with a new elementary-particle internal quantum number - isomer - and with pattern matches that associate with solutions to Diophantine equations. A new principle (conservation of degrees-of-freedom-related aspects) links the Diophantine equations to popular physics modeling that has bases in space-time coordinates. This paper suggests that nature includes six isomers of most known elementary particles. Five isomers associate with most dark matter. Solutions to Diophantine equations suggest means to catalog properties of objects, to interrelate properties of elementary particles and other objects, and to gain insight regarding interactions between objects. Modeling regarding charged lepton anomalous magnetic moments suggests advantages for modeling elementary particles as being other than point-like.

Keywords: elementary particles, dark matter, dark energy, galaxy formation, neutrino masses

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1. Introduction

This paper suggests explanations for data that physics seems not to otherwise explain. This paper suggests and uses principles and modeling that seem to provide useful advances regarding the following physics challenges.

1. Catalog properties of objects.
2. Catalog elementary particles.
3. Explain data that associate with the two-word term dark matter.
4. Explain data that associate with the two-word term dark energy.

This paper develops SUPP to supplement POST.

- POST is an acronym for POpular physics modeling based on ST. ST associates with notions of principles and modeling that associate with Space-Time coordinates.

- SUPP is an acronym for SUggested physics modeling based on PP. PP associates with notions of principles and modeling that associate with Particle Properties.

POST associates with continuous coordinates (such as space-time coordinates) and continuous variables (such as position and velocity). POST includes modeling that associates with discrete (or, quantized) values (such as for angular momentum). This paper associates the acronym STS with mathematical spaces that associate with POST. STS associates with notions of Space-Time and other associated coordinate Systems. Here, the notion of other includes coordinates that associate with velocity or momentum.

Table 1: Integers that SUPP uses and uses that SUPP makes of the integers. The symbol † denotes that a new (compared to POST) principle or quantum number pertains. Regarding n_Γ, a new principle associates changes in n_Γ with changes in numbers of degrees of freedom that associate with STS (as in mathematics that associate with Space-Time and other associated coordinate Systems). The acronym CODOFRA (as in Conservation Of DOF-Related Aspects) associates with the new principle and with both POST notions of DOF (as in Degrees Of Freedom) and SUPP notions of DOFLA (as in DOF-Like Aspects). Regarding R_I, a new (compared to POST) quantum number - isomer - associates dark matter counterparts of most ordinary matter elementary particles with the ordinary matter instances of elementary particles. Regarding N', SUPP hints that future work might suggest new elementary particle physics principles.

Integer	Range	†	SUPP use
Σ	>0	-	Catalog properties (such as charge and mass) of objects.
Σ	0	-	Catalog elementary particles.
n_Γ	≥1	†	Link SUPP and POST, based on CODOFRA (as in Conservation Of Degrees-Of-Freedom-Related Aspects).
R_I	1, 2, 6	†	Characterize interactions between ordinary matter and dark matter.
N'	-	?	Interrelate properties of elementary particles.

SUPP associates with integers and solutions to Diophantine equations. SUPP features new (compared to POST) modeling that associates with integers. This paper associates the acronym PPS with mathematical spaces that associate with SUPP. PP associates with notions of Particle Property Spaces.

POST has sought to address each one of the above four challenges for the most recent at least 80 years.

Table 1 previews some integers that SUPP uses and previews some uses that SUPP makes of the integers. Each one of some of the uses associates with a new (compared to POST) principle or quantum number.

This paper suggests that SUPP complements and does not disturb subsets of POST that comport with data. SUPP suggests some changes to some POST models that currently somewhat comport with data. SUPP suggests limitations regarding the applicability of some POST models that currently seem not to comport with data.

2. Perspective

This unit provides perspective regarding using physics patterns and physics principles and regarding context for SUPP methods and models.

2.1. Physics patterns and physics principles

This unit notes that physics advances can have bases in extrapolating from patterns or in extrapolating based on principles.

Sometimes, physics moves forward based on extrapolations from patterns. The following examples feature advances - based on patterns - that occurred before physics included principles that explained the patterns.

- Galileo Galilei and near-contemporaries to Galileo suggested patterns regarding the motions of free-falling objects and the motions of spherical objects that roll down ramps. For objects that start as having no motion, the distance traveled is proportional to the square of the time that elapses after the time at which motion starts. Publications of text such as text that reference [1] includes occurred decades before Newton proposed - in reference [2] - principles that might associate with such motions.
- Mendeleev suggested cataloging the then-known chemical elements based on similarity with respect to chemical interactions and on atomic weight. Mendeleev published the suggestions in reference [3] decades before there was enough atomic physics to explain chemical interactions and decades before there was enough nuclear physics to explain atomic weights.

Sometimes, physics moves forward based on extrapolations that have bases in mathematical models that associate with principles. Examples include the suggesting and discovering of the Higgs boson and of various antimatter counterparts to matter elementary particles.

2.2. The approach that this paper takes and results that this paper suggests

This unit discusses the approach that this paper takes, summarizes types of results that this paper suggests, and points to a compendium (in the concluding remarks section of this paper) of such results.

Regarding the approach that this paper takes, the following notions pertain. New modeling might explain observations that popular modeling seems not to explain. New modeling should be compatible with principles and methods that associate with successful popular modeling. New modeling should augment successful popular modeling. New modeling can have bases that might seem to differ markedly from bases for popular modeling. New modeling can introduce new principles or quantum numbers. New modeling can suggest insight regarding the range of applicability of popular modeling. New modeling can suggest ways to extend the range of applicability of popular modeling. A combination of new modeling and popular modeling should explain observations that popular modeling alone does not explain or should make verifiable predictions that popular modeling alone does not make. Such explanations or predictions should pertain to significant aspects of physics. Such explanations or predictions do not necessarily need to pertain to all seemingly relevant aspects within an area of physics. Such explanations or predictions do not necessarily need to pertain (at least directly) to seemingly complicated aspects within an area of physics. Such explanations or predictions can be fully quantitative or partly quantitative. Discussion of new approaches might benefit by including discussion-elements that might seem - in the context of popular modeling - to include contextual or philosophical notions.

This paper suggests advances, based on identifying and extrapolating from patterns.

This paper suggests new principles that underlie some of the patterns.

This paper suggests a new quantum number that associates with some of the principles and patterns.

This paper suggests other advances, based on using the new principles.

Regarding suggestions that this paper makes, the concluding remarks section of this paper summarizes quantitative predictions (that seem to be verifiable), qualitative predictions (that seem to be verifiable), notions regarding how to reduce seeming discrepancies between popular modeling and data, relationships between modeling that this paper suggests and popular modeling, new modeling principles, new quantum numbers, concepts for possible experiments and observations, and notions that might point to new aspects of physics.

2.3. Context for SUPP methods and models

This unit provides perspective about context for SUPP methods and models.

2.3.1. Popular (POST) modeling and suggested additional (SUPP) modeling

This paper intertwines the following modeling.

1. POST (as in POpular Space-Time modeling). POST has bases in mathematics related to STS (as in Space-Time and other associated coordinate Systems). POST includes bases for modeling, serves physics branches, and includes hypothesized attributes.
   (a) POST includes the following pair of bases for modeling. CM (or, classical mechanics) includes ND (or, Newtonian dynamics), SR (or, special relativity), and GR (or, general relativity). QM (or, quantum mechanics) includes QFT (or, quantum field theory).
   (b) Within the physics branch of elementary particles, POST includes the SM (or, the elementary particle Standard Model). The SM has bases in QFT.
   (c) Within the physics branch of cosmology and astrophysics, POST includes CC (or, concordance cosmology). CC includes notions about stars, solar systems, black holes, galaxies, galaxy clusters, and so forth. CC has bases in ND, SR, and GR.
   (d) POST includes the following trio of hypothesized attributes. OM (or, ordinary matter) associates (approximately) with stuff that associates directly with observations of light. DM (or, dark matter) associates with notions that suggest more gravitational attracting between objects than the gravitational attracting that POST associates with OM. DE (or, dark energy) associates with notions that suggest gravitational repelling between large objects that POST associates with OM plus DM.
   (e) The SM evolved - based on physics observations - based on proposals for new elementary particle internal quantum numbers (such as color charge) and proposals for new elementary particles such as quarks and gluons).
2. SUPP (as in SUggested physics modeling based on PP). SUPP has bases in integer mathematics related to PPS (as in principles and modeling that associate with Particle Properties). Mathematics that underlies PPS aspects of SUPP associates with the two-word term Diophantine equations.

(SUPP does not have direct bases in space-time coordinates, POST notions of tangent spaces to space-time spaces, or POST notions of phase spaces. SUPP points to properties - such as velocity - that associate with POST tangent spaces and with POST phase spaces.)

(a) SUPP suggests - based on CC observations - that nature includes six isomers (or, near copies) of all elementary particles except LRI (or, long-range interaction) elementary bosons. LRI elementary bosons include the (known) photon and the (might-be) graviton. (SUPP uses notation of the forms SL and ΣL to denote LRI elementary bosons. S or Σ associates with POST notions of spin. The symbol 1L associates with the word photon. The symbol 2L associates with the word graviton. Each one of the symbols 3L and 4L associates with a might-be elementary particle that SUPP suggests.)
(b) SUPP suggests an elementary-particle internal quantum number - isomer - that POST does not include. SUPP suggests that a notion of isomeric composition (or, the amounts of each of the six isomers) pertains regarding objects (including, for example, galaxies).
(c) SUPP proposes specifications for DM and DE. A notion of multipole expansions regarding gravity and the notion of six isomers (of most elementary particles) provide bases for the specifications for DM and DE.
(d) The specifications For DM and DE have inspirations in and seem to explain CC observations.
(e) Modeling that associates with multipole expansions and with isomers extrapolates to suggest various catalogs.
(f) SUPP outputs a catalog of properties (including charge and mass) of objects.
(g) SUPP outputs a catalog of known elementary particles (including the electron, the Z boson, and all other known elementary particles). By extrapolating based on that catalog, SUPP suggests possible new elementary particles.

POST and SUPP have direct links to each other based on notions that associate with POST notions of STS DOF (as in Degrees Of Freedom). POST and SUPP have direct links to each other based on the SUPP notion of isomers (or, in effect, reuses) of non-LRI elementary particles.

This paper suggests that the combination of the blending of properties that SUPP suggests with POST properties and the blending of elementary particles that SUPP suggests with POST elementary particles might provide insight about elementary particles and might explain (otherwise seemingly unexplained) cosmology-and-astrophysics data.

2.3.2. Information about POST and topics that this paper discusses

The following references provide information about topics that this paper discusses.

- Electromagnetism, gravity, physics constants, and physics properties.

Reference [4] explores notions of a coupling between electromagnetism and gravity. Reference [5] and reference [6] discuss Einstein-Maxwell equations that suggest combining electromagnetic stress-energy tensors and the Einstein field equations, which have origins in modeling regarding gravitation. References [7], [8], and [9] discuss gravitoelectromagnetism, which suggests similarities between gravity and electromagnetism.

Reference [10] and articles to which reference [10] alludes discuss, at least in the context of general relativity, possible relationships between mass and angular momentum.

Reference [11] discusses notions of repulsive components of gravity.

References [12], [13], [14], and [15] discuss experimental tests of theories of gravity.

- Elementary particles.

Reference [16] provides an overview of elementary particles and the elementary particle Standard Model.

Reference [17] lists some types of modeling that people have considered regarding trying to extend the elementary particle Standard Model, including trying to suggest elementary particles that people have yet to find. Reference [18] provides information about some of these types of modeling. References [19], [20], and [21] provide information about modeling and about experimental results. Reference [22] (including reviews numbered 86, 87, 88, 89, 90, and 94) provides other information about modeling and about experimental results.

Reference [23] suggests the notion of an inflaton field.

Reference [24] discusses the notion of a graviton.

Reference [25] discusses the notion of neutrino mass mixing.

Reference [25] discusses notions of sterile neutrinos and heavy neutrinos. References [26] and [27] discuss lower limits regarding masses of heavy neutrinos.

Reference [28] notes that quantum field theory suggests that massless elementary particles cannot have spins that exceed two.

A symmetry regarding Maxwell's equations suggests that nature might include magnetic monopoles. Reference [29] discusses theory. Reference [21] reviews modeling and experiments regarding magnetic monopoles. Reference [30] discusses a search - for magnetic monopoles - that did not detect magnetic monopoles.

Reference [19] reviews modeling and experiments regarding axions. Reference [19] also notes modeling that suggests that nature might include axions.

Reference [20] reviews modeling and experiments regarding leptoquarks.

Reference [25] discusses modeling and data about neutrino masses and neutrino oscillations.

- Cosmology and astrophysics.

Reference [31] provides an overview of concordance cosmology and related topics regarding general physics, dark matter, and elementary particles. Reference [32] provides an overview of cosmology. References [33], [34], [35], and [36] review aspects of concordance cosmology. Reference [37] discusses observational tests for cosmological models.

Reference [38] discusses possibilities leading up to a Big Bang.

References [39] and [34] discuss inflation.

Reference [40] discusses attempts to explain the rate of expansion of the universe.

References [41] and [42] discuss so-called tensions between cosmology models and cosmology data.

References [35], [43], [44], [45], and [46] discuss the notion that concordance cosmology underestimates recent increases in the rate of expansion of the universe. Reference [35] suggests that possible resolutions regarding such an underestimate might focus on phenomena early in the history of the universe.

References [47], [48], and [49] discuss possible types of dark matter.

Reference [48] notes that physics has yet to determine directly whether nature includes cold dark matter.

Reference [50] suggests that notions of warm dark matter might reduce discrepancies between data regarding clustering within galaxies and modeling that associates with cold dark matter.

Reference [51] suggests the following notions regarding dark matter. Models that associate with the two-word term modified gravity might pertain; but - to the extent that the models suggest long-range astrophysical effects - such models might prove problematic. Some modeling suggests limits on the masses of basic dark matter objects. Observations suggest small-scale challenges to the notion that all dark matter might be cold dark matter. People use laboratory techniques to try to detect dark matter. People use astrophysical techniques to try to infer properties of dark matter. (Reference [52] discusses astrophysical and cosmological techniques.)

Reference [53] suggests notions of dark matter charges and dark matter photons. Reference [54] discusses possible effects of dark matter photons.

References [55], [56], and [57] discuss the notion that dark matter might include atom-like objects.

Reference [58] suggests that dark matter might include hadron-like particles.

Reference [59] suggests evidence of non-gravitational interactions - in galaxies and in galactic clusters - between dark matter and ordinary matter.

Reference [60] discusses galaxy formation and evolution, plus contexts in which galaxies form and evolve. Reference [60] discusses parameters for classifying and describing galaxies. Reference [60] seems not to preclude galaxies that have few ordinary matter stars. Reference [60] seems not to preclude galaxies that have little ordinary matter.

Reference [61] suggests that concordance cosmology might not adequately explain gravitational interactions between neighboring galaxies.

- Multipole expansions.

Reference [62] discusses multipole expansions regarding electrostatics and the property of charge. Reference [63] discusses a multipole expansion regarding gravitation and the property of mass. Reference [64] discusses multipole expansions regarding acoustics.

3. Methods

This unit suggests new physics modeling (or, SUPP modeling) that includes a new physics modeling principle and that echoes and extrapolates from data, popular physics modeling (or, POST modeling), and patterns that associate with data and popular modeling.

3.1. Some bases for SUPP modeling

This unit discusses relationships between some bases for SUPP modeling and some aspects of POST modeling regarding electromagnetism and gravitation.

3.1.1. Notions about objects, properties of objects, and modeling

POST modeling includes the notion of objects (such as an elementary particle or an atom) that have properties (such as mass or charge). POST modeling includes the notion of observers (such as other objects) that sense effects that associate with properties of objects.

Regarding one object, one property, and one observer (or one set of coordinates in which a hypothetical observer is not moving), SUPP suggests the following types of modeling.

- T1 modeling associates with the following notions.
  - The property is or would be (in the context of POST) a scalar property or a vector property. Examples of scalar properties include charge and mass. Examples of vector properties include magnetic moment and spin (as in intrinsic angular momentum).
  - The modeling adequately separates aspects that associate with the property from aspects that associate with other properties.
  - The object models - with respect to observations that associate with the observer - as not accelerating. The object models - with respect to coordinates that associate with notions of position, coordinates that associate with notions of velocity, and coordinates that associate with notions of acceleration - as not associating with coordinates that associate with acceleration.
  - The property models as having a spherically symmetric distribution (in the rest-frame of the object) or as having a point-like distribution.
- T2 modeling associates with modeling (that has similarities to T1 modeling and) that does not necessarily comport with all of the T1 notions. T2 modeling includes T1 modeling.
- The following notions associate with T2BNT1 (as in T2 But Not T1).
  - The property does or would associate, in POST, with the notions of n-tensor and $n \geq 2$. (Here, $n = 0$ associates with the word scalar and $n = 1$ associates with the word vector.)
  - The property does or would associate, in POST, with a notion of a non-spherical distribution of the property or with the POST notion of anomalous property (as in anomalous magnetic moment). Regarding the property of magnetic moment and modeling that does not associate with notions of a point-like object, SUPP suggests that the specific property of T1 component of magnetic moment associates with a uniformly rotating (in the sense of angular velocity), spherically symmetric distribution of charge. The specific property of T2BNT1 component of the magnetic moment associates with the difference between the magnetic moment and the T1-component of magnetic moment. Regarding the T2BNT1 component of magnetic moment, the notion of uniformly rotating can pertain and, if so, the difference (between the magnetic moment and the T1-component of the magnetic moment) would associate with the notion that the distribution of charge is oblate.

3.1.2. Some electromagnetic properties of objects

POST ND includes notions of DOF (as in degrees of freedom). The following examples pertain. One STS DOF associates with temporal aspects (or, time). Three STS DOF associate with spatial aspects (or, position). Three STS DOF associate with velocity.

Across some POST ND modeling, various such STS DOF have relevance. For specific aspects of POST ND modeling, only one or a few such STS have relevance.

For an object that models as T1 (in the POST context of STS) with respect to charge and has nonzero charge, POST ND includes notions of electromagnetic potential energies V for which equation (1) pertains. Here, r denotes a radial coordinate that associates with a distance from the object. (If the object models as not point-like, the distance is from the center of the spherical distribution of charge.) n_V is a negative integer.

$$V(r) \propto r^{n_V} \tag{1}$$

$n_V = -1$ associates with ND notions of monopole potentials, monopole forces, and electric fields. The notion of monopole force pertains - regarding ND - for the property of charge and the electric field component of the electromagnetic field. POST ND associates position with three STS DOF. POST ND associates - with respect to an observer or a coordinate system - the position of the object with the notion of removing those three STS DOF. ($r = 0$ associates with the position of the object.) Other DOF - such as those associating with velocity - are relevant with respect to modeling (regarding electromagnetism) but are not relevant with respect to modeling regarding the electric field.

POST ND associates - with respect to an observer or a coordinate system - the location of the object with one temporal STS DOF. (Discussion below does not suggest additional temporal STS DOF.)

POST ND includes the series - of terms - position, velocity, (possibly) acceleration, and (possibly) so forth. Discussion below suggests relevance (with respect to modeling regarding the electromagnetic field) for additional (compared to just one temporal and three spatial) DOF.

$n_V = -2$ associates with ND notions of dipole forces and magnetic fields.

- The notion of dipole force pertains - regarding ND - for the property of charge-current and the magnetic field component of the electromagnetic field. POST ND associates velocity with three additional (compared to the case of charge and $n_V = -1$) STS DOF. POST ND associates - with respect to an observer or a coordinate system - the velocity of the object with the notion of removing those three STS DOF.
- The notion of dipole force pertains - regarding ND - for the property of magnetic moment and the magnetic field component of the electromagnetic field. POST ND associates angular velocity with three additional (compared to the case of charge and $n_V = -1$) STS DOF. (Discussion here assumes that the charge distribution models as rigid and as not point-like.) POST ND associates - with respect to an observer or a coordinate system - the angular velocity of the object with the notion of removing those three STS DOF.

SUPP uses the two-word term intrinsic property to describe each one of charge and magnetic moment. SUPP uses the two-word term extrinsic property to describe charge-current.

$n_V = -3$ associates with ND notions of quadrupole.

- The notion of quadruple might pertain - regarding ND - for the property of charge-acceleration (as in change in charge-current). However, POST ND associates - with respect to an observer or a coordinate system - the acceleration of the object with interactions between the object and fields that associate with other objects. The object models as, in effect, being part of a system of objects. In effect, the object (at least partly) loses the object's identity. The notion of fully independent does not pertain. The notion of T1 does not pertain.
- The notion of quadruple might pertain - regarding ND - for the property of velocity of the magnetic moment. (The magnetic moment moves along with the charge. Compared to the case of magnetic moment and $n_V = -2$, no new STS DOF pertain.) However, the combination of the angular velocity and the velocity associate with a two-tensor. The notion of T1 does not pertain.
- The notion of quadruple might pertain - regarding ND - for the property of precession of magnetic moment. (The Earth exhibits such precession. The period of precession is one day. This precession associates with aspects of the Earth and does not necessarily associate with influences of other objects. The POST notion of Larmor precession does not pertain regarding this discussion.) POST ND might associate - with respect to an observer or a coordinate system - the notion of (non-Larmor precession) precession of magnetic moment with three additional (compared to the case of magnetic moment and $n_V = -2$) STS DOF. SUPP considers that (non-Larmor) precessing magnetic moment is an intrinsic property of an object. However, POST ND might and SUPP does consider that this precession mixes electromagnetic aspects of the object and gravitational aspects of the object. In this case the gravitational aspects associate with spin (as in angular momentum). The notion of T1 does not pertain.
- For T1, SUPP does not suggest a third (or, quadrupole) element in the series for which the first two elements are monopole and dipole.
- SUPP does not suggest (and POST does not include) a third (or, quadrupole) element in the series for which the first two elements are electric (or, monopole) field and magnetic (or, dipole) field.

3.1.3. Some aspects of SUPP modeling regarding n_Γ and POST degrees of freedom

Regarding discussion immediately above, SUPP defines the integer n_Γ by the equation $n_\Gamma = -n_V$.

SUPP suggests that SUPP PPS notions of n_Γ pertain, independently from POST choices that include ND, SR, and GR. For POST choices other than ND, the notion of n_V does not necessarily pertain.

POST notions (that are relevant to this paper) of DOF pertain independently of POST choices that include ND, SR, and GR.

SUPP PPS uses of words such as monopole and dipole associate with values of n_Γ. For example, SUPP PPS use of the word quadrupole associates with $n_\Gamma = 3$. SUPP PPS uses of words such as monopole and dipole pertain, independently from POST choices that include ND, SR, and GR.

SUPP notes a pattern. For T2 modeling specific to one object property, $3n_\Gamma$ is a number of relevant STS DOF that do not associate with the one temporal DOF. Modeling that positions - with respect to an observer or a coordinate system - an object and the object's properties associates (regarding the one property) with engaging with and removing those $3n_\Gamma$ STS DOF from the overall set of STS DOF.

3.1.4. Some aspects of SUPP modeling regarding electromagnetism and gravitation

POST associates two circular-polarization modes - left-circular polarization and right-circular polarization - with POST (CM and QM) notions of electromagnetic fields. For each mode and for non-negative integers l, QM associates l units of excitation of the mode with an angular momentum that has a magnitude of $l\hbar$.

SUPP PPS modeling includes integer-arithmetic equations that echo aspects of two circular-polarization modes and aspects of ND monopole and dipole forces. The two-word term Diophantine equations pertains. The Diophantine equations have the form $s = \cdots$, in which s is an integer and in which the expression $\cdots$ involves sums of nonzero integers.

For the monopole electromagnetic force, $s = +1$ associates with one unit of left-circular polarization and $s = -1$ associates with one unit of right-circular polarization. For the monopole gravitational force, $s = +2$ associates with one unit of left-circular polarization and $s = -2$ associates with one unit of right-circular polarization.

For the dipole electromagnetic force, $s = -1 + 2 = +1$ associates with one unit of left-circular polarization and $s = +1 - 2 = -1$ associates with one unit of right-circular polarization.

SUPP PPS modeling extends the above notions in the following ways.

1. A solution-pair consists of a pair of solutions - $s = +(\cdots)$ and $s = -(\cdots)$ for which the two $\cdots$ expressions are identical.
2. For a $2^{n_\Gamma - 1}$-pole aspect, each $\cdots$ expression includes exactly one instance of each of n_Γ (nonzero) integers. For example, for a monopole (or, $n_\Gamma = 1$) aspect, there is one integer. For a dipole (or, $n_\Gamma = 2$) aspect there are two integers. For a quadrupole (or, $n_\Gamma = 3$) aspect there are three integers.
3. Regarding electromagnetism and gravity, SUPP extends the above notions in the following ways.
    (a) For an electromagnetic aspect (other than an aspect that POST QM would associate with the word anomalous), each solution-pair associates with $|s| = 1$ and with the notion that the integer 1 appears in the expression $\cdots$. (POST QM associates the word anomalous with the notion of anomalous magnetic moment.)
    (b) For a gravitational aspect (other than an aspect that POST QM might - if POST QM associates with gravity - associate with the word anomalous), each solution-pair associates with $|s| = 2$ and with the notion that the integer 2 appears in the expression $\cdots$.
    (c) For a monopole aspect, each solution-pair associates with an intrinsic property of objects.
    (d) For a $2^{n_\Gamma - 1}$-pole aspect for which $n_\Gamma \geq 2$, each solution-pair might associate with an intrinsic property of objects. (Regarding STS modeling that pertains to motion and an intrinsic property of objects, there are $3n_\Gamma$ relevant STS DOF.)
        i. For $|s| \neq 2$ and $n_\Gamma \geq 3$ (regarding intrinsic solution-pairs), each solution-pair mixes at least two of the notions of electromagnetic property, gravitational property, and other property.
    (e) For a $2^{n_\Gamma - 1}$-pole aspect for which $n_\Gamma \geq 2$, each solution-pair might associate with an extrinsic property of objects. (However, regarding STS modeling that pertains to motion, across all such extrinsic properties, there are only three relevant STS DOF.)
4. Beyond electromagnetism and gravity, SUPP PPS modeling extends the above notions in other ways.

3.2. Bases for SUPP catalogs of object-properties and of elementary particles

This unit defines aspects of SUPP uses of Diophantine equations, provides a principle that links SUPP uses of Diophantine equations with POST STS modeling, and discusses modeling that leads to catalogs of properties (of objects) and a catalog of elementary particles.

3.2.1. Mathematical bases for SUPP PPS solutions

Equation (2) shows a term in which k is an integer and s_k can be one of minus one, zero, or plus one.

$$ks_k \tag{2}$$

SUPP multipole mathematics has bases in sums of the form that equation (3) shows. The symbol Z denotes a set of positive integers. An integer k appears no more than once in each such sum. The symbol $\in$ denotes the set theory notion of being a member of a set.

$$s = \sum_{k \in Z} ks_k \tag{3}$$

Regarding sums of the form that equation (3) shows, the symbol k_{max} denotes the largest value of k for which $|s_k| = 1$.

Equation (4) defines Σ.

$$\Sigma \equiv |s| \tag{4}$$

For each solution that associates with equation (3), there is exactly one different solution for which, for each $k \in Z$, the negative of the value s_k replaces s_k. For the second solution, $-s$ replaces s. SUPP uses the one-element term solution-pair to denote such a pair of solutions.

Equation (5) shows notation that SUPP associates with solution-pairs. The letter g is a convenience regarding notation. (Some applications of SUPP associate $\Sigma = 1$ with electromagnetic properties of objects and $\Sigma = 2$ with gravitational properties of objects. Regarding $\Sigma = 1$ and the letter g, one might think of the two-word term gamma rays. Regarding $\Sigma = 2$ and the letter g, one might think of the word gravity.) The symbol Γ denotes a list - in ascending order - of the positive integers k for which $k \in Z$ (and, therefore, $|s_k| = 1$).

$$\Sigma \mathrm{g} \Gamma \tag{5}$$

Regarding equation (5), SUPP uses the symbol Z_Γ to denote the set of positive integers k for which $k \in Z$ (and, therefore, $|s_k| = 1$). The symbol n_Γ denotes the number of positive integers k for which $k \in Z$ (and, therefore, $|s_k| = 1$).

Table 2 alludes to all $s = \sum_{k \in Z}(ks_k)$ expressions for which $1 \leq k \leq k_{max} \leq 4$.

SUPP includes solution-pairs for which integers k for which $k \geq 5$ pertain. For each of those solution-pairs, $k_{max} \geq 5$ pertains. In general, the following notions pertain.

SUPP suggests that each relevant solution-pair comports with equation (6).

$$1 \in Z_\Gamma \text{ or } 2 \in Z_\Gamma \text{ or } 3 \in Z_\Gamma \text{ or } 4 \in Z_\Gamma \tag{6}$$

For each solution-pair $\Sigma \mathrm{g} \Gamma$, equation (7) defines k_{n_0}.

$$k_{n_0} \equiv \max\{k | 1 \leq k \leq 4 \text{ and } k \in Z_\Gamma\} \tag{7}$$

For each solution-pair $\Sigma \mathrm{g} \Gamma$, equation (8) computes n_0. The symbol $\notin$ denotes the set theory notion of not being a member of a set.

$$n_0 = \text{ the number of } k \text{ for which } 1 \leq k \leq k_{n_0} \text{ and } k \notin Z_\Gamma \tag{8}$$

Equation (6) and equation (8) imply that the range $0 \leq n_0 \leq 3$ pertains regarding n_0.

For $n_\Gamma \geq 4$, each one of some combinations of Γ and Σ can associate with more than one solution-pair. For a combination of Γ and Σ that associates with more than one solution-pair, equation (9) shows a symbol that SUPP uses.

$$\Sigma \mathrm{g} \Gamma \mathrm{x} \tag{9}$$

Table 2: $\Sigma = |s| = |\sum_{k\in Z}(ks_k)|$ solution-pairs for which $1 \le k \le k_{max} \le 4$. The columns labeled $1 \cdot s_1$ through $4 \cdot s_4$ show contributions that associate with terms of the form ks_k. Each entry in the column with the label Σ alludes to a unique solution-pair. Regarding table 2, the integer n_0 equals the number of k for which $1 \le k \le k_{max} \le 4$ and $s_k = 0$. The integer n_Γ equals the number of k for which k appears in the list Γ. The number n_{sp} equals $2^{n_\Gamma - 1}$ and states the number of solution-pairs. The column for which the one-element label is PPS-pole associates mathematically with the number of solution-pairs. For a row for which exactly one solution-pair pertains, the column shows the word monopole. For a row for which exactly two solution-pairs pertain, the column shows the word dipole. For a row for which exactly four solution-pairs pertain, the column shows the word quadrupole. For a row for which exactly eight solution-pairs pertain, the column shows the word octupole. For the case of octupole, each one of $\Sigma = 2$ and $\Sigma = 4$ associates with two solution-pairs. Regarding $\Sigma = 2$, $|-1+2-3+4| = 2 = |-1-2-3+4|$. Regarding $\Sigma = 4$, $|-1-2+3+4| = 4 = |+1+2-3+4|$.

k_{max}	Γ	$1 \cdot s_1$	$2 \cdot s_2$	$3 \cdot s_3$	$4 \cdot s_4$	Σ	n_0	n_Γ	n_{sp}	PPS-pole
1	1	±1	-	-	-	1	0	1	1	Monopole
2	2	0	±2	-	-	2	1	1	1	Monopole
2	1‘2	±1	±2	-	-	1,3	0	2	2	Dipole
3	3	0	0	±3	-	3	2	1	1	Monopole
3	1‘3	±1	0	±3	-	2,4	1	2	2	Dipole
3	2‘3	0	±2	±3	-	1,5	1	2	2	Dipole
3	1‘2‘3	±1	±2	±3	-	0,2,4,6	0	3	4	Quadrupole
4	4	0	0	0	±4	4	3	1	1	Monopole
4	1‘4	±1	0	0	±4	3,5	2	2	2	Dipole
4	2‘4	0	±2	0	±4	2,6	2	2	2	Dipole
4	3‘4	0	0	±3	±4	1,7	2	2	2	Dipole
4	1‘2‘4	±1	±2	0	±4	1,3,5,7	1	3	4	Quadrupole
4	1‘3‘4	±1	0	±3	±4	0,2,6,8	1	3	4	Quadrupole
4	2‘3‘4	0	±2	±3	±4	1,3,5,9	1	3	4	Quadrupole
4	1‘2‘3‘4	±1	±2	±3	±4	0,2,2,4,4,6,8,10	0	4	8	Octupole

3.2.2. Some sets of SUPP solution-pairs

SUPP uses symbols of the form Σg’ to denote solution-pairs for which the integer Σ is positive and $\Sigma \in Z_\Gamma$. For example, 1g1‘2 associates with 1g’.

SUPP uses symbols of the form Σg” to denote solution-pairs for which the integer Σ is positive and $\Sigma \notin Z_\Gamma$. For example, 3g1‘2 associates with 3g”. (Here, one solution associates with $s = +1 + 2 = +3$.)

SUPP uses symbols of the form Σg to denote solution-pairs for which the integer Σ is positive. For example, 1g1‘2 associates with 1g. 3g1‘2 associates with 3g.

SUPP uses the symbol 0g to denote solution-pairs for which the integer Σ is zero. The solution-pair 0g1‘2‘3 provides an example. (Here, one solution associates with $s = -1 - 2 + 3 = 0$.) Arithmetically, 0g associates with $n_\Gamma \ge 3$. SUPP associates the symbol n_{Γ^0} with n_Γ that associate with 0g solution-pairs.

3.2.3. Cascades that interrelate SUPP solution-pairs

SUPP includes the notion of adding - to one Γ - one new positive integer and thereby producing a new Γ. SUPP associates the word cascade with the notion that, for an original solution-pair Σ_1gΓ_1 and a resulting solution-pair Σ_2gΓ_2, $\Sigma_2 = \Sigma_1$.

For one original solution-pair, more than one cascade solution-pair might pertain.

SUPP also associates the word cascade with a network of solution-pairs that cascade (from each other) based on multiple cascade steps that ensue from one solution-pair. The solution-pair 1g1‘2 associates with a first step in a cascade that starts with the solution-pair 1g1. A next cascade step provides the 1g1‘2‘4 solution-pair. A next cascade step produces two 1g1‘2‘4‘6 solution-pairs and the 1g1‘2‘4‘8 solution-pair.

3.2.4. Notions that associate with POST notions of scalar property, vector property, and so forth

For ΣgΓ solution-pairs for which $\Sigma \ge 1$, the following notions pertain. $n_\Gamma = 1$ associates with POST notions of scalar (or, order 0 tensor) properties of objects. $n_\Gamma = 2$ associates with POST notions of vector (or, order 1 tensor) properties of objects. And so forth.

Equation (10) pertains. The symbol $\leftrightarrow$ associates with the two-word phrase associates with.

$$\text{For } \Sigma \ge 1,\ n_\Gamma \leftrightarrow (n_\Gamma - 1)\text{-tensor properties of objects} \tag{10}$$

3.2.5. Notions that associate with objects and with multi-component systems

POST includes various modeling techniques and a notion of physical objects.

Regarding POST modeling that treats an object as having components, SUPP associates the word system (or, the two-element phrase multi-component system) with the object.

SUPP suggests that objects associate with observations and with $\Sigma = 0$ (and, thus, with 0g). Arithmetically, $\Sigma = 0$ associates with $n_{\Gamma^0} \geq 3$. (Table 9 pertains regarding elementary particles. Discussion related to equation (26) pertains regarding multi-component systems.)

3.2.6. Notions that link SUPP PPS modeling with POST STS DOF and symmetries

SUPP suggests links between POST STS DOF symmetries and aspects related to the SUPP PPS notion of n_Γ.

This paper's discussion regarding such links starts from bases that discussion related to equation (1) establishes. This paper's discussion continues here, starting with discussion related to a charged object and the components that the charged object contributes to the electromagnetic field. Discussion here associates with the notion of T1.

For $n_\Gamma = 1$, SUPP suggests that the three removed STS DOF associate with the position of the object.

For $n_\Gamma = 1$, only 1g1 pertains and only $s_1 = \pm 1$ pertains. Each one of the $s_1 = +1$ mode (of the electromagnetic field) and the $s_1 = -1$ mode (of the electromagnetic field) associates with one PPS DOFLA (as in DOF-Like Aspect). (SUPP suggests the term DOFLA - and not DOF - in recognition that an integer variable - and not a continuous variable - pertains.) Per POST QM, modeling for the excitations of each mode associates with harmonic oscillator mathematics. For each mode, an integer variable - the number of excitations - pertains. SUPP suggests - regarding PPS - that, across the two modes, mathematics for a two-dimensional harmonic oscillator pertains. The ground state of the two-dimensional harmonic oscillator associates with $SU(2)$ symmetry. (Per reference [65], for $n \geq 2$, $SU(n)$ symmetry associates with the ground state of an n-dimensional harmonic oscillator.) SUPP suggests that $s_1 = 0$ associates with a third PPS DOFLA and with mathematics for a one-dimensional harmonic oscillator. Mathematics associates a $U(1)$ symmetry with a one-dimensional harmonic oscillator. SUPP PPS associates the $U(1)$ symmetry with the notion of a specific magnitude of spin. Here, the magnitude of spin is $1\hbar$ and not some other integer multiple of $\hbar$. Regarding PPS, a total of three (as in two - for $s_1 = \pm 1$ - plus one - for $s_1 = 0$) DOFLA pertains. Two DOFLA associate with $s = \pm 1$ (for electromagnetism). One DOFLA associates with spin-one.

For $n_\Gamma = 2$, only 1g1'2 pertains. SUPP suggests that extrinsic use of 1g1'2 associates with three additional (compared to the case for which $n_\Gamma = 1$) removed STS DOF that associate with the velocity of the object. SUPP suggests that intrinsic use of 1g1'2 associates with three additional (compared to the case for which $n_\Gamma = 1$) removed STS DOF that associate with the angular velocity of the object. (Regarding this intrinsic use of 1g1'2, the notion of T1 associates with notions of a rigid - but not necessarily point-like - object.)

For each one of extrinsic use of 1g1'2 and intrinsic use of 1g1'2, SUPP suggests that three new (compared to $n_\Gamma = 1$) PPS DOFLA pertain. Two of the new DOFLA associate with $s_2 = \pm 1$ and with a new (compared to $n_\Gamma = 1$) PPS $SU(2)$ symmetry. The other new DOFLA associates with the magnitude of spin $2\hbar$ and with a new (compared to $n_\Gamma = 1$) $U(1)$ symmetry. Across extrinsic use and intrinsic use, only those three new DOFLA pertain.

Per POST, the electromagnetic field does not necessarily convey information that would point to just one of charge, charge-current, and magnetic moment as associating with an excitement of a photon. Regarding electromagnetism (and not the individual relevant solution-pairs, such as 1g1'2), SUPP suggests that the association (of s_2) with $2\hbar$ and the notion that $|1s_1 + 2s_2| = 1$ associate with - in effect - keeping the number of DOFLA at three. Two DOFLA associate with $s = \pm 1$ for electromagnetism. One DOFLA associates with spin-one.

3.2.7. Conservation of DOF-Related Aspects

SUPP suggests the following generalization from discussion (just above) regarding STS DOF and PPS DOFLA that associate with electromagnetism. (Discussion immediately below includes examples of the uses of the generalization. Table 5 shows examples of uses for the generalization.)

Adding one to n_Γ (in contexts that associate with equation (3)) or engaging with a new value of k (in contexts that do not associate with equation (3)) associates with the following.

- Modeling engages with a new STS space that associates with three DOF. (For electromagnetism and adding one to an original $n_\Gamma = 1$, the new STS space associates with one of - respectively - velocity and angular velocity.) Engagement associates with one other STS DOF and with a notion of $U(1)$ symmetry.

- Modeling associates with selecting a point in the new STS space. (For electromagnetism and adding one to an original $n_\Gamma = 1$, the point associates with - respectively - a specific velocity or a specific angular velocity.) The selection associates with notions of shutting down the three STS DOF and the one other STS DOF.
- Modeling associates with adding - with respect to PPS - a so-called triad and a so-called monad. The triad associates with three values of a property or three types of related properties. (For electromagnetism and adding one to an original $n_\Gamma = 1$, the point associates with - respectively - a specific value of charge-current or a specific value of magnetic moment. For electromagnetism and the case of engaging with a new value - $k = 1$ - of k, the triad associates with the three values $Q = 1/3$, $Q = 2/3$, and $Q = 1$. Q denotes a magnitude of charge divided by the magnitude of the charge of an electron. The three values associate with the three values that POST suggests for elementary particles that have nonzero charge.) The triad associates with a notion of three PPS DOFLA. The monad associates with the extent to which the triad has relevance. (For electromagnetism and the case of engaging with a new value - $k = 1$ - of k, the monad associates with whether an elementary particle has nonzero charge or zero charge.) The monad associates with a notion of one other PPS DOFLA.

SUPP associates the word tetrad with the combination of a triad and the related monad.

Equation (11) expresses a notion that SUPP associates with the acronym CODOFRA (as in Conservation Of DOF-Related Aspects). The symbol $\triangle$ associates with the two-word phrase change in.

$$\triangle(\text{number of STS DOF}) + \triangle(\text{number of PPS DOFLA}) = 0 \tag{11}$$

3.2.8. Notions that associate with nonnegative integers n_V and nonpositive integers k

Aspects of POST ND associate with some nonnegative integer values of n_V.

$n_V = 2$ associates with ND notions of physical harmonic oscillators.

$n_V = 1$ associates with ND notions that might associate with the long-range potential that associates with the strong interaction. POST uses the two-word term asymptotic freedom.

$n_V = 0$ associates with an electromagnetic potential that an observer would attribute to a uniform background of charge, assuming that the background extends adequately broadly with respect to an observation from within the uniform background of charge. (Compared to the case of 1g1, the three triad-related STS DOF that associate with position remain as DOF.)

For $n_V \geq 0$, SUPP suggests that equation (12) might associate nonpositive values of k with some aspects of POST.

$$k = -n_V \tag{12}$$

SUPP does not extend some notions that pertain for $k \geq 1$ to cases for which $k \leq 0$. Notions that associate with equation (3) do not pertain regarding $k \leq 0$.

SUPP extends some notions that pertain for $k \geq 1$ to cases for which $k \leq 0$. The notion of CODOFRA (and the related equation (11)) pertains. For each relevant nonpositive integer k, the following notions pertain. A PPS triad pertains. A PPS monad pertains. (Three rows in table 5 pertain.)

Regarding $k = 0$, SUPP suggests that the triad associates with a notion of three isomer-pairs. Per discussion (in this paper) that leads to equation (21), SUPP suggests that nature includes six isomers of elementary particles other than the 1L elementary particle (or, photon), the 2L possible elementary particle (or, graviton), a possible 3L elementary particle, and a possible 4L elementary particle. One of the isomers associates with the POST CC notion of ordinary matter. Five of the isomers associate with some POST CC notions of dark matter. Modeling that associates with six isomers associates with the notion that six equals three times two. The factor of three (as in the number of isomer-pairs) associates with the three members of the PPS triad. The factor of two associates with the SUPP notion that each non-SL elementary particle associates with a solution-pair for which $\Sigma = 0$. The notion that a solution-pair associates with two solutions associates with the factor of two in the number, six, of isomers. Within each isomer-pair, one isomer associates with the POST notion of left-handedness (as in left-handed matter elementary particles) and one isomer associates with the POST notion of right-handedness (as in right-handed matter elementary particles). Here, the word matter associates with notions of being more prevalent, compared to antimatter. SUPP suggests that isomer is a new (compared to POST) property. SUPP suggests that isomer is a new (compared to POST) quantum number. SUPP suggests that the PPS monad associates with the notion that some elementary particles (the SL elementary particles) do not necessarily associate with just one isomer.

Regarding $k = -1$, SUPP suggests that three removed STS DOF associate with the $SU(3)$ symmetry that POST QM associates with the strong interaction. The PPS triad associates with the three POST QM color-charges (red, blue, and green). The PPS monad associates with the notion that some objects (quarks) exhibit the property of color charge and other objects either (or both, depending on notions regarding modeling) exhibit no color charge or exhibit clear (or white) color charge.

Regarding $k = -2$, SUPP suggests the following notions. The PPS triad associates with the notion of three STS spatial dimensions. The PPS monad associates with the notion of one STS temporal dimension.

3.2.9. Notions that associate with solution-pairs for which $\Sigma = 0$

Solution-pairs for which $\Sigma \geq 1$ can associate with pinpointing and cataloging some properties (such as electromagnetic properties and gravitational properties, but not strong interaction properties) of objects.

Table 2 alludes to solution-pairs for which $\Sigma = 0$. 0g1'2'3 is an example.

Solution-pairs for which $\Sigma = 0$ associate with $n_{\Gamma^{o}} \geq 3$. (For $1 \leq n_{\Gamma} \leq 2$, it is not possible arithmetically to have $\Sigma = 0$.)

SUPP suggests the following cases.

1. Elementary particles associate with intrinsic uses of solution-pairs that associate with zero-step cascades or more-than-zero-step cascades from solution-pairs for which $\Sigma = 0$, $n_{\Gamma^{o}} = 3$, $\{1, 3\} \subset Z_{\Gamma}$, and $\{5, 7\} \cap Z_{\Gamma} = \emptyset$. (The symbol $\subset$ denotes the set-theory notion of being a subset. The symbol $\cap$ denotes the set-theory notion of the intersection of two sets. The symbol $\emptyset$ denotes the empty set - or, a set with no elements.)
2. Spin states of two-component systems can associate with intrinsic uses of solution-pairs that associate with zero-step cascades or more-than-zero-step cascades from solution-pairs for which $\Sigma = 0$, $n_{\Gamma^{o}} = 4$, and $\{5, 7\} \cap Z_{\Gamma} \neq \emptyset$.

For each one of the above cases, SUPP suggests the following notions. (For case 1, table 9 and discussion related to table 9 illustrate the notions. For case 2, discussion related to equation (26) illustrates the notions.)

- Boson or fermion? - Equation (13) pertains.

$$\text{The object is a fermion} \;\Leftrightarrow\; 6 \in (\text{intrinsic})Z_{\Gamma} \tag{13}$$

- Magnitude of spin? - For boson elementary particles, intrinsic use of a solution-pair associates with the spin that equation (14) computes. For fermion elementary particles, intrinsic use of a solution-pair associates with the spin that equation (15) computes.

$$\text{If the object is a boson, } S = |(\text{intrinsic})n_{\Gamma^{o}} - 4| \tag{14}$$

$$\text{If the object is a fermion, } S = |(\text{intrinsic})n_{\Gamma^{o}} - 4.5| \tag{15}$$

- Charge or no charge? - Equation (16) pertains. Q denotes the magnitude of the charge (of the object) divided by the magnitude of the charge of the electron.

$$Q > 0 \Leftrightarrow 4 \notin (\text{intrinsic})Z_{\Gamma} \;\text{ and }\; Q = 0 \Leftrightarrow 4 \in (\text{intrinsic})Z_{\Gamma} \tag{16}$$

Regarding the case of elementary particles, SUPP suggests the following notions. Compared to $n_{\Gamma} = 0$, (intrinsic)$n_{\Gamma^{o}} = 3$ associates with the removal of three sets of STS DOF. Removal of one of the sets of STS DOF associates with locating at a spatial point the electromagnetic properties of the object. The associated PPS triad associates with the three nonzero units of charge - $Q = 1/3$, $Q = 2/3$, and $Q = 1$. (Table 5 alludes to this trio of nonzero units of charge and to other similar trios.) The associated PPS monad associates with the notion that - for some elementary particles - $Q = 0$. Removal of another one of the sets of STS DOF associates with locating at a spatial point the gravitational properties of the object. The associated PPS triad associates with the three categories of mass for non-SL elementary particles - $m_b = 0$, $m_b > 0$, and $m_f > 0$. m_b denotes the mass of a boson object (in this discussion, a boson non-SL elementary particle). m_f denotes the mass of a fermion object (in this discussion, a fermion elementary particle). The associated PPS monad associates with the notion that - for SL elementary particles - $m = 0$. Removal of the other one of the sets of STS DOF associates (regarding three STS DOF) with co-locating the spatial point that associates with the gravitational properties of the object and the spatial point that associates with the electromagnetic properties of the object and (regarding one STS DOF)

Table 3: Some SUPP suggestions regarding associations between solution-pairs and nature. The leftmost two columns specify a set of solution-pairs. The rightmost column describes physics notions (usually POST-related notions) that SUPP suggests associate with some of the solution-pairs. Some physics notions do not associate with the associations that the table suggests. (For example, SUPP associates the QFT property of anomalous magnetic moment with intrinsic use of the solution-pair 3g1‘2, which associates with 3g” and does not associate with 1g’.) For 3g and 4g, SUPP suggests properties that associate with numbers of elementary fermions. The symbol $\subset$ denotes the set-theory notion of being a subset. The symbol $\cap$ denotes the set-theory notion of the intersection of two sets. The symbol $\emptyset$ denotes the empty set (or, a set with no elements). For $\Sigma = 1$ and $\Sigma \in Z_\Gamma$, another property is charge-current. For $\Sigma = 2$ and $\Sigma \in Z_\Gamma$, another property is momentum. For $\Sigma = 3$ and $\Sigma \in Z_\Gamma$, properties associate with POST notions of elementary-particle handedness and with SUPP notions of isomer. For $\Sigma = 4$ and $\Sigma \in Z_\Gamma$, properties associate with POST notions of $B - L$ (or, baryon number minus lepton number) and with SUPP notions of isomer.

Σg$\cdots$	Other constraints	Associations
1g’	-	Electromagnetic properties (such as charge) of objects
2g’	-	Gravitational properties (such as energy) of objects
3g’	-	SUPP-suggested (handedness-related) properties of objects
4g’	-	Baryon number and lepton number properties of objects
Σg’	$1 \leq \Sigma \leq 4$, $n_\Gamma \leq 2$	LRI-related conservation laws (such as conservation of energy)
Σg”	$n_\Gamma = 2$	Anomalous properties of objects
0g	$\{1,3\} \subset Z_\Gamma$, $\{5,7\} \cap Z_\Gamma = \emptyset$	Elementary particles (such as the electron and the Z boson)
0g	$\{5,7\} \cap Z_\Gamma \neq \emptyset$	Spin-centric states of multi-component systems

with co-locating the temporal point that associates with charge with the temporal point that associates with mass.

Regarding the case of two-component systems, SUPP suggests the following notion. The difference between the minimum of (intrinsic)$n_{\Gamma^0} = 4$ for this case and the minimum of (intrinsic)$n_{\Gamma^0} = 3$ for the case of elementary particles associates with the co-locating of the two components of a two-component system.

3.2.10. Notions that associate with the range of integers k

Equation (17) specifies the values of k that have relevance for this paper.

$$-2 \leq k \leq 8, \text{ or } \log_2(k) - 3 \text{ is a positive integer} \tag{17}$$

3.3. SUPP catalogs of object-properties, elementary particles, and POST conservation laws

This unit lists SUPP catalogs - of object-properties, elementary particles, and some POST conservation laws - that associate with solutions to equation (3).

Table 3 previews some SUPP suggestions regarding associations between aspects of nature and solution-pairs that associate with solutions to equation (3). Discussion related to table 5 provides information regarding the scalar properties that associate with 1g’, 2g’, 3g’, and 4g’.

Table 3 does not directly allude to aspects that associate with the strong interaction or the weak interaction. The 0g association with elementary particles associates with some aspects of the strong interaction and with some aspects of the weak interaction. The notion of $k = -1$ associates with some aspects of the strong interaction.

3.4. Associations between multipole contributions to forces

This unit discusses notions that - for, for example, electromagnetism and gravity - effects that associate with non-scalar properties add to or subtract from effects that associate with scalar properties such as charge and energy.

3.4.1. Some electromagnetic properties of objects

This unit associates with the row - in table 3 - that associates with 1g’ and electromagnetic properties.

SUPP associates intrinsic use of the solution-pair 1g1 with the property of charge.

SUPP associates extrinsic use of the solution-pair 1g1‘2 with the property of charge-current.

POST SR associates charge and charge-current with a 4-vector.

An observer that senses an object as non-moving and non-rotating observes - regarding contributions by the object to the electromagnetic field - that $E_1 \propto |q|$ and $B_1 = 0$. $|q|$ denotes the magnitude of the charge of the object. E_1 denotes the magnitude of the electric field. B_1 denotes the magnitude of the magnetic field. (The subscript $_1$ anticipates notions of E_Σ and B_Σ in which - for example - $\Sigma = 2$ associates with gravitational fields. The case of $\Sigma = 2$ might associate with CM notions of gravitoelectromagnetism.

For the case of $\Sigma = 2$, SUPP associates the symbol $E_{2:n_\Gamma}$ with solution-pairs that associate with both E_2 and a particular value of n_Γ. For the case of $\Sigma = 2$, SUPP associates the symbol $B_{2:n_\Gamma}$ with solution-pairs that associate with both B_2 and a particular value of n_Γ.)

For an observer that senses an object for which $|q| > 0$ as moving and not rotating, $B_1 > 0$.

SR associates equation (18) with Lorentz invariance. c denotes the speed of light.

$$(E_1)^2 - c^2(B_1)^2 = \text{ a constant } \geq 0 \tag{18}$$

Per equation (18), an observer that senses the object as moving senses a larger value of E_1 than does an observer that senses the object as not moving. An observer that senses the object as moving senses a larger value of $|q|$ than does an observer that senses the object as not moving.

SUPP suggests that - in effect - effects of extrinsic 1g1‘2 subtract from effects of intrinsic 1g1.

SUPP associates intrinsic use of the solution-pair 1g1‘2 with the property of T1 magnetic moment.

Similarly (to the case of extrinsic 1g1‘2), SUPP suggests that - in effect - effects of intrinsic 1g1‘2 subtract from effects of intrinsic 1g1. (One might consider that - for a model that considers the magnetic moment to associate with charges that revolve around an axis - the motion associating with such revolving associate with nonzero charge-currents.)

The notion of T1 modeling associates with spherical distributions of properties and can associate with the notion of a limit as $r_{Object} \to 0$ (or, the radius of the object approaches zero). In nature, an object with a uniform distribution of non-zero magnetic moment would have an oblate (as in compressed near the poles and expanded near the equator), not spherical, distribution of the property of magnetic moment. SUPP suggests that 3g1‘2 associates with correcting (compared to just using 1g1‘2) for the oblateness. SUPP associates 3g1‘2 with T2BNT1. SUPP suggests that (for physics modeling, including CM), the 3g1‘2 component of magnetic moment is non-negative. Effects that associate with 3g1‘2 do not subtract from effects that associate with 1g1‘2.

SUPP suggests that intrinsic magnetic moment associates with - in effect a sum of - intrinsic T1 magnetic moment (and intrinsic 1g1‘2) and intrinsic T2BNT1 magnetic moment (and intrinsic 3g1‘2).

SUPP suggests the notion that B_1 associates with extrinsic 1g1‘2, intrinsic 1g1‘2, and intrinsic 3g1‘2. SUPP uses the notation 1_3g1‘2 to associate with the notion that intrinsic use of 3g1‘2 associates with B_1 and with the magnetic field component of the electromagnetic field.

SUPP suggests that E_1 associates with $\Gamma = 1$. SUPP suggests that B_1 associates with $\Gamma = 1‘2$.

SUPP does not suggest needs to consider a component - of strictly just the electromagnetic field (and not, for example, also of the gravitational field) - that would extend the series monopole E_1 and dipole B_1 to include a quadrupole item (which, presumably, would associate with - for example - $\Gamma = 1‘2‘4$).

3.4.2. Force-related associations between multipole contributions

SUPP extrapolates from discussion related to equation (18).

SUPP suggests that - for an object that exhibits a nonzero value of the property that associates with (either an extrinsic or an intrinsic) use of a Σg solution-pair $\Sigma \text{g}\Gamma_2$ that cascades in one step from a solution-pair $\Sigma \text{g}\Gamma_1$ - the nonzero value associates with dilution of the effects (of the object) that associate with intrinsic use of the solution-pair $\Sigma \text{g}\Gamma_1$. (For cases in which $\Sigma \text{g}\Gamma_1$ is not a monopole solution-pair, the effects that associate with $\Sigma \text{g}\Gamma_2$ propagate - via solution-pairs from which $\Sigma \text{g}\Gamma_1$ cascades - toward the relevant monopole $\Sigma \text{g}\Gamma$.)

For any one value of Σ, equation (19) and equation (20) pertain.

$$\text{dipole, octupole, } \cdots \text{ effects subtract from monopole effects} \tag{19}$$

$$\text{quadrupole, 16-pole, } \cdots \text{ effects add to monopole effects} \tag{20}$$

Regarding gravity, the following notions pertain. SUPP suggests needs to consider components - of the gravitational field - that would extend the series monopole $E_{2:1}$ and dipole $B_{2:2}$ to include at least a quadrupole item that associates with $\Gamma = 1‘2‘3$). Intrinsic use of solution-pairs that associate with $E_{2:1}$, intrinsic use of solution-pairs that associate with $E_{2:3}$, and so forth associate with gravitational attraction. Intrinsic use of solution-pairs that associate with $B_{2:2}$, intrinsic use of solution-pairs that associate with $B_{2:4}$, and so forth associate with gravitational repulsion. Intrinsic uses of solution-pairs that associate with $E_{2:\geq 3}$ or with $B_{2:\geq 4}$ associate with $n_\Gamma - 1$-tensors for which $n_\Gamma - 1 \geq 2$ and do not associate with notions of scalar properties or vector properties.

3.5. *Dark matter and the notion of six isomers of most elementary particles*

This unit discusses the specification - that SUPP suggests - for dark matter. (This paper elsewhere - for example, in discussion related to table 15 - motivates the specification and tests the specification against observational data.)

POST does not yet provide an established description of dark matter.

POST associates with exactly one use of the set of all known elementary particles.

SUPP uses the two-word term isomeric set to denote the set of all (known and yet-to-be-found) elementary particles except the elementary particles that do (if known) intermediate LRI or might (if found) intermediate LRI. (Table 9 points to all known elementary particles and to yet-to-be-found elementary particles that SUPP suggests. 1L is known; associates with LRI; and is, thus, not a member of the isomeric set. Each one of 2L, 3L, and 4L is not known, would associate with LRI, and would not be a member of the isomeric set.)

SUPP associates the word isomer with each one of six uses (that SUPP suggests that nature includes) of the isomeric set.

SUPP suggests that most DM associates with five new (compared to POST) uses of the isomeric set (of elementary particles).

SUPP uses the one-element term isomer-zero to denote the isomer that generally associates with OM. (SUPP does not rule out some CC notions, such as notions of primordial black holes, of OM objects that might measure as DM.)

SUPP suggests that - generally (but not always) - effects that associate with the five non-isomer-zero uses of the isomeric set measure - from the perspective of POST - as effects of DM. (The following notions associate with the use - in the previous sentence - of the word generally and with the non-use in the previous sentence of a word such as always. SUPP suggests that OM objects detect light emitted by atoms that associate with one DM isomer. SUPP suggests that OM objects would not detect - at least other than with marginal effectiveness - thermal electromagnetic radiation from DM stars.)

Table 15a, table 15b, and table 15c associate with the SUPP suggestion that SUPP notions regarding dark matter might help explain data that POST otherwise does not necessarily explain.

3.6. *Isomer-related properties of elementary particles*

This unit discusses differences - among the six isomers - regarding properties of elementary particles.

Regarding each LRI field, for each Σg solution-pair, one solution associates with $s > 0$ and with the POST notion of a left-circular polarization mode of the field and one solution associates with $s < 0$ and with the POST notion of a right-circular polarization mode of the field.

POST notions of left-handedness pertain for (at least) all known elementary particles - possibly except for neutrinos - that have nonzero mass and nonzero spin. (LRI elementary particles associate with zero mass. POST associates the notion of polarization - and not necessarily the notion of handedness - with all known elementary particles that have zero mass.)

Table 3 suggests that 0g solution-pairs for which $\{1,3\} \subset Z_\Gamma$ and $\{5,7\} \cap Z_\Gamma = \emptyset$ associate with elementary particles.

For each 0g solution-pair that associates with a non-LRI elementary particle, SUPP suggests that the notion of two solutions associates with a notion of a pair of isomers. SUPP associates the one-element term isomer-pair with such a pair of isomers. SUPP suggests that one isomer associates with POST notions of left-handedness and that one isomer associates with POST notions of right-handedness.

SUPP suggests the notion of three-isomer pairs.

SUPP names the isomers with one-element terms - isomer-zero, isomer-one, ..., and isomer-five. The three isomer-pairs associate, respectively, with isomer-zero and isomer-three, isomer-one and isomer-four, and isomer-two and isomer-five. The notion of left-handed particles (in the context of the three-word phrase particle and antiparticle) associates with each one of isomer-zero, isomer-two, and isomer-four. The notion of right-handed particles (in the context of the three-word phrase particle and antiparticle) associates with each one of isomer-one, isomer-three, and isomer-five.

SUPP suggests that POST associates with isomer-zero. For example, POST non-LRI elementary particles that have nonzero mass and nonzero spin associate with left-handedness. POST does not associate with the other five isomers that SUPP suggests.

SUPP suggests that the masses of counterpart non-LRI elementary particles do not vary between isomers. Per discussion related to table 12, SUPP suggests the following notions about flavours regarding isomeric counterparts of known isomer-zero nonzero-charge fermion elementary particles.

- For each one of isomer-zero and isomer-three, the flavour of the lowest-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the highest-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the intermediate-mass charged lepton equals the remaining quark flavour.
- For each one of isomer-one and isomer-four, the flavour of the lowest-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the intermediate-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the highest-mass charged lepton equals the remaining quark flavour.
- For each one of isomer-two and isomer-five, the flavour of the intermediate-mass charged lepton matches the flavour of the highest-mass quark. The flavour of the highest-mass charged lepton matches the flavour of the two lowest-mass quarks. The flavour of the lowest-mass charged lepton equals the remaining quark flavour.

3.7. Reaches that associate with SUPP multipole solution-pairs

This unit discusses the extent to which various components of LRI forces associate with interactions between isomers.

3.7.1. Notions that associate with solution-pairs that associate with LRI fields

SUPP uses the word instance and the word reach to describe aspects of the extent to which $\Sigma \geq 1$ LRI solution-pairs associate with interactions within and between isomers.

SUPP suggests that equation (21) pertains for each $\Sigma \geq 1$ LRI solution-pair. The positive integer n_I denotes a number of instances of a solution-pair. The positive integer R_I denotes the reach - in number of isomers - that associates with one instance of the solution-pair.

$$n_I R_I = 6 \tag{21}$$

POST suggests that, to a first approximation, DM appears - to OM - to be electromagnetically dark. SUPP suggests that, to a first approximation, each isomer appears - to each other isomer - to be electromagnetically dark. For the solution-pair 1g1, SUPP suggests that $n_I = 6$ and $R_I = 1$. SUPP points to six instances of the property of charge. Thereby, SUPP points to six instances of intrinsic use of the solution-pair 1g1.

1g1 associates with $n_0 = 0$. (Equation (8) defines n_0.) SUPP suggests that $R_I = 1$ pertains for all cases for which (intrinsic)$n_0 = 0$.

POST suggests that gravity associates with interactions between OM and OM, interactions between OM and DM, and interactions between DM and DM. For the solution-pair 2g2, SUPP suggests that $n_I = 1$ and $R_I = 6$. SUPP points to one instance of the property of energy.

2g2 associates with $n_0 = 1$. SUPP suggests that $R_I = 6$ pertains for all cases for which (intrinsic)$n_0 = 1$.

SUPP notes that - for each k for which $1 \leq k \leq k_{max}$ and the sum that equation (3) lacks - the sum lacks effects that associate with the pair $s_k = +1$ and $s_k = -1$. SUPP associates that lack with a notion of a lack of two DOFLA.

SUPP suggests that equation (22) pertains for $1 \leq n_0 \leq 3$. SUPP uses the one-element notation gen(GX) to denote the number of generators of the group GX.

$$R_I = \mathrm{gen}(SU(7))/\mathrm{gen}(SU(2 \times ((\mathrm{intrinsic})n_0) + 1)) \tag{22}$$

Thus, SUPP suggests the following reaches for intrinsic uses of LRI solution-pairs - $R_I = 1$ for $n_0 = 0$, $R_I = 6$ for $n_0 = 1$, $R_I = 2$ for $n_0 = 2$, and $R_I = 1$ for $n_0 = 3$.

SUPP suggests that the R_I for an extrinsic use of an LRI solution-pair equals the R_I for the intrinsic use of the solution-pair from which the extrinsic solution-pair cascades in one step. (Otherwise, the notion of object would not pertain.)

Equation (23) shows SUPP notation for the notion that a reach of R_I associates with each instance of a $\Sigma \geq 1$ solution-pair ΣgΓ.

$$\Sigma(R_I)\mathrm{g}\Gamma \tag{23}$$

3.7.2. Notions that associate with solution-pairs that associate with elementary particles

SUPP extends the use of the notation (that equation (23) shows) to include $\Sigma(R_I)\Phi\Gamma$, in which Φ associates with the notion of a family of elementary particles. (Table 9 and discussion related to table 9 provide details regarding families of elementary particles.) Φ =L associates with LRI fields. Other than for Φ =L, SUPP suggests that $n_I = 6$ and that $R_I = 1$ for $0(R_I)\Phi\Gamma$.

3.7.3. Reaches for long-range interaction Sg' solution-pairs

This unit associates with the rows - in table 3 - that associate with Σg' and do not mention conservation laws.

Table 4 shows reaches that associate with Σg' solution-pairs for which $1 \leq \Sigma \leq 4$, $1 \leq k_{max} \leq 8$, and $\{5,7\} \cap Z_\Gamma = \emptyset$.

SUPP suggests that T2BNT1 associates with solution-pairs (to which the intrinsic column in table 4 allude) for which $n_\Gamma \geq 3$ and table 4 shows a value of R_I.

The notion that - for intrinsic use of the 1g1'2'4 solution-pair - $R_I = 6$ associates with the notion that SUPP suggests that detectors that have only OM physical components can detect electromagnetic radiation that - for $l_I \geq 1$ - isomer-l_I stuff radiates.

3.7.4. Minimal observable nonzero values for some properties of objects

POST notions regarding three spatial coordinates, one temporal coordinate, and velocity associate with mathematical notions of continuous (as opposed to discrete). Notions of nonzero minimum values of nonzero values do not pertain.

For each one of charge, mass, momentum, and angular momentum, CM (including ND, SR, and GR) does not necessarily associate with a minimum magnitude for nonzero values.

The SM associates with minimal nonzero magnitudes for nonzero values of some intrinsic properties, such as charge. For nonzero-charge objects other than quarks, $|q_{min}| = |q_e|$. q_e denotes the charge of an electron. For some quarks, $|q_{min}| = (1/3)|q_e|$. For the other quarks, $|q_{min}| = (2/3)|q_e|$.

Based on the notions that velocity is continuous and charge is one component of a 4-vector, SR associates with the three non-overlapping ranges of values for observable charge (q) that equation (24) shows.

$$-q \leq -|q_{min}|,\ q = 0,\ \text{and}\ q \geq |q_{min}| \tag{24}$$

3.7.5. Tetrads and elementary-particle aspects other than angular momentum states

SUPP suggests that the following notions pertain (compared to the notion of no object or the notion of not an elementary particle) regarding intrinsic monopole solution-pairs to which table 4 alludes. For each case, a trio of DOF-related aspects pertain and a fourth DOF-related aspect pertains. For each case, the following notions pertain. SUPP uses the word triad to denote the trio. SUPP uses the word monad to associate with the fourth DOF-related aspect. SUPP uses the word tetrad to associate with the combination of the triad and the monad.

- The following notions pertain regarding 1g1. For each one of the six isomers, there are three DOF-related aspects - $Q = 1/3$, $Q = 2/3$, and $Q = 1$. For SL elementary particles and other zero-charge elementary particles, there is one DOF-related aspect - $Q = 0$.

- The following notions pertain regarding 2g2. Across the six isomers, there are three DOF-related aspects - $m_b = 0$, $m_b > 0$, and $m_f > 0$. For SL elementary particles, there is one DOF-related aspect - $m_b = 0$. SL elementary particles do not associate with single isomers.

- The following notions pertain regarding 3g3. For each one of the three isomer-pairs, there are three DOF-related aspects - $f_{l-r} = +1$, $f_{l-r} = 0$, and $f_{l-r} = -1$. The symbol f_{l-r} denotes - for fermion elementary particles - whether a matter particle (in the context of matter particle and antimatter particle) elementary fermion is left-handed ($f_{l-r} = +1$), does not associate with handedness ($f_{l-r} = 0$, in which case the elementary fermion is its own antiparticle), or is right-handed ($f_{l-r} = -1$). For example, the isomer-zero electron associates with $f_{l-r} = +1$ and the isomer-zero positron associates with $f_{l-r} = -1$. POST suggests the possibility that neutrinos are Majorana fermions. For Majorana neutrinos, $f_{l-r} = 0$ would pertain. For non-fermion elementary particles, there is one DOF-related aspect - $f_{l-r} = 0$.

Table 4: Reaches that associate with Σg' solution-pairs for which $1 \leq \Sigma \leq 4$, $1 \leq k_{max} \leq 8$, and $\{5, 7\} \cap Z_\Gamma = \emptyset$. The symbol † alludes to the notion that the intrinsic solution-pairs do not cascade from other intrinsic solution-pairs that the table shows. The column with the one-word label extrinsic shows solution-pairs that cascade in one step from the intrinsic solution-pairs. The symbol ‡ alludes to the notion that the solution-pairs appear more than once in the column that lists extrinsic solution-pairs. The three rightmost columns designate rows that show SUPP-relevant pairs of one intrinsic solution-pair and one extrinsic solution-pair. (An intrinsic solution-pair can associate with more than one extrinsic solution-pair.) The symbol SL associates with a known LRI elementary particle or a might-be LRI elementary particle and with the value of S equals Σ. The symbol ⦸ associates with the notion that - for the intrinsic solution-pairs - no extrinsic solution-pairs pertain. SUPP assumes that each one of extrinsic ⦸ and intrinsic $6 \in Z_\Gamma$ associates with the notion that intrinsic uses of the solution-pairs that the column labeled intrinsic lists are not relevant regarding SUPP. (Per equation (13), SUPP suggests that - regarding elementary particles - intrinsic use of solution-pairs for which $6 \in Z_\Gamma$ associates with fermion elementary particles and does not associate with boson elementary particles. The notion of SL associates with boson elementary particles and does not associate with fermion elementary particles.) The next-to-rightmost column shows the number of instances for each pair - of one intrinsic solution-pair and one extrinsic solution-pair - that is relevant regarding SL. The rightmost column shows the reach for each pair - of one intrinsic solution-pair and one extrinsic solution-pair - that is relevant regarding SL.

†	Intrinsic	Extrinsic	SL	n_I	R_I
†	1g1	1g1‘2	1L	6	1
-	1g1‘2	1g1‘2‘4	1L	6	1
-	1g1‘2‘4	1g1‘2‘4‘8, 1g1‘2‘4‘6x	1L	1	6
-	1g1‘2‘4‘8	1g1‘2‘4‘6‘8x ‡	1L	1	6
-	1g1‘2‘4‘6‘8x ‡	⦸	-	-	-
-	1g1‘2‘4‘6x	1g1‘2‘4‘6‘8x ‡	-	-	-
†	1g1‘4‘6	1g1‘4‘6‘8	-	-	-
-	1g1‘4‘6‘8	1g1‘2‘4‘6‘8x ‡	-	-	-
†	2g2	2g2‘4	2L	1	6
-	2g2‘4	2g2‘4‘8	2L	3	2
-	2g2‘4‘8	⦸	-	-	-
†	2g1‘2‘3	2g1‘2‘3‘4x, 2g1‘2‘3‘6	2L	6	1
-	2g1‘2‘3‘4x	2g1‘2‘3‘4‘8x	2L	6	1
-	2g1‘2‘3‘4‘8x	2g1‘2‘3‘4‘6‘8x ‡	2L	6	1
-	2g1‘2‘3‘4‘6‘8x ‡	⦸	-	-	-
-	2g1‘2‘3‘6	2g1‘2‘3‘6‘8x	-	-	-
-	2g1‘2‘3‘6‘8x	2g1‘2‘3‘4‘6‘8x ‡	-	-	-
†	3g3	3g3‘6	3L	3	2
-	3g3‘6	⦸	-	-	-
†	3g2‘3‘4	3g2‘3‘4‘8, 3g2‘3‘4‘6	3L	1	6
-	3g2‘3‘4‘8	3g2‘3‘4‘6‘8 ‡	3L	1	6
-	3g2‘3‘4‘6	3g2‘3‘4‘6‘8 ‡	-	-	-
-	3g2‘3‘4‘6‘8 ‡	⦸	-	-	-
†	4g4	4g4‘8	4L	6	1
-	4g4‘8	⦸	-	-	-
†	4g1‘2‘3‘4x	4g1‘2‘3‘4‘6x	4L	6	1
-	4g1‘2‘3‘4‘6x	4g1‘2‘3‘4‘6‘8x ‡	-	-	-
-	4g1‘2‘3‘4‘6‘8x ‡	⦸	-	-	-
†	4g1‘2‘3‘4‘8x	4g1‘2‘3‘4‘6‘8x ‡	4L	6	1

Table 5: Some tetrads that SUPP suggests regarding elementary particles. The row for which $k = -2$ associates with notions of space-time dimensions that are relevant for CM modeling and QM modeling. The acronym EP abbreviates the two-word term elementary particle. The acronym EF abbreviates the two-word term elementary fermion. The symbol † associates with the POST notion of Majorana fermion.

Parameter	Triad notion	n_I	R_I	Monad notion
$k = -2$	Three spatial dimensions	1	$6\uplus$	One temporal dimension
$k = -1$	Color charges (red, blue, green)	6	1	The EP is not an EF
$k = 0$	Three isomer-pairs	3	2	The EP associates with one SL
$\Sigma = 1$	Charge ($Q = 1/3$, $Q = 2/3$, $Q = 1$)	6	1	The EP associates with $Q = 0$
$\Sigma = 2$	Mass ($m_b = 0$, $m_b > 0$, $m_f > 0$)	1	6	The EP associates with one SL
$\Sigma = 3$	EF f_{l-r} ($+1$, $0^{\dagger}$, -1)	3	2	The EP is not an EF
$\Sigma = 4$	EF $\lvert f_{B-L}\rvert$ ($0^{\dagger}$, 1/3, 1)	6	1	The EP is not an EF
$k = 6$	Three EF flavours	6	1	The EP is not an EF

- The following notions pertain regarding 4g4. For each one of the six isomers, there are three DOF-related aspects - $|f_{B-L}| = 0$, $|f_{B-L}| = 1/3$, and $|f_{B-L}| = 1$. The symbol f_{B-L} denotes - for fermion elementary particles - the POST notion of baryon number minus lepton number. POST suggests the possibility that neutrinos are Majorana fermions. For Majorana neutrinos, $f_{B-L} = 0$ would pertain. Regarding elementary fermions other than Majorana neutrinos, $|f_{B-L}| = 1/3$ for quarks and $|f_{B-L}| = 1$ for leptons. For non-fermion elementary particles, there is one DOF-related aspect - $f_{B-L} = 0$.

Paralleling discussion related to equation (24), SUPP suggests that - for SR - f_{l-r} associates with notions of being the scalar component of a 4-vector. Paralleling discussion related to equation (24), SUPP suggests that - for SR - f_{B-L} associates with notions of being the scalar component of a 4-vector.

Equation (25) shows notation that SUPP uses to describe a reach that includes all six isomers and all LRI phenomena.

$$R_I = 6\uplus \tag{25}$$

Table (5) lists some tetrads that SUPP suggests regarding elementary particles.

SUPP suggests relevance - regarding the following - for the notion of the existence of two solutions per one solution-pair.

- Two isomers (in the context of isomer-pair).
- Two handednesses (in the context of isomers).
- Two handednesses (in the context of nonzero spin, nonzero mass elementary particles).
- Two circular polarizations (in the context of zero-mass elementary particles).

3.7.6. Properties and reaches that associate with intrinsic uses of some long-range-interaction solution-pairs

Table 6 shows properties and reaches that associate with intrinsic uses of some Σg solution-pairs for which $1 \leq \Sigma \leq 4$, $1 \leq k_{max} \leq 8$, and $\{5, 7\} \cap Z_{\Gamma} = \emptyset$.

3.8. Gravitational properties of objects

This unit discusses reaches that associate with various gravitational properties of objects.

This unit associates primarily with the row - in table 3 - that associates with 2g' and with the two-word term gravitational properties.

GR associates a notion of repulsion between objects with a notion of pressure. POST associates a notion of DE with such a pressure.

Per table 4, equation (19), and equation (20), SUPP suggests that an excitation of a field (such as the 2L field - or, the gravitational field) encodes knowledge of the isomer-related instances of the properties that associate with the excitation. In effect, the gravitational field carries knowledge of the isomers that associate with the excitation.

Regarding the active gravitational properties of an object A, the following notions can pertain. The number of instances, n_I, of gravitationally attractive intrinsic monopole properties is one. The number of instances of gravitationally diluting intrinsic dipole properties can be as many as three. The number of

Table 6: Properties and reaches that associate with intrinsic uses of some Σg solution-pairs for which $1 \leq \Sigma \leq 4$, $1 \leq k_{max} \leq 8$, and $\{5,7\} \cap Z_\Gamma = \emptyset$. The symbol SL associates with a known LRI elementary particle or a might-be LRI elementary particle and with the value of S equals Σ. Here, Σ associates with the notion (as in the leftmost column in the table) of either one of ΣgΓ or Σ...gΓ and with one of E_Σ, B_Σ, $E_{\Sigma:n_\Gamma}$, and $B_{\Sigma:n_\Gamma}$. The next-to-rightmost column shows the number of instances for each pair - of one intrinsic solution-pair and one (one cascade-step) extrinsic solution-pair - that is relevant regarding SL. The rightmost column shows the reach for each pair - of one intrinsic solution-pair and one (one cascade-step) extrinsic solution-pair - that is relevant regarding SL.

Intrinsic	Property	E_Σ or B_Σ	SL	n_I	R_I
1g1	Charge	E_1	1L	6	1
1g1‘2	Magnetic moment, assuming a spherical charge distribution	B_1	1L	6	1
1_3g1‘2	Magnetic moment correction for an oblate charge distribution	B_1	1L	6	1
2g2	Energy	$E_{2:1}$	2L	1	6
2g2‘4	Angular momentum, assuming a spherical energy distribution	$B_{2:2}$	2L	3	2
2_6g2‘4	Angular momentum correction for an oblate energy distribution	$B_{2:2}$	2L	3	2
2g1‘2‘3	Two distinct axes of moments of inertia	$E_{2:3}$	2L	6	1
2g···	···	···	2L	···	···
3g3	f_{l-r}	E_3	3L	3	2
4g4	f_{B-L}	E_4	4L	6	1

instances of gravitationally additive intrinsic quadrupole properties can be as many as six. The number of instances of gravitationally diluting intrinsic octupole properties can be as many as six. The number of instances of gravitationally additive intrinsic 16-pole properties can be as many as six.

Regarding a one-isomer object, there are six possibilities regarding isomer. The following examples pertain regarding interactions between a sun (as a one-isomer object A) and a planet (as a one-isomer object C).

- To the extent that the sun exhibits only gravitationally active $R_I = 6$ aspects (or, monopole intrinsic gravitational properties), SUPP suggests that POST modeling is not necessarily inadequate.
- To the extent that the sun exhibits gravitationally active $R_I \neq 6$ aspects, SUPP suggests that POST modeling might be inadequate. Nonzero rotation of an isomer-zero sun provides a basis for an example. Intrinsic use of the solution-pair 2g2‘4 associates with nonzero rotation. A one-isomer planet that associates with isomer-zero or with isomer-three senses the rotation of the sun. The rotation affects the trajectory of the planet. A one-isomer planet that associates with isomer-one, isomer-two, isomer-four, or isomer-five does not sense the rotation of the sun. The rotation does not affect the trajectory of the planet.

In POST, the effective active gravitational properties of object A depend only on aspects of object A.

SUPP suggests that the effective active gravitational properties (of object A) that one object C senses depend on aspects of object A and on aspects of object C.

For a general T1 case of a point-like (and possibly multi-isomer) object C interacting with the 2L field that associates with an object A, object C senses all (nonzero value of property) 2gΓ solution-pair components that associate with the 2L field that associates with object A. The weighting that associates with any one intrinsic solution-pair associates with the geometric factor of the pole (monopole, dipole, or so forth) that associates with the intrinsic solution-pair and with an orientation factor that associates with a tensor-like notion (scalar for monopole, vector for dipole, and so forth) that associates with the intrinsic solution-pair. (SUPP uses the word weighting to avoid possibly inappropriate conflation with POST notions such as probability and amplitude. This paper does not operationally define the one-word term weighting.) For ND, the geometric factor associates with r^{-n_Γ}. Generally, possibly, effects that associate with one geometric factor or with two geometric factors dominate compared to effects that associate with other geometric factors.

SUPP suggests that the passive gravitational properties of an object equal the active gravitational properties of the object.

3.9. Inertial properties of objects

This unit discusses relationships between inertial properties of objects and gravitational properties of objects.

Discussion as to the extent to which inertial properties of objects equal gravitational properties of objects dates to around the 1680s and reference [2].

Table 7: Some conserved quantities that POST includes or that SUPP suggests. The word intrinsic abbreviates the three-word phrase intrinsic use of. The word extrinsic abbreviates the three-word phrase extrinsic use of. The symbol 2_6g2‘4 associates with notions - for rotating objects - of oblateness and with the solution-pair 6g2‘4. The acronym EF abbreviates the one-element term elementary-fermion. Conserved notions that SUPP suggests and POST does not necessarily include are numbers of dimensions and EF f_{l-r}. Regarding each row for which $1 \leq \Sigma \leq 4$ pertains, equation (10) pertains regarding notions of scalar properties and vector properties. The symbol † associates with the POST notion that conservation of EF f_{B-L} would not pertain to the extent that neutrinos model as Majorana fermions.

(a) SUPP interpretation.

Parameter	Conserved triad notion	n_I	R_I	Conserved monad notion
$k=-2$	Three spatial dimensions	1	$6\uplus$	One temporal dimension
$k=-1$	Color charge	6	1	-
$\Sigma=1$ - Intrinsic 1g1	Charge	6	1	-
$\Sigma=2$ - Intrinsic 2g2	Energy	1	$6\uplus$	-
$\Sigma=2$ - Extrinsic 2g2‘4	Momentum	1	$6\uplus$	-
$\Sigma=2$ - Intrinsic 2g2‘4 and 2_6g2‘4	Angular momentum	1	$6\uplus$	-
$\Sigma=3$ - Intrinsic 3g3	EF f_{l-r}	3	2	-
$\Sigma=4$ - Intrinsic 4g4	EF f_{B-L} †	6	1	-

(b) POST-like interpretation. POST does not necessarily include notions that associate with (at least) $n_I \geq 3$ and f_{l-r}.

Parameter	Conserved vector notion	n_I	R_I	Conserved scalar notion
$k=-2$	Three spatial dimensions	1	$6\uplus$	One temporal dimension
$k=-1$	-	6	1	Color charge
$\Sigma=1$ - Intrinsic 1g1	-	6	1	Charge
$\Sigma=2$ - Intrinsic 2g2	-	1	$6\uplus$	Energy
$\Sigma=2$ - Extrinsic 2g2‘4	Momentum	1	$6\uplus$	-
$\Sigma=2$ - Intrinsic 2g2‘4 and 2_6g2‘4	Angular momentum	1	$6\uplus$	-
$\Sigma=3$ - Intrinsic 3g3	-	3	2	EF f_{l-r}
$\Sigma=4$ - Intrinsic 4g4	-	6	1	EF f_{B-L} †

Based on the notion that $R_I = 6$ for intrinsic use of the solution-pair 2g2, SUPP suggests the following notions. For POST ND, inertial mass equals active gravitational mass and equals passive gravitational mass. For POST SR, inertial energy equals active gravitational energy and equals passive gravitational energy.

Based on the notion that $R_I = 6$ for extrinsic use of the solution-pair 2g2‘4 solution-pair, SUPP suggests the following notion. For POST ND and for POST SR, inertial momentum equals active gravitational momentum and equals passive gravitational momentum.

3.10. Conservation laws

This unit suggests new - compared to POST notions - conserved quantities and discusses numbers of instances of applicability of various conservation laws.

Table 7 lists some conserved quantities that POST includes or that SUPP suggests.

For example, for an object and absent interactions that exchange charge with other objects, each of the six instances of charge is conserved.

Each of the rows - in table 7 - for which the parameter column notes a value of Σ associates with n_I instances of a quantity that adds across components (including SL aspects) of a multi-component system.

3.11. Some spin-related properties of a two-component system and its two components

This unit discusses SUPP modeling regarding two-component systems.

This unit associates with the row - in table 3 - that associates with 0g and with $\{5, 7\} \cap Z_\Gamma \neq \emptyset$.

3.11.1. Spins S regarding two-component systems

The notion of 0g pertains. The notion of an upper limit on k_{max} does not pertain. Regarding spin S, the notion that $2S$ is a nonnegative integer pertains. The range $0 \leq S < \infty$ pertains.

Per equation (14) and equation (26), the solution-pair 0g1‘3‘5‘7 associates with $S = 0$.

$$0 = |+1-3-5+7| \tag{26}$$

For an integer l, SUPP uses the notation $+l\hat{}$ to denote the series to which equation (27) alludes. Each item in the series totals to l.

$$+l\hat{} \text{ denotes the series } +l,\ -l+2l,\ -l-2l+4l,\ -l-2l-4l+8l,\ \cdots \tag{27}$$

SUPP uses the notation $+l_n\hat{}$ to denote the item in equation (27) that includes exactly n terms. For example, $+l_2\hat{}$ denotes , $-l+2l$.

The following three sets of solution-pairs associate with $\Sigma = 0$ and with $4 \notin Z_\Gamma$. Equation (27) defines $+8\hat{}$ and $+16\hat{}$. Regarding the notions of 0g and ΣgΓ, for each solution-pair, the integers shown below (or alluded to by the series just above) appear in Γ and no other integers appear in Γ. For each set, for other than the first solution-pair, each solution-pair cascades from the first solution-pair.

1. $|-1-2-5+8|$, $|-1-2-5-8+16\hat{}|$,$|+1+2-5-6+8|$, and $|+1+2-5-6-8+16\hat{}|$
2. $|+2-3-7+8|$, $|+2-3-7-8+16\hat{}|$, $|+2+3-6-7+8|$, and $|+2+3-6-7-8+16\hat{}|$.
3. $|+1-3-5+7|$, $|-1-3+5-7+8\hat{}|$, $|-1-3+5-6+7|$, and $|-1-3-5+6-7+8\hat{}|$.

For each set, SUPP suggests that equation (14) pertains regarding the first two of the four expressions. Thus, each set includes exactly one expression for each nonnegative S for which $2S$ is an even integer. For each set, SUPP suggests that equation (15) pertains regarding the second two of the four expressions. Thus, each set includes exactly one expression for each nonnegative S for which $2S$ is an odd integer.

SUPP suggests, regarding a two-component system, that $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$ can associate with one component, $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$ can associate with the other component, and $5 \in Z_\Gamma$ and $7 \in Z_\Gamma$ can associate with the system.

Removal of (just) the criterion that $4 \notin Z_\Gamma$ results in the following notions. Regarding $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$, $n_{\Gamma^0} = 4$ solution-pairs that might associate with $S = 0$ associate with $|+1-2-4+5|$, $|-1-2-5+8|$, $|+1-4-5+8|$, and $|+2-3-4+5|$. Regarding $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$, $n_{\Gamma^0} = 4$ solution-pairs that might associate with $S = 0$ associate with $|-1-2-4+7|$, $|+2-3-7+8|$, and $|+3-4-7+8|$. Regarding $5 \in Z_\Gamma$ and $7 \in Z_\Gamma$, $n_{\Gamma^0} = 4$ solution-pairs that might associate with $S = 0$ associate with $|+1-3-5+7|$ and $|+2-4-5+7|$. The numbers of $S = 0$ solution-pairs are four for the case of $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$, three for the case of $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$, and two for the case of $5 \in Z_\Gamma$ and $7 \in Z_\Gamma$.

Discussion above de-emphasizes the notion that a solution-pair associates with two solutions. SUPP suggests - but this paper does not explore - the following notion. For some modeling, one linear combination of the two solutions might associate with S - as in the POST notion of $S(S+1)\hbar^2$ - and another (perhaps orthogonal to the previous) linear combination of the two solutions might associate with s - as in the POST notion of the $s \in \{-S, -S+1, \cdots, S-1, S\}$ that pertains regarding measuring an angular momentum $s\hbar$ with respect to a spatial axis.

3.11.2. Spins S regarding atoms

SUPP suggests that modeling can treat an atom as a two-component system. One component is the nucleus of the atom. The other component is the electron cloud.

For an atom, each one of the nucleus and the electron cloud has nonzero charge. Based on extrapolating from the notion that - for 0g solution-pairs that associate with elementary particles - $4 \in Z_\Gamma$ associates with zero charge, SUPP suggests that the following notions might pertain. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|-1-2-5+8|$ associate with spins S of the electron cloud. The solution-pair that associates with $|+1-2-4+5|$ associates with the spin $S = 0$ state of the electron cloud if the atom is an ion that has no electrons. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|+2-3-7+8|$ associate with spins S of the nucleus. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|+2-4-5+7|$ associate with spins S of the atom, if the atom (is not an ion and thus) has a charge of zero. Solution-pairs that associate with (zero-step or more-than-zero-step) cascades from the solution-pair that associates with $|+1-3-5+7|$ associate with spins S of the atom, if the atom (is an ion and thus) has a nonzero charge.

3.12. Electromagnetic properties and events that associate with atoms or stars

This unit suggests that detectors based on OM stuff can detect electromagnetic atomic radiation (but not necessarily electromagnetic thermal radiation) emitted by stuff that associates with one DM isomer.

This unit associates primarily with the row - in table 3 - that associates with 1g' and with the two-word term electromagnetic properties. This unit associates also with the row - in table 3 - that associates with 0g and $\{5, 7\} \cap Z_\Gamma \neq \emptyset$.

Table 8: Some aspects of some cascades that associate with some Σg" solution-pairs for which $n_\Gamma \geq 4$, $\Sigma = 1$, and $1 \leq k_{max} \leq 8$. The column with the two-element label $n_\Gamma = 4$ solution-pair associates with states of a two-component system. The column with the two-element label one-step cascade associates with states to which the $n_\Gamma = 4$ solution-pair state can transit via an interaction with the electromagnetic field. (Electromagnetic interactions can also associate with transitions from one-step cascade states to $n_\Gamma = 4$ solution-pair states.) The column with the one-element label R_I shows the reach for each $n_\Gamma = 4$ solution-pair and for each one-step cascade solution-pair. SUPP suggests that notions - in this table - of $n_\Gamma = 4$ solution-pair and of one-step cascade solution-pair can associate with properties of objects that model as being parts of multi-component systems. For example, for an atom, $5 \in Z_\Gamma$ and $7 \notin Z_\Gamma$ can associate with internal energy-related and angular-momentum-related properties of the atom's electron cloud. $5 \notin Z_\Gamma$ and $7 \in Z_\Gamma$ can associate with possible external influences on the atom's electron cloud. $n_\Gamma = 4$ solution-pair use of $6 \in \Gamma$ can associate with aspects that associate with single electrons. $n_\Gamma = 4$ solution-pair use of 1g2‘4‘5‘8 can associate with the atom's fine-structure state. $n_\Gamma = 4$ solution-pair use of 1g2‘4‘7‘8 can associate with the atom's hyperfine state. The column with the two-element label atom-related event suggests phenomena - related to an atom - that can associate with excitations or de-excitations of the 1L (or, electromagnetic) LRI field.

$n_\Gamma = 4$ solution-pair	One-step cascade	SL	R_I	Atom-related event
1g2‘4‘5‘6	1g2‘4‘5‘6‘8x	1L	2	An electron transits to a new principal energy.
1g2‘4‘5‘8	1g2‘4‘5‘6‘8x	1L	2	An atomic fine-structure change occurs.
1g2‘4‘6‘7	1g2‘4‘6‘7‘8x	1L	2	The atom adds or subtracts an electron.
1g2‘4‘7‘8	1g2‘4‘6‘7‘8x	1L	2	An atomic hyperfine transition occurs.

3.12.1. Electromagnetic events that associate with two-component systems such as atoms

SUPP suggests that, for $n_\Gamma \geq 4$, 1g" solution-pairs can associate with electromagnetic properties of objects that model as being parts of multi-component systems.

Table 8 shows some aspects of some cascades that associate with some Σg" solution-pairs for which $n_\Gamma \geq 4$, $\Sigma = 1$, and $1 \leq k_{max} \leq 8$. (Discussion related to equation (26) pertains. The notion that $4 \in Z_\Gamma$ associates with zero charge pertains regarding solution-pairs for which $\Sigma = 0$ and does not necessarily pertain regarding table 8.)

3.12.2. Electromagnetic events that associate with stars

SUPP suggests that stars tend to associate with single isomers. For a one-isomer star, SUPP suggests the following contributions to the electromagnetic field.

If the star has a net nonzero charge, some contributions to the electromagnetic field associate with the star's intrinsic 1g1 and extrinsic 1g1‘2. The notion of 1g' pertains. The relevant reach, R_I, for the emitted electromagnetic radiation is one isomer.

SUPP suggests that stellar radiation (such as thermal radiation) might associate - for example - with intrinsic 1g1‘2‘3‘4‘5x and with extrinsic 1g1‘2‘3‘4‘5‘6x or extrinsic 1g1‘2‘3‘4‘5‘8x. (Another example associates with intrinsic 1g1‘2‘3‘4‘7x. Another example associates with 1g", with intrinsic 1g4‘5, and with extrinsic 1g2‘4‘5. For the last example, the extrinsic solution-pair - in effect - inherits a reach, $R_I = 1$, from the intrinsic solution-pair.) The relevant reach, R_I, for the emitted electromagnetic radiation is one isomer.

3.12.3. Implications regarding cosmic background radiation and sensing dark matter

Per table 8, components of cosmic (electromagnetic) background radiation that associate with creation (of electromagnetic radiation) via atomic phenomena can associate with a reach R_I of two isomers. Components of cosmic (electromagnetic) background radiation that associate with creation via other phenomena can associate with a reach R_I of one isomer.

SUPP suggests that electromagnetic radiation that associates with creation via single-isomer atomic phenomena can associate with a reach R_I of two. SUPP suggests that detectors that have bases in OM stuff can detect such radiation created by isomer-zero atomic phenomena or by isomer-three atomic phenomena. Discussion related to table 15a suggests that detectors that have bases in OM stuff may have detected electromagnetic radiation created by isomer-three atomic phenomena.

4. Results

This unit suggests (based on POST modeling and SUPP modeling) explanations for data that POST modeling alone does not necessarily explain. This unit suggests possible future data.

4.1. A catalog of elementary particles

This unit shows a catalog of all elementary particles - of which people know or that SUPP suggests - and discusses symmetries that SUPP suggests associate with elementary particles and with conservation of DOFRA.

This unit associates with the row - in table 3 - that associates with 0g, $\{1,3\} \subset Z_\Gamma$, and $\{5,7\} \cap Z_\Gamma = \emptyset$. This unit associates with perspective and terminology centric to OM. (Generally, regarding the other five isomers, similar notions pertain.)

SUPP uses the following notions to catalog elementary particles. A symbol of the form $S\Phi$ associates with a so-called family of elementary particles. Each elementary particle associates with one family. Each family associates with one of one, three, or eight elementary particles. For a family, the value S denotes the spin (in units of $\hbar$) for each elementary particle in the family. S associates with the POST expression $S(S+1)\hbar^2$ that associates with angular momentum. Regarding POST, known values of S include 0, 0.5, and 1. The symbol Φ associates with a symbol of the form X_Q, in which X is a capital letter and Q is the magnitude of the charge (in units of $|q_e|$, in which q_e denotes the charge of an electron) for each particle in the family. For cases for which $Q = 0$, SUPP omits - from the symbols for families - the symbol Q.

Table 9 catalogs all known elementary particles and some elementary particles that SUPP suggests nature might include.

SUPP suggests the following notions regarding properties of elementary particles that associate with intrinsic solution-pairs that table 9 shows. (These notions supplement notions that discussion related to equation (13) suggests.)

- Mass or no mass? - Equation (28) pertains. The symbol m associates with the property of mass.

$$m = 0 \Leftrightarrow (6 \notin (\text{intrinsic})Z_\Gamma \text{ and } 8 \in (\text{intrinsic})Z_\Gamma) \tag{28}$$

- Number of fermion flavours? - For fermion elementary particles, each intrinsic use of a solution-pair associates with $6 \in Z_\Gamma$ and with three flavours.
- The magnitude of charge (that associates with a relevant solution-pair)? Equation (29) pertains (and is more specific than is equation (16)).

$$Q = 1 \Leftrightarrow 4 \notin (\text{intrinsic})Z_\Gamma \;\text{ and }\; Q = 0 \Leftrightarrow 4 \in (\text{intrinsic})Z_\Gamma \tag{29}$$

- LRI elementary particle or not an LRI elementary particle? For elementary particles, equation (30) pertains if and only if the elementary particle is an LRI elementary particle. The symbol $\subset$ denotes the four-word phrase is a subset of. The symbol $\cap$ denotes the notion of the intersection of two sets. The symbol $\emptyset$ denotes the empty set (or, the set that has zero members).

$$(\{1,3,4,8,16\} \subset (\text{intrinsic})Z_\Gamma) \text{ and } (\{2,6\} \cap (\text{intrinsic})Z_\Gamma = \emptyset) \tag{30}$$

SUPP suggests the following notion regarding properties of elementary particles that associate with extrinsic (or, one-step cascade) solution-pairs that table 9 shows.

- Can model (in POST) as existing independently of other elementary particles? - Equation (31) pertains.

$$\text{The object cannot model as independent } \Leftrightarrow \{1,2,3,4,6,8\} \subset (\text{one-step cascade})Z_\Gamma \tag{31}$$

Each one of the quarks, the gluons, and the might-be jay boson associates with the same pair of one-step-cascade solution-pairs and associates with the strong interaction. Regarding the gluons and the jay boson, the two one-step-cascade solution-pairs associate with eight gluons and one might-be jay boson. (Here, each one-step-cascade solution-pair might associate with three DOF-related aspects. Two instances of three DOF-related aspects might associate with nine - as in three times three - elementary particles.) SUPP suggests that the jay boson might associate with repulsive aspects of the residual strong force. SUPP suggests that the jay boson might associate with Pauli repulsion between like fermions, whether the like fermions are elementary fermions or are not elementary fermions.

SUPP suggests the following statements regarding interactions that involve elementary particles.

- Any elementary boson that associates with a one-step-cascade solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$ can transform into a pair of elementary bosons that are similar to each other and not similar to the original elementary boson. For example, a Z boson can transform into two photons. For the W boson, there is no one-step-cascade solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$. The W boson cannot transform into two (hypothetical) elementary particles for which each of the two produced elementary particles would associate with $Q = 0.5$.

Table 9: All known elementary particles and some elementary particles that SUPP suggests nature might include. The leftmost three columns provide information about elementary particles. The three charged leptons are the electron, the muon, and the tau. n_{EP} denotes the number of elementary particles. Q is a magnitude of charge (in units of $|q_e|$, in which q_e denotes the charge of an electron). m denotes mass. Regarding the $0.5\mathrm{Q}_{2/3}$ family of three quarks, a notion of two-thirds times the $Q = 1$ intrinsic solution-pair plus one-third times the $Q = 0$ intrinsic solution-pair pertains. Regarding the $0.5\mathrm{Q}_{1/3}$ family of three quarks, a notion of one-third times the $Q = 1$ intrinsic solution-pair plus two-thirds times the $Q = 0$ intrinsic solution-pair pertains. The symbol † denotes that the elementary particles are as-yet unfound. The word inflaton associates with POST notions of a possible inflaton elementary particle. 2L cascades from 1L, 3L cascades from 2L, and so forth. The acronym TBD abbreviates the three-word phrase to be determined. Equation (27) and related remarks define notation of the form $+16_n\hat{}$. The table de-emphasizes (but SUPP does not necessarily rule out) the possibilities that - for each one of some $S \geq 1$ - intrinsic use of the solution-pair $|+1-2-3-4-8+16_S\hat{}|$ associates with an elementary boson that has spin $S + 1$. Such elementary bosons might associate with notions of $(S + 1)$J families. The table de-emphasizes (but SUPP does not necessarily rule out) the possibilities that - for each one of some $S \geq 1$ - intrinsic use of combinations of the solution-pair $|-1+3-4-6-8+16_S\hat{}|$ and the solution-pair $|-1-2-3-4-6+16_{(S-1)}\hat{}|$ associate with elementary fermions that have spins of $S + 0.5$. Such elementary fermions might associate with notions of $(S + 0.5)$Q families. In table 9, each intrinsic solution-pair for which $n_{\Gamma 0} \geq 4$ cascades (in $n_{\Gamma 0} - 3$ steps) from (at least) one of solution-pair $|-1-3+4|$ and solution-pair $|-1-2+3|$.

Intrinsic	Names	Families	n_{EP}	Q	m	One-step cascades
$\vert -1-3+4\vert$	Z	1Z	1	0	>0	$\vert +1-2-3+4\vert$; $\vert -1-3-4+8\vert$; $\vert +1-3-4+6\vert$.
$\vert -1-2+3\vert$	W	$1\mathrm{W}_1$	1	1	>0	$\vert +1-2-3+4\vert$; $\vert -1-2-3+6\vert$.
$\vert +1-2-3+4\vert$	Higgs boson	0H	1	0	>0	$\vert +1-2-3-4+8\vert$; $\vert -1+2-3-4+6\vert$.
$\vert -1-3-4+8\vert$	Inflaton	0I	1 †	0	=0	$\vert +1-2-3-4+8\vert$; $\vert -1-3-4-8+16\vert$; $\vert -1+3-4-6+8\vert$.
$\vert +1-3-4+6\vert$	Neutrinos	0.5N	3	0	>0	$\vert -1+3-4-6+8\vert$; $\vert -1+2-3-4+6\vert$.
$\vert -1-2-3+6\vert$	Charged leptons	$0.5\mathrm{C}_1$	3	1	>0	$\vert -1+3-4-6+8\vert$; $\vert -1+2-3-4+6\vert$.
$\vert -1+2-3-6+8\vert$ and $\vert -1+2-3-4+6\vert$	Quarks	$0.5\mathrm{Q}_{2/3}$; $0.5\mathrm{Q}_{1/3}$	6	1, 0	>0	$\vert -1-2-3+4-6+8\vert$, $\vert +1-2+3-4-6+8\vert$.
$\vert +1-2-3-4+8\vert$	Gluons	1G	8	0	=0	$\vert +1-2+3-4-6+8\vert$, $\vert -1-2-3+4-6+8\vert$.
$\vert +1-2-3-4+8\vert$	Jay	1J	1 †	0	=0	$\vert +1-2+3-4-6+8\vert$, $\vert -1-2-3+4-6+8\vert$.
$\vert -1-3-4-8+16_1\hat{}\vert$	Photon	1L	1	0	=0	$\vert -1-3-4-8+16_2\hat{}\vert$, $\vert +1-2-3-4-8+16_1\hat{}\vert$, $\vert -1+3-4-6-8+16_1\hat{}\vert$.
$\vert -1-3-4-8+16_S\hat{}\vert$, with S being the S in SL	Graviton, TBD, …	2L, 3L, …	1 †, 1 †, …	0, 0, …	=0, =0, …	$\vert -1-3-4-8+16_{(S+1)}\hat{}\vert$, $\vert +1-2-3-4-8+16_S\hat{}\vert$, $\vert -1+3-4-6-8+16_S\hat{}\vert$

- Any elementary boson that does not associate with a one-step-cascade solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$ cannot directly transform into a pair of elementary bosons that are similar to each other and not similar to the original elementary boson. For the gluons, there is no one-step-cascade solution-pair for which $8 \in Z_\Gamma$ and $6 \notin Z_\Gamma$. Gluons do not transform directly into, for example, pairs of photons.
- Any elementary boson that associates with a one-step-cascade solution-pair for which $6 \in Z_\Gamma$ can transform into a pair of elementary fermions. For example, a Z boson can transform into two elementary fermions that are antiparticles to each other. The W boson can transform into a pair of fermions (for example, an electron and a neutrino). The W boson is the only elementary boson that cannot transform into two elementary fermions that are antiparticles to each other.

SUPP associates the symbol that equation (32) shows with a possible maximum spin S for LRI elementary particles.

$$S_{max,\mathrm{L}} \tag{32}$$

Each one of the following two notions might suggest that $S_{max,\mathrm{L}}$ is no greater than four. The first notion associates with the following sentence. Discussion related to equation (22) suggests the limit $n_0 \leq 3$ and hence a limit of $\Sigma \leq 4$ regarding the relevance of ΣgΓ solution-pairs for which Σ is the only element in the list Γ. The second notion associates with the following sentences. Equation (41) suggests that the solution-pair 1g1 associates with an interaction strength that includes a factor of four and that the solution-pair 2g2 associates with an interaction strength that includes a factor of three. Extrapolation suggests that the solution-pair 3g3 associates with an interaction strength that includes a factor of two, that the solution-pair 4g4 associates with an interaction strength that includes a factor of one, and that the solution-pair 5g5 would associate with an interaction strength that includes a factor of zero.

Reference [28] notes that QFT suggests that zero-mass elementary particles might not have spins that exceed two. SUPP suggests that each phenomenon for which SUPP suggests an explanation has at least one SUPP-based explanation that can comport with the possibility that $S_{max,\mathrm{L}} \leq 2$. For example, a basis for baryon asymmetry can associate with equation (69) and at least one 1g' solution-pair (such as 1g1'2'4). (The notion that 3g3 might not associate with an elementary particle does not disturb the possibility that equation (69) associates with a mechanism that produced baryon asymmetry. The notion that 3g3 and 4g4 might not associate with elementary particles does not disturb notions that tables 3, 5, and 7 discuss.) This paper does not explore the extent to which multi-isomer analogs to QFT might suggest limits other than $S_{max,\mathrm{L}} \leq 2$ regarding the spins of zero-mass elementary particles.

4.2. Relationships among properties of boson elementary particles

This unit discusses interrelationships that SUPP suggests pertain regarding properties of boson elementary particles and points to a notion - that associates with integers N' - that might portend new physics that this paper does not fully develop.

This unit associates with perspective and terminology centric to OM. (Generally, regarding the other five isomers, similar notions pertain.)

4.2.1. Relationships between the masses of nonzero mass elementary bosons

SUPP suggests that equation (33) pertains regarding the masses of the nonzero-mass elementary bosons.

$$(m_\mathrm{W})^2 : (m_\mathrm{Z})^2 : (m_\mathrm{Higgs})^2 :: 7 : 9 : 17 \tag{33}$$

Equation (33) is not inconsistent with data. Based on information that reference [66] provides, the following notions pertain. The most accurately known of the three masses is m_Z. Based on the nominal value of m_Z, the nominal value (that equation (33) suggests) for m_Higgs is within 0.5 experimental standard deviations of m_Higgs. Based on the nominal value of m_Z, the nominal value (that equation (33) suggests) for m_W is within 3.6 experimental standard deviations of m_W. Based on information that reference [67] provides, the following notions pertain. Based on the nominal value of m_Z, the nominal value (that equation (33) suggests) for m_W is within 1.1 experimental standard deviations of m_W. (Reference [67] does not provide new information about m_Higgs.) Based on the nominal value of m_Z that reference [67] suggests and on information that reference [66] provides about m_Higgs, the nominal value that equation (33) suggests for m_Higgs is within 0.5 experimental standard deviations of m_Higgs.

SUPP suggests that equation (33) might point to possible insight regarding - and a possible extension to - the POST notion of the weak mixing angle.

Table 10: SUPP-suggested relationships between properties of all known elementary bosons and some elementary bosons that SUPP suggests. . The symbol M' denotes the mass (in units of $m_Z/3$) of the elementary boson. The symbol S' denotes the spin that POST associates with the expression $S'(S'+1)\hbar^2$. The symbol Q' denotes the magnitude (in units of the magnitude $|q_e|$ of the charge - q_e - of the electron) of the charge of the elementary boson. The symbol μ' denotes the magnitude of the magnetic moment divided by the magnetic moment of the W boson. The column with the label $\sum(X')^2$ shows the sum of the squares of the numbers to the left of the column. The symbol T' denotes one, if the mass is nonzero. (Here, POST suggests that the notion of longitudinal polarization pertains). The symbol T' denotes zero, if the mass is or would be zero. (Here, POST suggests or presumably would suggest that the notion of longitudinal polarization does not pertain.) The symbol N' denotes a positive integer for which $\sum(X')^2$ equals sum of squares of the numbers to the right of the $\sum(X')^2$ column. The table omits the possible 2G, 3G, and 4G elementary bosons. The table downplays possibilities for SL or SJ bosons for which S exceeds four.

Elementary boson(s)	Family	Status	M'	S'	Q'	μ'	$\sum(X')^2$	T'	N'
Higgs boson	0H	Known	$\sqrt{17}$	0	0	0	17	1	4
Z	1Z	Known	$\sqrt{9}$	1	0	0	10	1	3
W	$1W_1$	Known	$\sqrt{7}$	1	1	1	10	1	3
Photon	1L	Known	0	1	0	0	1	0	1
Gluons	1G	Known	0	1	0	0	1	0	1
Inflaton	0I	SUPP-suggested	0	0	0	0	0	0	0
Jay	1J	SUPP-suggested	0	1	0	0	1	0	1
Graviton	2L	SUPP-suggested	0	2	0	0	4	0	2
TBD	3L	SUPP-suggested	0	3	0	0	9	0	3
TBD	4L	SUPP-suggested	0	4	0	0	16	0	4

4.2.2. Relationships between properties of all known and SUPP-suggested elementary bosons

Table 10 SUPP-suggested relationships between properties of all known elementary bosons and some elementary bosons that SUPP suggests.

SUPP suggests that equation (34) pertains for each elementary boson to which table 9 alludes.

$$(N')^2 \equiv (M')^2 + (S')^2 + (Q')^2 + (\mu')^2 - (T')^2 \tag{34}$$

For each known or SUPP-suggested elementary boson, the notion that N' is an integer is not inconsistent with data.

SUPP suggests that equation (35) and equation (36) pertain for boson elementary particles. (SUPP does not suggest any elementary bosons for which (intrinsic)$n_{\Gamma^0} \leq 2$.)

$$M' > 0 \Leftrightarrow N' = (\text{intrinsic})n_{\Gamma^0} \ ; \quad M' = 0 \Leftrightarrow N' = (\text{intrinsic})n_{\Gamma^0} - 4 \tag{35}$$

$$N' \in \{0, 1, 2, 3, 4\} \tag{36}$$

For each elementary boson to which table 9 alludes, SUPP suggests that equation (37) pertains. (SUPP does not suggest any elementary bosons for which $M' > 0$ and $N' \leq 2$.)

$$M' > 0 \Leftrightarrow N' = 4 - S' \geq 3 \ ; \quad M' = 0 \Leftrightarrow N' = S' \tag{37}$$

4.2.3. Notions that might have relevance regarding notions of integers N' for elementary bosons

SUPP suggests that modeling each term on the right side of equation (34) with an expression of the form that equation (38) shows. Here, $D'_b = 1$ pertains. Equation (34) pertains for each isomer and for the LRI elementary bosons (which can associate with multiple isomers).

$$(X')^2 \propto \int_0^{X'} x^{D'_b} dx \tag{38}$$

Regarding QM raising and lowering operators for elementary boson states, the following notions pertain. States associate with the range $0 \leq n_b$ for integers n_b. For the raising operator, a factor that equation (39) shows pertains. For the lowering operator, a factor of $(n_b)^{1/2}$ pertains.

$$(1 + D'_b n_b)^{1/2} = (1 + n_b)^{1/2} \tag{39}$$

4.3. Relationships among properties of fermion elementary particles

This unit discusses interrelationships that SUPP suggests pertain regarding properties of fermion elementary particles and points to a notion - that associates with integers N' - that might portend new physics that this paper does not fully develop.

This unit associates with perspective and terminology centric to OM. (Generally, regarding the other five isomers, similar notions pertain. However, SUPP suggests that - for DM counterparts to OM charged leptons - different associations between flavour and mass can pertain.)

4.3.1. A relationship between the tau mass, electron mass, and strengths of two forces

Regarding charged leptons, SUPP suggests a link between the strength of electromagnetism and the strength of gravity.

Equation (40) and equation (41) define, respectively, β' and β. m_τ denotes the mass of the tau particle (which is a charged lepton). m_e denotes the mass of the electron (which is a charged lepton). The right-hand side of equation (41) is the ratio of the electrostatic repelling between two electrons to the gravitational attracting between the two electrons. The ratio does not depend on the distance between the two electrons.

$$\beta' \equiv m_\tau / m_e \tag{40}$$

$$(4/3) \cdot (\beta^2)^6 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2) \tag{41}$$

Based on data, $\beta \approx 3477.1891 \pm 0.0226$. (Reference [66] provides the relevant underlying data.) The standard deviation associates almost entirely with the standard deviation for G_N, the gravitational constant.

Equation (42) shows an equality that SUPP suggests.

$$\beta' = \beta \tag{42}$$

Equation (43) results from equation (42). The standard deviation associates almost entirely with the standard deviation for G_N.

$$m_{\tau,\text{ calculated}} \approx 1776.8400 \pm 0.0115 \text{ MeV}/c^2 \tag{43}$$

Equation (43) comports with data. More than eight standard deviations fit within one standard deviation from the data-based nominal value for m_τ.

4.3.2. A formula that might approximately link the masses of all elementary fermions

SUPP suggests a formula that might approximately link the masses of all elementary fermions.

Equation (44) defines $m(l_m, l_q)$ and has bases in the equations that immediately follow equation (44). Equation (45) defines the fine-structure constant. m_μ denotes the mass of the muon, which is a charged lepton. Equation (50) has bases in trying to fit data.

$$m(l_m, l_q) \equiv m_e \cdot (\beta^{1/3})^{l_m + j_m d_m} \cdot (\alpha^{-1/4})^{(1+l_m)n_q + j_q d_q(l_m)} \tag{44}$$

$$\alpha = ((q_e)^2/(4\pi\varepsilon_0))/(\hbar c) \tag{45}$$

$$j_m = 0, +1, 0, -1 \text{ for, respectively, } l_m \bmod 3 = 0, 1, 3/2, 2\text{; with } 3/2 \bmod 3 \equiv 3/2 \tag{46}$$

$$d_m = 2 - (\log(m_\mu/m_e)/\log(\beta^{1/3})) \approx 3.840613 \times 10^{-2} \tag{47}$$

$$n_q = 0, 3/2, 3/2, 3/2, 3/2, \text{ for, respectively, } l_q = 3, 2, 3/2, 1, 0 \tag{48}$$

$$j_q = 0, -1, 0, +1, +3 \text{ for, respectively, } l_q = 3, 2, 3/2, 1, 0 \tag{49}$$

$$d_q(0) \sim 0.324,\ d_q(1) \sim -1.062,\ d_q(2) \sim -1.509 \tag{50}$$

Table 11: Approximate values of $\log_{10}(m(l_m, l_q)/m_e)$ for all known charged fermion elementary particles. Regarding flavour, this table generalizes, based on POST terminology that associates with charged leptons and with neutrinos. (For example, POST uses the term electron-neutrino.) In table 11, the symbol l_f numbers the three flavours. The "l_f (0.5C$_1$)" terms pertain for fermions in the 0.5C$_1$ family. The symbol 0.5Q$_{>0}$ denotes the pair 0.5Q$_{2/3}$ and 0.5Q$_{1/3}$. The "l_f (0.5Q$_{>0}$)" terms pertain for quarks (or, elementary particles in the two families 0.5Q$_{2/3}$ and 0.5Q$_{1/3}$). l_m is an integer parameter. The domain $-6 \leq l_m \leq 18$ might have relevance regarding modeling. Q denotes the magnitude of charge, in units of $|q_e|$. The family 0.5C$_1$ associates with $Q = 1$. The family 0.5Q$_{2/3}$ associates with $Q = 2/3$. The family 0.5Q$_{1/3}$ associates with $Q = 1/3$. Regarding table 11, $l_q = 3Q$ pertains. Regarding the rightmost four columns, items show $\log_{10}(m(l_m, l_q)/m_e)$ and - for particles that nature includes - the name of an elementary fermion. For each † case, no particle pertains. Each number in the column with the label $Q = 1/2$ equals the average of the number in the $Q = 2/3$ column and the number in the $Q = 1/3$ column. The notion of geometric mean pertains regarding the mass of the $Q = 2/3$ particle and the mass of the $Q = 1/3$ particle. Regarding each † case, equation (44) provides the number $m(l_m, l_q)$.

l_f (0.5C$_1$)	l_f (0.5Q$_{>0}$)	l_m	$Q = 1$	$Q = 2/3$	$Q = 1/2$	$Q = 1/3$
1 (Electron)	1 (Up, Down)	0	0.00 Electron	0.63 Up	0.80 †	0.97 Down
-	2 (Charm, Strange)	1	1.23 †	3.40 Charm	2.83 †	2.26 Strange
2 (Mu)	3 (Top, Bottom)	2	2.32 Muon	5.53 Top	4.72 †	3.91 Bottom
3 (Tau)	-	3	3.54 Tau	-	-	-

$$d_q(l_m) = 0 \text{ for } l_m \leq -1 \text{ and for } l_m \geq 3 \tag{51}$$

SUPP suggests that the number of seemingly independent irrational numbers input into the above equations is seven. For example, the list consisting of m_e, m_μ, β, α, $d_q(0)$, $d_q(1)$, and $d_q(2)$ includes seven irrational numbers.

4.3.3. Nominal properties of known charged elementary fermions

Table 11 shows information about properties of all known charged fermion elementary particles. (Reference [66] provides the data that underlie table 11.) Regarding similar tables for each one of isomer-one, isomer-two, isomer-four, and isomer-five, SUPP suggests (per table 12) that the values of l_f that table 11 shows for the charged leptons are not appropriate. For example, for isomer-two, the l_f values in the leftmost column would be 3 (for the row for which - for quarks - $l_f = 1$), blank (for the row for which - for quarks - $l_f = 2$), 1 (for the row for which - for quarks - $l_f = 3$), and 2 (for the remaining row).

For each charged elementary fermion except the top quark, equation (44) suggests a mass that is within one experimental standard deviation of the nominal mass that reference [66] reports. Reference [66] alludes to three estimates for the mass of the top quark. Equation (44) provides a mass (for the top quark) that is within 4.4 standard deviations below the nominal mass that associates with direct measurements, within 4.3 upward standard deviations above the nominal mass that associates with cross-section measurements, and within 1.6 standard deviations below the nominal mass that associates with the four-element phrase pole from cross-section measurements.

4.3.4. Notions that might have relevance regarding notions of integers N' for elementary fermions

Equation (34) interrelates the properties of elementary bosons.

Compared to equation (34), equations relating mass, charge, and other properties for elementary fermions would include two new (compared to for elementary bosons) aspects - flavour and fractional charge. Flavour associates with one new (compared to for elementary bosons) tetrad. (Notions related to the $k = 6$ row in table 5 pertain.) Fractional charge associates with one new (compared to for elementary bosons) tetrad. (Notions related to the $\Sigma = 1$ row in table 5 pertain.)

SUPP suggests that the change of two regarding a relevant number of tetrads (as per table 5) might associate with notions that associate with equation (52).

$$D'_f = D'_b - 2 = -1 \tag{52}$$

SUPP notes the following notions, which may represent a coincidence or may point to possibilities for future physics principles. Regarding QM raising and lowering operators for elementary fermion states, the following notions pertain. States associate with the range $0 \leq n_f \leq 1$ for integers n_f. For the raising operator, a factor that equation (53) shows pertains. For the lowering operator, a factor of $(n_f)^{1/2}$ pertains. This paper does not explore these notions further.

$$(1 + D'_f n_f)^{1/2} = (1 - n_f)^{1/2} \tag{53}$$

4.3.5. Relationships - known elementary fermions associating with a common magnitude of charge

SUPP suggests that equation (54) might provide a parallel - for the three elementary fermions that associate with any one value of Q' - to equation (34), which pertains for elementary bosons. Regarding known elementary fermions, SUPP suggests that equation (55) lists one possible set $\{X'\}$ of properties X'. (SUPP suggests that a notion of S' need not pertain here, because one value of spin pertains across the set of all known elementary fermions. SUPP suggests that a notion of T' need not pertain here, because - likely - each known elementary fermion has nonzero mass.) SUPP suggests that equation (44) suggests possible applicability for equations (56) and (57). SUPP suggests that equations (52), (56) and (57) pertain. SUPP suggests that, for each one of 0.5C$_1$ (or, charged leptons), 0.5Q$_{2/3}$, 0.5Q$_{1/3}$, and 0.5N (or, neutrinos), the set of X_{ref} can associate with the properties of any one of the three elementary fermions. For neutrinos, $\mu' = 0$.

$$N' = \sum_{\{X'\}} log(X'/X_{ref}) \tag{54}$$

$$\{X'\} = \{M', \mu'\} \tag{55}$$

$$log(X'/X_{ref}) = \int_{X_{ref}}^{X'} x^{D'_f} dx, \text{ for a property for which the } X' \text{ value is nonzero} \tag{56}$$

$$log(X'/X_{ref}) \equiv 0, \text{ for a property for which the } X' \text{ value is zero} \tag{57}$$

Modeling that would pertain regarding more than one magnitude of charge would add Q' to the set $\{X'\}$ and would involve non-integer values of N'. For example, for the up quark and modeling in which Q_{ref} associates with the down quark, the notion of $log(X'/X_{ref}) = log(2)$ would pertain. (Discussion related to equation (67) pertains regarding relationships that associate with more than one value of charge.)

4.3.6. Anomalous magnetic moments of charged leptons

This unit associates with the row - in table 3 - that associates with Σg" and with $n_\Gamma = 2$.

QFT associates with two complementary aspects of magnetic moment - nominal magnetic moment and anomalous magnetic moment. QFT calculates anomalous magnetic moments that match data regarding the electron and the muon. The calculations feature notions of virtual photons.

SUPP suggests that POST notions of nominal magnetic moment associate with SUPP notions of the T1 component of magnetic moment. SUPP suggests that POST notions of anomalous magnetic moment associate with SUPP notions of the T2BNT1 component of magnetic moment.

QFT suggests equation (58). The term μ_{cl} associates with the notion of (total) magnetic moment. The factor $|g_{cl}|$ associates with the property of charge. The term that associates the number two associates with the notion of nominal magnetic moment and the term that associates with a_{cl} associates with the notion of anomalous magnetic moment.

$$\mu_{cl} \propto |q_{cl}| m_{cl} g_{cl} = |q_{cl}| m_{cl} (2 + a_{cl}) \tag{58}$$

SUPP makes the following notational associations. For the electron, cl can be either e or (as in $l_m = 0$) zero. For the muon, cl can be either μ or (as in $l_m = 2$) two. For the tau, cl can be either τ or (as in $l_m = 3$) three.

SUPP suggests that - regarding equation (44) - equation (59) recasts the term $j_m d_m$ for each of $l_m = 0$, $l_m = 2$, and $l_m = 3$. The notation $x_1 \leftarrow x_2$ associates with the three-element phrase x_2 replaces x_1.

$$j_m d_m \leftarrow l_m - 3(\log(\mu_{l_m}/\mu_0)/\log(\mu_3/\mu_0)) \tag{59}$$

Here (as in equation (44)), $j_m d_m$ is zero for each one of $l_m = 0$ and $l_m = 3$. For $l_m = 2$, SUPP suggests that the replacement $j_m d_m$ equals the $j_m d_m$ that equation (47) suggests.

Along with the notion of a_3 that equation (58) suggests and based on equation (59), SUPP suggests that - for $x = -d_2$ - equation (60) pertains.

$$g_3 = (\mu_0/m_3)(\mu_2/\mu_0)^{3/(2+x)}, \text{ with } x = -d_2 \tag{60}$$

Reference [66] provides the data $a_0 \approx 0.00115965$ and $a_2 \approx 0.00116592$. Based on $x = -d_2$ and equation (60), SUPP suggests the result for the tauon anomalous magnetic moment that equation (61) shows.

$$a_{\tau,\mathrm{SUPP}} = a_\tau = a_3 \approx 0.00116926 \tag{61}$$

Reference [68] provides, based on the SM, a first-order result - which SUPP calls $a_{\tau,\mathrm{SM}}$ - for a_τ. Here, SM denotes the two-word term Standard Model. The result is $a_{\tau,\mathrm{SM}} \approx 1.177 \times 10^{-3}$ and leads to a value of $(a_{\tau,\mathrm{SUPP}} - a_{\tau,\mathrm{SM}})/a_{\tau,\mathrm{SM}}$ of approximately -0.0066. Each one of $a_{\tau,\mathrm{SUPP}}$ and $a_{\tau,\mathrm{SM}}$ comports with data that reference [66] provides.

For each one of the charged leptons, SUPP suggests that equation (62) pertains, if one associates the values of X_{ref} with properties of the electron. The equation shows a sum of four terms. Each term associates with the respective term that equation (34) shows.

$$N' = \sum_{\{X'\}} log(X'/X_{ref}) = l_m + 0 + 0 + l_m - 0 = 2l_m \tag{62}$$

4.3.7. Notions that associate with the lack of a charged lepton that would associate with $l_m = 1$

Extrapolating - from work that associates with equation (61) - to the case $l_m = 1$ would calculate a negative value for a_1. SUPP suggests - per remarks that associate with equation (18) and per a SUPP suggestion that the component of magnetic moment that associates with the word oblate associates with the POST QFT notion of anomalous magnetic moment - anomalous magnetic moments are nonnegative.

4.3.8. Notions that associate with neutrino oscillations

The SM suggests that neutrino oscillations associate with a notion that flavour eigenstates do not necessarily match mass eigenstates. Flavour eigenstates associate with the creation of neutrinos via the weak interaction. Mass eigenstates associate with the motion - after the creation of a neutrino - of a neutrino.

SUPP associates the weak interaction bosons (and the weak interaction) with 0g solution-pairs. Per table 9, neutrino flavour eigenstates associate with intrinsic use of 0g1‘3‘4‘6. SUPP associates mass - and mass eigenstates - with intrinsic use of 2g2.

SUPP suggests the possibility that intrinsic use of the 6g2‘4 solution-pair (and extrinsic use of 6g2‘4‘8) associates with a notion of anomalous angular momentum (including for zero-charge elementary fermions). Paralleling discussion regarding equation (58) and anomalous magnetic moments for charged leptons, there might be differences - among the three neutrino mass eigenstates - regarding anomalous angular momentum. SUPP suggests that such differences might associate with SM notions of differences - regarding masses - between neutrinos.

Reference [25] notes that observations explore the extent to which neutrino oscillations associate with interactions - between neutrinos and the environments through which neutrinos pass. SUPP suggests that such interactions might associate with 2g” (and notions that associate with mass), with 3g” (and notions that associate with flavour), or with 4g” (and notions that associate with flavour). SUPP suggests that properties and events that associate with interactions between neutrinos and their environments might associate (for example) with intrinsic use of 2g1‘4‘5‘6 (for which 2g1‘4‘5‘6‘8 is a one-step cascade), with intrinsic use of solution-pair 3g2‘4‘5‘6 (for which 3g2‘4‘5‘6‘8 is a one-step cascade), or with intrinsic use of solution-pair 4g1‘2‘5‘6 (for which 4g1‘2‘5‘6‘8 is a one-step cascade). (These notions parallel notions that table 8 suggests regarding 1g” - or, electromagnetic - properties and events.)

4.3.9. Neutrino masses

SUPP suggests neutrino masses.

Reference [22] suggests that data point to the notion that the sum of the three neutrino rest energies is at least approximately 0.06 eV and not more than approximately 0.12 eV. Reference [69] discusses data and modeling regarding upper bounds for the sum of the masses of the three neutrinos. Reference [70] discusses a lower bound of 0.06 eV, an upper bound of 0.15 eV, and a possible upper bound of 0.12 eV. Reference [66] suggests that an upper bound might be approximately 0.10 eV.

Neutrinos associate with $Q = 0$. SUPP suggests that some $m(l_m, 0)$ solutions associate with neutrino masses. For $l_m \leq -1$ and for $l_m \geq 3$, no quarks pertain and SUPP suggests that $d_q(l_m) = 0$.

Equation (63) shows a result from equation (44).

$$mc^2 = m(-4, 0)c^2 \approx 3.448 \times 10^{-2} \text{ eV} \tag{63}$$

SUPP suggests the following two possibilities.

1. $mc^2 = m(-4,0)c^2 \approx 3.448 \times 10^{-2}$ eV pertains for each of the three neutrinos.
2. $mc^2 = m(-4,0)c^2 \approx 3.448 \times 10^{-2}$ eV pertains for each of two neutrinos. For one neutrino, one of $m(-6,0)c^2 \approx 4.2 \times 10^{-6}$ eV and $m(-5,0)c^2 \approx 4.4 \times 10^{-4}$ eV might pertain.

This paper does not try to explore the extent to which SUPP notions - such as notions regarding anomalous angular momentum and the 6g2'4 solution-pair or such as notions regarding interactions that associate with Σg" properties for which $n_\Gamma \geq 3$ - might suffice to explain neutrino oscillations, including for the case in which just one mass pertains for all three neutrinos.

Regarding neutrinos and equation (54), for each neutrino, $N' = l_m$. For at least two neutrinos, SUPP suggests that $N' = -4$.

4.3.10. Relationships among properties of all known charged elementary fermions

SUPP extends notation that associates with equation (44) to denote the magnetic moments $g(l_m, l_q)$ for all elementary fermions.

Quark flavours associate with the range $0 \leq l_m \leq 2$. SUPP defines - for each one of the three quark flavours - the ratio that equation (64) shows. SUPP suggests the notion that - for each quark flavour - equation (65) pertains.

$$r'(l_m) = (m(l_m,3)g(l_m,3))/(m(l_m,2)g(l_m,2)) \tag{64}$$

$$m(l_m,l_q)g(l_m,l_q) = (r'(l_m))^{\Theta'} m(l_m,3)g(l_m,3) \tag{65}$$

Based on information that table 11 shows, an assumption that there is not much harm in assuming that the various $g(l_m,3)$ are approximately two, and the associating of Θ_{ref} with $l_q = 3$ (which, generally associates with charged leptons), SUPP suggests the following notions.

For quark flavour 1 (or, $l_m = 0$), the following notion pertains. For $Q' = 1$ (or, $l_q = 3$) , $Q' = 2/3$ (or, $l_q = 2$), and $Q' = 1/3$ (or, $l_q = 1$), SUPP suggests that, respectively, Θ' is 0, $+2$, and $+3$.

For quark flavour 2 (or, $l_m = 1$), the following notion pertains. For $Q' = 1$ (or, $l_q = 3$) , $Q' = 2/3$ (or, $l_q = 2$), and $Q' = 1/3$ (or, $l_q = 1$), SUPP suggests that, respectively, Θ' is 0, -2, and -1.

For quark flavour 3 (or, $l_m = 2$), the following notion pertains. For $Q' = 1$ (or, $l_q = 3$) , $Q' = 2/3$ (or, $l_q = 2$), and $Q' = 1/3$ (or, $l_q = 1$), SUPP suggests that, respectively, Θ' is 0, -2, and -1.

For each one of the three flavours, $|\Theta'| = 2$ pertains for $Q' = 2/3$ (or, $l_q = 2$). SUPP suggests that equation (66) pertains, with the possibility that the sign of $log(r'(l_m))$ associates with the SUPP suggestion that anomalous contributions must be nonnegative. This paper does not explore this notion further.

$$log((m(l_m,2)g(l_m,2))/(m(l_m,3)g(l_m,3))) = 2|log(r'(l_m))| \tag{66}$$

SUPP suggests the possibility that along with equation (54) - equation (67) - links the properties of all known charged elementary fermions.

$$\{X'\} = \{M', \mu', \Theta'\} \tag{67}$$

This paper does not explore the extent to which work associating with equation (67) suggests a reduction - for example from the number, seven, that seems to associate with equation (44) - in a number of independent physics constants.

4.4. Differences - between isomers - regarding properties of fermion elementary particles

This unit discusses differences - regarding the flavours of charged leptons - between isomers.

This unit associates with perspective and terminology centric to OM and with perspective and terminology centric to the five DM counterparts to OM.

If the stuff that associates with each of the five all-DM isomers evolved similarly to the stuff that associates with isomer-zero, SUPP suggestions regarding DM might not adequately comport with observations regarding the Bullet Cluster collision of two galaxy clusters. (Discussion - below - that cites reference [71] provides more information.)

SUPP uses the symbol l_I to number the isomers. The notion of isomer-l_I pertains.

Per discussion (including discussion regarding table 11) above, regarding each l_I that is at least one, SUPP suggests that the instances (of elementary particles) that associate with isomer-l_I match - with respect to mass - the instances (of the counterpart elementary particles) that associate with isomer-zero.

Table 12: Matches between masses and flavours, for isomers of charged elementary fermions. The symbol l_I denotes the isomer number. The symbol f_{l-r} denotes - for fermion elementary particles - whether a matter particle (in the context of matter particle and antimatter particle) elementary fermion is left-handed ($f_{l-r} = +1$), does not associate with handedness ($f_{l-r} = 0$, in which case the elementary fermion is its own antiparticle), or is right-handed ($f_{l-r} = -1$). The symbol $0.5\mathrm{Q}_{>0}$ denotes the pair $0.5\mathrm{Q}_{1/3}$ and $0.5\mathrm{Q}_{2/3}$. As in table 11, here the symbol l_f numbers the three flavours.

l_I	f_{l-r}	l_m ($0.5\mathrm{Q}_{>0}$)	Respective l_f ($0.5\mathrm{Q}_{>0}$)	l_m ($0.5\mathrm{C}_1$)	Respective l_f ($0.5\mathrm{C}_1$)
0	$+1$	$0, 1, 2$	1,2,3	$0, 2, 3$	1,2,3
1	-1	$3, 4, 5$	1,2,3	$3, 5, 6$	3,1,2
2	$+1$	$6, 7, 8$	1,2,3	$6, 8, 9$	2,3,1
3	-1	$9, 10, 11$	1,2,3	$9, 11, 12$	1,2,3
4	$+1$	$12, 13, 14$	1,2,3	$12, 14, 15$	3,1,2
5	-1	$15, 16, 17$	1,2,3	$15, 17, 18$	2,3,1

For modeling regarding flavours (and not - for $0 \leq l_I \leq 5$ - for modeling regarding masses), SUPP associates the quarks in isomer-l_I with three values of l_m. The values are $3l_I + 0$, $3l_I + 1$, and $3l_I + 2$. (Table 11 shows the associations for $l_I = 0$.) Across the six isomers, quarks associate with each value of l_m that is in the range $0 \leq l_m \leq 17$. Regarding quarks and flavours, SUPP suggests that - within isomer-l_I - flavour 1 associates with $l_m = 3l_I$, flavour 2 associates with $l_m = 3l_I + 1$, and flavour 3 associates with $l_m = 3l_I + 2$.

Aspects of table 11 point to the notion that means for matching flavours and masses for charged leptons do not match means for matching flavours and masses for quarks. For charged leptons, isomer-zero does not have a charged lepton that associates with $l_m = 1$ and does have a charged lepton that associates with $l_m = 3$. SUPP suggests that - for each l_I - a charged lepton associates with each of $l_m = 3l_I + 0$, $l_m = 3l_I + 2$, and $l_m = 3l_I + 3$.

SUPP suggests that - for each isomer-l_I such that $1 \leq l_I \leq 5$ - the charged-lepton flavour that associates with $l_m = 3(l_I) + 0$ equals the flavour that associates with the isomer-($l_I - 1$) charged lepton that associates with the same value of l_m and - thus - with $l_m = 3(l_I - 1) + 3$. SUPP suggests that, across the six isomers, one cyclical order pertains regarding flavours for charged leptons.

Table 12 shows, for isomers of charged elementary fermions, matches between masses and flavours.

4.5. Isomer, as a new internal quantum number for elementary particles

This unit discusses the notion that l_I (as in isomer number) associates with the notion of a new (compared to POST QM) internal quantum number for elementary particles.

In table 4, each row that associates with $n_I > 1$ associates with the SUPP notion that isomer is a property of objects.

Regarding elementary particles for which $\Phi \neq$L, SUPP suggests that $n_I = 6$.

Table 12 suggests associations between some internal quantum numbers (for some elementary fermions) and l_I (as in isomer number).

SUPP suggests that the SUPP notion of l_I associates with a new (compared to POST QM) internal elementary-particle quantum number.

4.6. Possibilities for conversions between isomers

This unit discusses a mechanism that might transfer energy between isomer and a mechanism that might transfer net elementary fermion handedness between the two isomers in an isomer-pair.

Equation (68) symbolizes an interaction in which an isomer-l_I object transits from a state X_{l_I} to a state Y_{l_I} and the interaction produces a pair of isomer-l'_I elementary fermions. The symbol FLH denotes a left-handed fermion elementary particle and the symbol FRH denotes a right-handed fermion elementary particle. The left-handed fermion elementary particle and the right-handed fermion elementary particle are antiparticles for each other.

$$X_{l_I} \to Y_{l_I} + FLH_{l'_I} + FRH_{l'_I} \tag{68}$$

For each one of 1L, 2L, and 3L, extending table 8 would result in at least one intrinsic Σg" solution-pair for which $R_I = 6$, there is a one-step-cascade solution-pair for which $6 \in Z_\Gamma$, and there is a one-step-cascade solution-pair for which $8 \in Z_\Gamma$. Examples of such intrinsic solution-pairs include 1g2‘3‘4‘5‘7x, 2g1‘3‘4‘5‘7x, and 3g1‘2‘4‘5‘7x.

SUPP suggests that interactions that associate with equation (68) can result in any isomer-l_1 creating stuff that associates with any isomer-l'_1. The notion that 16-pole pertains regarding the relevant solution-pairs might suggest that such interactions occur mainly in situations in which stuff is dense (or, mainly, in the early history of the universe).

Equation (69) symbolizes an interaction in which an isomer-l_I matter-and-antimatter pair of similar elementary fermions produce an isomer-l_I left-handed elementary particle and an isomer-l'_I right-handed similar elementary particle. (Similar interactions could produce $FLH_{l'_I} + FRH_{l_I}$.) Here, based on the notion - in table 7 - of conservation (for each one of the three isomer-pairs) of elementary fermion f_{l-r}, SUPP suggests that $|l_I - l'_I| = 3$.

$$FLH_{l_I} + FRH_{l_I} \rightarrow FLH_{l_I} + FRH_{l'_I} \tag{69}$$

For each one of 1L and 3L, table 4 lists at least one intrinsic Σg' solution-pair for which both $R_I \geq 2$ and there is a one-step-cascade solution-pair for which $6 \in Z_\Gamma$. Examples of such intrinsic solution-pairs include 1g1'2'4 and 3g3. For 1g1'2'4, there is also a one-step-cascade solution-pair for which $8 \in Z_\Gamma$.

For l_I=0 and $l'_I = 3$, equation (69) associates with a decrease in the number of isomer-zero right-handed elementary fermions and an increase in the number of isomer-three right-handed elementary fermions.

4.7. Eras in the evolution of the universe

This unit discusses SUPP-suggested notions regarding eras - including possible eras before inflation and known eras after inflation - in the evolution of the universe.

Reference [72] discusses CC notions regarding cyclic cosmology. SUPP includes the possibility that the present universe arose from an implosion of energy. SUPP does not yet consider either aspects that may have created the energy that would have imploded or whether the present universe might eventually implode.

Reference [38] discusses possibilities that might lead to a Big Bang.

CC suggests three eras in the rate of expansion of the universe. The eras feature, respectively, rapid expansion; continued expansion, with the rate of expansion decreasing; and continued expansion, with the rate of expansion increasing.

SUPP suggests using the notion of eras regarding the separating from each other of clumps - that, today, POST would consider to be large - of stuff. Examples of such clumps might include galaxy clusters and even larger clumps. SUPP suggests (based on equation (19) and equation (20)) that, for a pair of similar objects that always move away from each other, the dominating gravitational effects transit (over time) all or a portion of the following sequence: 16-pole attracting, octupole repelling, quadrupole attracting, dipole repelling, and monopole attracting.

Table 13 discusses six possible eras regarding the rate of separating of large clumps. (References [73], [23], [39], and [74] discuss the possible inflationary epoch. References [75], [76], [77], and [78] provide data and discussion about the two multi-billion-years eras. Reference [40] discusses attempts to explain the rate of expansion of the universe.)

SUPP suggests that some SUPP notions regarding eras that follow the inflationary epoch might not necessarily depend significantly on SUPP notions regarding the inflationary epoch or regarding eras that might precede the inflationary epoch.

This paper does not try to explore the possibility that (or to estimate a time at which) a transition - for the largest observable objects - from repelling based on 2g2'4 to attracting based on 2g2 might occur.

4.8. Baryon asymmetry

This unit discusses a SUPP-suggested mechanism that might have led to POST notions of baryon asymmetry.

The two-word term baryon asymmetry associates with the POST notion that - regarding known stuff - there are many more left-handed (or matter) fermion elementary particles than right-handed (or antimatter) fermion elementary particles. CC suggests that baryon asymmetry arose early in the history of the universe. From the perspective of SUPP, such known stuff associates with isomer-zero.

Discussion related to equation (69) points to a means that may have produced baryon asymmetry. Possibly, POST notions of lasing pertained regarding relevant excitations of LRI fields. SUPP suggests that processes leading to baryon asymmetry led to isomer-three stuff having fewer left-handed (or antimatter, from the perspective of isomer-three) fermion elementary particles than right-handed (or matter, from the perspective of isomer-three) fermion elementary particles.

Table 13: Six possible eras regarding the rate of separating of large clumps. The rightmost three columns suggest eras. The leftmost four columns describe phenomena that SUPP suggests as noteworthy causes for the eras. Generally, a noteworthy cause associates with notions of acceleration. Generally, an era associates with a range of velocities. The symbol → associates with the notion that a noteworthy cause may gain prominence before an era starts. The two-element term intrinsic s-p abbreviates the two-element phrase intrinsic solution-pairs. Subsequent rows associate with later eras. CC suggests notions of a Big Bang (or - at least - of a time that CC associates with the two-word term Big Bang). The symbol † denotes a possible association between the relevant era and some CC notions of a Big Bang. SUPP points to the possibility for the first two eras that the table discusses. Intrinsic use of the solution-pair 0g1‘2‘3‘4‘8 associates with the Pauli exclusion principle (and with the might-be jay boson). The other intrinsic solution-pairs to which the table alludes associate with gravitation. CC uses the word inflation (or, the two-word term inflationary epoch) to name the era that associates with the third row in the table. CC suggests that the inflationary epoch started about 10^{-36} seconds after the Big Bang. CC suggests that the inflationary epoch ended between 10^{-33} seconds after the Big Bang and 10^{-32} seconds after the Big Bang. Possibly, no direct evidence exists for the inflationary epoch. TBD denotes to be determined. The following notions pertain regarding the column with the one-word label notes. The symbol 1 denotes the notion that CC interpretations of data support the notions of each one of the two billions-of-years eras. The symbol 2 denotes the notion that CC suggests the era. The symbol 3 denotes the notion that SUPP suggestions regarding resolving CC tensions (between data and modeling) that associate with the fifth row do not necessarily depend on the existence of the era.

Force	Intrinsic s-p	PPS-pole	R_I	→	Rate of separating	Duration	Notes
Attractive	2g1‘2‘3‘4‘8x	16-pole	6	→	Is negative	TBD	3
Repulsive	0g1‘2‘3‘4‘8	16-pole	1	→	Turns positive †	TBD	3
Repulsive	2g1‘2‘3‘4x	Octupole	1	→	Increases rapidly	Less than a second	2, 3
Attractive	2g1‘2‘3	Quadrupole	1	→	Decreases	Billions of years	1
Repulsive	2g2‘4	Dipole	2	→	Increases	Billions of years	1
Attractive	2g2	Monopole	6	→	Would decrease	-	3

This paper does not address the topic of the extent to which steps leading to a predominance in isomer-zero stuff of left-handed elementary particles (and not to a predominance of right-handed elementary particles) have a basis other than random chance.

4.9. Evolution of stuff that associates with dark matter isomers

This unit discusses SUPP-suggested notions about the evolution of stuff that associates with the five isomers that SUPP associates with dark matter.

SUPP uses the two-element term isomer-l_I stuff to denote objects (including hadron-like particles, atom-like objects, and stars) that associate with the isomer-l_I.

4.9.1. Evolution of isomer-1, isomer-2, isomer-4, and isomer-5 stuff

Here, SUPP uses the one-element term alt-isomer to designate an isomer other than isomer-zero and isomer-three.

For each one of the six isomers, a charged baryon that includes exactly three flavour 3 quarks is more massive than the counterpart zero-charge baryon that includes exactly three flavour 3 quarks. (For example, the hadron that includes just two tops and one bottom has a larger total mass than does the hadron that includes just one top and two bottoms.)

Per table 11 and table 12, alt-isomer flavour 3 charged leptons are less massive than isomer-zero flavour 3 charged leptons. When flavour 3 quark states are much populated (and based on interactions mediated by W bosons), the stuff that associates with an alt-isomer converts more charged baryons to zero-charge baryons than does the stuff that associates with isomer-zero. Eventually, regarding the stuff that associates with the alt-isomer, interactions that entangle multiple W bosons result in the stuff that associates with the alt-isomer having more neutrons and fewer protons than does the stuff that associates with isomer-zero. The sum of the mass of a proton and the mass of an alt-isomer flavour 1 charged lepton exceeds the mass of a neutron. Compared to isomer-zero neutrons, alt-isomer neutrons scarcely decay. The IGM (or, intergalactic medium) that associates with the alt-isomer scarcely interacts with itself via electromagnetism.

4.9.2. Evolution of isomer-3 stuff

The following two possibilities pertain. In one possibility, the evolution of isomer-three stuff parallels the evolution of isomer-zero stuff. In the second possibility, the evolution of isomer-three stuff does not parallel the evolution of isomer-zero stuff. The second possibility might associate with - for example - a difference in handedness - with respect to charged leptons or with respect to W bosons - between isomer-three and isomer-zero.

This paper nominally assumes that the evolution of isomer-three stuff parallels the evolution of isomer-zero stuff.

4.10. Tensions - among data and models - regarding large-scale phenomena

This unit discusses SUPP-suggested notions that might help resolve so-called tensions - between data and POST CC - regarding the rate of expansion of the universe, regarding large-scale clumping of matter, and regarding gravitational interactions between neighboring galaxies.

4.10.1. The rate of expansion of the universe

Table 13 lists two known eras in the history of the universe.

CC underestimates - for the second multi-billion-years era - increases in the rate of expansion of the universe. (References [43], [44], [45], [46], [79], [80], [81], and [82] provide further information. Reference [83] suggests that the notion that DM is similar to OM might help resolve the relevant tension. Reference [84] discusses various possible resolutions. Reference [85] provides data about the Hubble constant.)

SUPP suggests the following explanation for such underestimates.

When using modeling based on GR, CC might try to extend the use of an equation of state (or the use of a cosmological constant) that works well regarding early in the first multi-billion-years era. Regarding the first multi-billion-years era, SUPP suggests dominance by attractive effects that associate with intrinsic use of the 2g1‘2‘3 component of gravity. The notion of a reach of one pertains. The symbol 2(1)g1‘2‘3 pertains. SUPP suggests that - later in the first multi-billion-years era - repulsive effects that associate with intrinsic use of 2(2)g2‘4 become significant. Dominance by intrinsic 2(2)g2‘4 pertains by the time the second multi-billion-years era starts. However, use of an equation of state that has roots in the time period in which intrinsic 2(1)g1‘2‘3 dominates might - at best - extrapolate based on a notion of intrinsic 2(1)g2‘4 (and not based on a notion of intrinsic 2(2)g2‘4) and would underestimate the strength of the key gravitational driver - of expansion - by a factor of two.

SUPP points - conceptually - to the following possible remedy.

CC might change (regarding the stress-energy tensor or the cosmological constant) the aspects that would associate with repelling and the 2g2‘4 component of gravity. The contribution - to the pressure - that associates with intrinsic use of 2g2‘4 might need to double (compared to the contribution that would associate with intrinsic use of 2(1)g2‘4).

4.10.2. Large-scale clumping of matter

CC overestimates large-scale clumping of matter - OM and DM. (References [86], [87], [88], and [46] provide data and discussion.)

SUPP suggests that CC modeling associates with a repulsive component - 2(1)g2‘4 - of gravity. SUPP suggests that 2(2)g2‘4 pertains. (That is, for each instance of 2g2‘4, a reach of two isomers pertains.) The additional (compared to CC modeling) repelling might explain the overestimating that CC suggests.

4.10.3. Effects - within galaxies - of the gravity associated with nearby galaxies

CC might not account for some observations about effects - within individual galaxies - of the gravity associated with nearby galaxies. (Reference [61] provides further information.)

SUPP suggests that CC modeling associates primarily with an attractive component - 2(6)g2 - of gravity. SUPP suggests, regarding the repulsive component - 2g2‘4 - of gravity, that POST CC modeling associates with SUPP notions of intrinsic 2(6)g2‘4. SUPP suggests that intrinsic 2(2)g2‘4 pertains. The reduced (compared to CC modeling) repelling might explain at least some aspects of the data that reference [61] discusses.

4.11. Formation, evolution, and composition of galaxies

This unit discusses SUPP-suggested notions regarding the formulation, evolution, and composition of galaxies.

4.11.1. Mechanisms regarding the formation and evolution of galaxies

Reference [48] suggests that galaxies form around early clumps of stuff. Reference [48] associates the word halo with such clumps.

SUPP suggests that each one of many early halos associates with one isomer. SUPP associates with such early halos the three-element term one-isomer original clump. Clumping occurs based on gravitational effects. Differences - between the evolution of stuff associating with any one of isomer-zero and isomer-three and the evolution of stuff associating with any one of isomer-one, isomer-two, isomer-four, and isomer-five are not necessarily significant regarding this gravitationally based clumping. The six isomers might form such clumps approximately equally.

Table 14: Stages and other information regarding the evolution of a galaxy for which a notion of a one-isomer original clump pertains. The table suggests stages, with subsequent rows associating with later stages. The next-to-rightmost column describes aspects of the stage. The leftmost three columns in the table describe a component of 2L that is a noteworthy cause for the stage. The two-element term intrinsic s-p abbreviates the two-element phrase intrinsic solution-pairs. The symbol → associates with the notion that a noteworthy cause may gain prominence before a stage starts. Table 14 associates with a scenario in which a galaxy forms based on one original one-isomer clump and initially does not significantly collide with other galaxies. The galaxy might retain some stuff that associates with the repelled isomer. The rightmost column in table 14 suggests terminology regarding the evolution of galaxies. (A galaxy can include stuff from more than one earlier galaxy.)

Force	Intrinsic s-p	R_I	→	Stage	Aspects of the stage	Era
Attractive	2g1‘2‘3	1	→	1	A one-isomer original clump forms.	First
Repulsive	2g2‘4	2	→	2	The original clump repels (some) stuff that associates with the isomer that associates with the original clump and (most) stuff that associates with one other isomer.	First
Attractive	2g2	6	→	3	The original clump attracts stuff that associates with the four not-repelled isomers and stuff that associates with the isomer that associates with the original clump.	Second
Attractive	2g2	6	→	4	Another galaxy subsumes the original clump and might subsequently merge with yet other galaxies.	Third

Table 14 discusses SUPP suggestions regarding the formation and evolution of a galaxy for which a notion of a one-isomer original clump pertains.

Presumably, some galaxies form based on two or more clumps, for which all the clumps associate with just one isomer. Possibly, some galaxies form based on two or more clumps, for which some clumps associate with isomers that are not the same as the isomers that associate with some other clumps.

4.11.2. Aspects regarding the evolution of galaxies

Table 14 suggests three eras regarding the evolution of galaxies. The first era associates with the first two rows in table 14. The second era associates with the 2g2 attractive force that associates with the third row in table 14. The third era associates with collisions between and mergers of galaxies.

SUPP suggests the possibility that some galaxies do not exit the first era and do not significantly collide with other galaxies.

SUPP suggests that some galaxies result from aspects associating with the 2g2 attractive force that associates with the third row in table 14. Here, this paper discusses three cases. (Mixed cases and other cases might pertain.)

- Each one of some era-one galaxies does not collide with other galaxies. Such a galaxy accumulates (via 2g2 attracting) stuff associating with various isomers that have representation in nearby IGM. The galaxy becomes an era-two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.

- Each one of some era-two galaxies merges (via 2g2 attracting) mainly just with galaxies that feature the same five isomers. The galaxy that merged, in effect, loses it status of being a galaxy. The resulting larger object is an era-two galaxy. The galaxy might include stuff that significantly associates with as many as five isomers.

- Each one of some era-one or era-two galaxies merges (via 2g2 attracting) with other galaxies. The galaxy that merged, in effect, loses its status of being a galaxy. The resulting larger object is an era-three galaxy. The galaxy might include stuff that significantly associates with as many as six isomers.

4.11.3. Amounts of dark matter in galaxies

POST CC suggests the possibility that DM associates with an explanation for phenomena that associate with the three-element term galaxy rotation curves. Absent an explanation, CC modeling would suggest that stuff rotating around the center of a galaxy would associate with lower velocities than the velocities that observations suggest.

SUPP is not incompatible with the possibility that DM associates with such an explanation. However, SUPP suggests that models based on POST CC might overestimate the amount of DM in some galaxies.

From the perspective of SUPP, POST CC modeling (that associates with galaxy rotation curves) associates with $R_I = 6$ for all (2g’) components that contribute to gravitational phenomena. However, SUPP suggests that - for intrinsic use of 2g2‘4 - $R_I = 2$. SUPP associates intrinsic use of 2g2‘4 with rotation and with dilution of the attraction that associates with 2g2. $R_I = 2$ would associate with less effect than would $R_I = 6$. Here, less effect associates with more gravitational attraction.

SUPP suggests that POST might consider the greater gravitational attraction to be a MOND (as in MOdified Newtonian Dynamics) adjustment (regarding gravity) that might account for some aspects that associate with galaxy rotation curves.

SUPP suggests - for each one of some galaxies - that the galaxy might include - within almost any (not adequately small) radial distance from the center of the galaxy - less DM than POST would currently estimate based on the observed rotation speed that associates with the distance.

4.12. Ratios of dark matter effects to ordinary matter effects

This unit suggests that SUPP notions comport with and explain various observed ratios of dark matter effects to ordinary matter effects. POST CC seems not to explain the ratios.

Table 15 lists observed ratios of dark matter effects to ordinary matter effects and alludes to SUPP notions that seem to explain the ratios. Discussion below defines terms such as dissimilar evolution regarding isomeric stuff, isomer-zero dark matter, and misinterpreted data.

4.12.1. Ratios that might pertain regarding the cosmic electromagnetic background

Table 15a lists ratios - that pertain to light that dates to about 400,000 years after the Big Bang - of observed effects to effects that POST modeling estimates. The acronym CMB abbreviates the three-word term cosmic microwave background (or, the four-word term cosmic microwave background radiation). (References [89], [90], [91], and [92] provide data and discussion regarding the amount of cosmic optical background. References [93], [94], and [95] provide data and discussion regarding absorption of CMB.)

The following two paragraphs provide SUPP-suggested explanations for the observations to which table 15a alludes.

The three-word phrase cosmic optical background associates with now nearly-optical light remaining from early in the universe. CC suggests that atomic transitions produced radiation that today measures as cosmic (optical and microwave) background radiation. SUPP associates POST atomic transitions with isomer-zero. Observations found twice as much light as CC expected. SUPP suggests that isomer-one, isomer-two, isomer-four, and isomer-five stuff did not result in much stuff that is similar to isomer-zero atoms. SUPP suggests that isomer-three stuff evolved similarly to isomer-zero stuff. For four types of changes in atomic energy levels, table 8 alludes to 1L-producing events that associate with $R_I = 2$. SUPP suggests that such events explain the two-to-one observed-to-expected ratios regarding the cosmic optical background. Isomer-zero (or, OM) stuff produced half of the observed light. Isomer-three (or, DM) stuff produced half of the observed light.

The four-element phrase some absorption of CMB associates with the notion that measurements of some specific depletion of CMB indicate twice as much depletion as CC expected based solely on hyperfine interactions with (isomer-zero) hydrogen atoms. SUPP suggests (per table 8) that isomer-three (or, DM) hydrogen-like atoms account for the half of the absorption for which isomer-zero (or, OM) hydrogen atoms do not account.

4.12.2. Ratios that pertain for some galaxies

Table 15b suggests explanations for some ratios - that pertain to some galaxies - of DM effects to OM effects. (References [96] and [97] provide data and discussion. Reference [96] influenced the choice - that this paper reflects - of a time range to associate with the word early. Regarding the combination of 0^+:1 and later, references [98], [99], [100], [101], [102], [103], and [104] provide data and discussion. Reference [105] discusses a galaxy that might have started as containing mostly OM. Reference [106] discusses a DM-deficient galaxy. Regarding observed DM galaxies, references [48], [107], [108], and [109] provide data and discussion. Current techniques might not be capable of observing early DM galaxies. References [110] and [111] suggest, regarding galaxy clusters, the existence of clumps of DM that might be individual galaxies. Extrapolating from results that references [48] and [112] discuss regarding ultrafaint dwarf galaxies that orbit the Milky Way galaxy might suggest that the universe contains many DM:OM $1 : 0^+$ later galaxies. Reference [113] discusses a trail of galaxies for which at least two galaxies have little DM. Reference [113] suggests that the little-dark-matter galaxies result from a collision that would

Table 15: Observed ratios of dark matter effects to ordinary matter effects and SUPP notions that seem to explain the ratios.

(a) Ratios - that pertain to light that dates to about 400,000 years after the Big Bang - of observed effects to effects that POST estimates. The three-word phrase cosmic optical background associates with radiation that - recently - measures as optical radiation or measures as close (with respect to wavelengths) to optical radiation. The acronym CMB associates with radiation that - recently - measures as cosmic microwave background radiation. DM:OM denotes a ratio of DM effects to OM effects that this paper suggests.

Aspect	Observed : POST-CC-expected	SUPP-suggested DM:OM
Amount of cosmic optical background	2 : 1	1 : 1
Some absorption of CMB	2 : 1	1 : 1

(b) Suggested explanations for some ratios - that pertain to some galaxies - of DM effects to OM effects. DM:OM denotes a ratio of DM effects to OM effects. Inferences of DM:OM ratios come from interpreting data. Regarding galaxies, the notion of early associates with observations that pertain to galaxies that associate with (or, would, if people could detect the galaxies, associate with) high redshifts. High might associate with $z > 7$ and possibly with smaller values of z. Here, z denotes redshift. The word later associates with the notion that observations pertain to objects later in the history of the universe. The two-element term DM galaxy denotes a galaxy that contains much less OM than DM. Possibly, people have yet to directly detect early DM galaxies. Table 14 provides information about the explanations.

Objects	DM:OM	Examples	Explanation
Some early galaxies	$0^+ : 1$	Reported	OM original clump. Stage-1 or stage-2.
Some later galaxies	$0^+ : 1$	Reported	OM original clump. Stage-1 or stage-2.
Some early galaxies	$1 : 0^+$	No known reports	DM-isomer(s) original clump. Stage-1 or stage-2.
Some later galaxies	$1 : 0^+$	Reported	DM-isomer(s) original clump. Stage-1 or stage-2.
Some later galaxies	$\sim 4 : 1$	Reported	Non-isomer-three original clump. Stage-3.
Many later galaxies	$5^+ : 1$	Reported	Any-isomer(s) original clump(s). Stage-4.

(c) Suggested explanations for observed ratios - that pertain to larger-than-galaxies-scale phenomena - of DM effects to OM effects. DM:OM denotes a ratio of DM effects to OM effects. More than one explanation might pertain. If none of these or other explanations pertained, SUPP would suggest DM:OM ratios of 5 : 1.

Aspect	DM:OM	Comment
Densities of the universe	$5^+ : 1$	Each one of dissimilar evolution regarding isomeric stuff, isomer-zero dark matter, and misinterpreted data might pertain.
Some galaxy clusters	$5^+ : 1$	SUPP suggests that galaxy clusters (that have not collided with other galaxy clusters) associate with DM:OM ratios that are similar to DM:OM ratios for densities of the universe. (Similarity to DM:OM ratios for many stage-4 galaxies also pertains.)

have some similarities to the Bullet Cluster collision. Regarding galaxies for which DM:OM ratios of ∼4:1 pertain, references [114] and [115] provide data and discussion. Regarding later galaxies for which DM:OM ratios of 5^+:1 pertain, reference [48] provides data and discussion. References [116] and [117] provide data about collisions of galaxies.)

Table 15b does not rule out the notion that galaxies somewhat fully populate DM:OM ranges within the interval of $0 : 1$ to, say, $6 : 1$. For DM:OM ratios of less than (say) ten, table 14 suggests that each range of DM:OM ratios to which table 15b alludes might stand out statistically (in terms of numbers of galaxies) from ranges (of positive-number ratios) near to the range to which table 15b alludes.

Table 15b does not rule out the notion that galaxies somewhat fully populate a DM:OM range of $10^{p_1} : 1$ to $10^{p_2} : 1$ for which SUPP does not suggest a value of p_1; p_1 exceeds, say, three; p_2 exceeds p_1; and SUPP does not suggest a value of p_2.

4.12.3. Ratios that pertain regarding phenomena that are bigger than galaxies

SUPP suggests the following aspects that might contribute toward the notion that measurements of large-scale presences of DM might exceed five times measurements of large-scale presences of OM.

- Dissimilar evolution regarding isomeric stuff. Here, the term alt-isomer refers to isomer-one, isomer-two, isomer-four, or isomer-five. The evolution of alt-isomer stuff might deviate - compared to the evolution of isomer-zero stuff - early enough that (nominally) isomer-zero high-energy excitations of the electromagnetic field produce alt-isomer stuff significantly more copiously than (nominally) alt-isomer excitations of the electromagnetic field produce isomer-zero stuff. (Discussion related to equation (68) pertains.)
- Isomer-zero dark matter. POST CC suggests notions - such as notions of primordial black holes or yet-to-be-found elementary particles - of stuff that might measure as DM and (in the context of SUPP) associate mainly with isomer-zero stuff. (SUPP does not necessarily suggest isomer-zero elementary particles that would associate with notions of DM.)
- Misinterpreted measurements. Interpretations of measurements might - based on notions that, for example, the $R_I = 2$ for intrinsic 2g2‘4 differs from the $R_I = 6$ for intrinsic 2g2 and for extrinsic 2g2‘4 - might lead to inferred ratios of DM effects to OM effects that do not associate exactly with actual ratios of DM stuff to OM stuff.

Table 15c suggests explanations for observed ratios - that pertain to larger-than-galaxies-scale phenomena - of DM effects to OM effects. (Reference [66] provides data and discussion regarding densities of the universe. References [118], [119], [120], and [121] provide data and discussion regarding galaxy clusters.)

4.12.4. Aspects related to collisions of pairs of galaxy clusters

Reference [71] discusses the Bullet Cluster collision of two galaxy clusters.

CC suggests two general types of trajectories for stuff. Most DM - from either one of the clusters - exits the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. Also, OM stars - from either cluster - exit the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. However, OM IGM - from either cluster - lags the cluster's OM stars and DM. CC suggests that the OM IGM interacted electromagnetically with the other cluster's OM IGM, as well as gravitationally with the other cluster.

SUPP suggests that SUPP might comport (regarding each cluster) with the interpretations of data, with one possible exception. The possible exception associates with the notion that SUPP suggests that isomer-three IGM interacts electromagnetically and follows trajectories that are consistent with OM IGM trajectories.

Regarding the possible exception, at least three possibilities arise.

For one possibility, per table 8, the light that CC associates with OM IGM might include light that SUPP associates with OM IGM and light that SUPP associates with isomer-three IGM.

For one possibility, isomer-three IGM measures as DM and CC does not adequately report (or otherwise account for) lagging isomer-three IGM.

For one possibility, isomer-three IGM follows trajectories that are consistent with other DM trajectories.

SUPP suggests that interpretations of data may not be sufficient to rule out each one of the first two possibilities or to rule out a combination of the first two possibilities.

SUPP notions of DM are not necessarily incompatible with constraints - that have bases in observations of collisions of galaxy clusters - regarding DM.

5. Discussion

This unit suggests possible additional (compared to previous units) associations between possible future data and SUPP modeling. This unit suggests possible additional (compared to previous units) associations between POST modeling and SUPP modeling. This unit suggests (based on POST modeling and SUPP modeling) bases that might point toward additional (compared to previous units) principles or other notions that – in the future – might underlie physics or physics modeling.

5.1. *Possible elementary particles that POST has yet to include*

This unit suggests possible insight regarding the existence and properties of some as-yet-unfound elementary particles that POST hypothesizes or that SUPP might suggest.

5.1.1. *Elementary particles that SUPP suggests*

Table 9 catalogs as-yet-unfound elementary particles that SUPP suggests.

5.1.2. *Right-handed W boson*

Reference [122] discusses a fraction of decays - of OM top quarks for which the decay products include W bosons - that might produce right-handed W bosons. The fraction, f_+, is 3.6×10^{-4}. Reference [22] provides a confidence level of 90 percent that the rest energy of a might-be W_R (or, right-handed W boson) exceeds 715 GeV. Reference [123] provides other information.

SUPP suggests that W_R bosons associate only with isomers one, three, and five. SUPP suggests possibilities for inter-isomer interactions and conversions.

Aspects of SUPP might approximately reproduce the above result that SM modeling suggests.

Aspects related to equation (44) suggest values of calculated masses that do not associate with masses of known or suggested elementary particles. For example, SUPP does not suggest that $m(5,3)$ associates with the inertial mass of an isomer-one charged lepton. However, perhaps such mass-like quantities associate with some measurable aspects of nature. For charged leptons and $0 \leq l_I \leq 4$ and $0 \leq l'_f \leq 3$, $m(3(l_I+1)+l'_f,3) = \beta m(3(l_I+0)+l'_f,3)$. One might conjecture that isomer-zero observations of some aspects of isomer-one phenomena associate with notions of non-inertial mass-like quantities that are β times the inertial masses for isomer-zero elementary particles (and that are β times inertial masses for the counterpart isomer-one elementary particles).

Based on notions of scaling that might calculate non-inertial mass-like quantities, SUPP might suggest that $f_+ \sim e^{(\beta^{-1})} - 1 \approx \beta^{-1} \approx 2.9 \times 10^{-4}$. This estimate might not be incompatible with results that reference [122] discusses. A notion of $m_{\text{non-inertial,W}_R\text{isomer one}}c^2 = \beta m_{\text{W}}c^2 \approx 2.8 \times 10^5$ GeV might pertain. Here, the notion of a non-inertial mass-like quantity might associate with data that associate with interactions that associate with 1L or 1W_1. The interactions do not necessarily associate directly with 2L.

5.1.3. *Magnetic monopole*

Table 2 seems not to suggest a 1L interaction with a monopole other than an electric monopole. SUPP does not suggest a property that would associate with a magnetic monopole.

5.2. *Phenomena that might involve the SUPP-suggested jay boson elementary particle*

This unit discusses phenomena that might associate with the SUPP-suggested jay boson.

5.2.1. *Pauli repulsion*

POST includes the notion that two identical fermions cannot occupy the same state. Regarding QM, one notion is that repelling between identical fermions associates with overlaps of wave functions. Another QM notion features wave functions that are antisymmetric with respect to the exchange of two identical fermions.

SUPP might be compatible with such aspects of POST and, yet, not necessitate - regarding POST dynamics modeling - the use of wave functions. QM based on jay bosons might suffice. CM based on potentials that would associate with effects of jay bosons might suffice.

SUPP suggests that QM or CM based on jay bosons might suggest that the prevention of two identical fermions from occupying the same state might associate with, in effect, interactions - mediated by jay bosons - that try to change aspects related to the fermions. Notions of changing a spin orientation might pertain. For elementary fermions, notions of changing a flavour might pertain.

5.2.2. Energy levels in positronium

Reference [124] discusses the transition - between two states of positronium - characterized by the expression that equation (70) shows.

$$2^3S_1 \to 2^3P_0 \tag{70}$$

Four standard deviations below the nominal observed value of the energy that associates with the transition approximately equals four standard deviations above the nominal value of the energy that POST suggests.

SUPP notions regarding jay bosons might explain the might-be discrepancy regarding positronium. Compared to QFT, a new notion of virtual charge exchange or a new notion of virtual flavour change might pertain.

To the extent that QFT does not suffice to explain positronium energy levels, SUPP notions related to the jay boson might help to close the gap between observations and modeling.

5.2.3. Pauli crystals

Reference [125] reports detection of Pauli crystals. SUPP suggests that modeling based on the notion of jay bosons might help explain relevant phenomena.

5.3. Some phenomena that associate with galaxies

This unit discusses SUPP-suggested notions that might help explain some phenomena that associate with some galaxies.

5.3.1. Some quenching of star formation

Some galaxies seem to stop forming stars. (Reference [126] and reference [127] discuss examples.) Such quenching might take place within three billion years after the Big Bang, might associate with a lack of hydrogen atoms, and might (per reference [127]) pertain to half of the galaxies that associate with the notion of a certain type of galaxy.

SUPP suggests that some such quenching might associate with repelling that associates with 2g2‘4. Some quenching might associate with galaxies for which original clumps featured isomer-zero stuff or isomer-three stuff.

5.3.2. Some stopping of the accrual of matter

Reference [128] discusses a galaxy that seems to have stopped accruing both OM and DM about four billion years after the Big Bang.

The galaxy that reference [128] discusses might (or might not) associate with the notion of significant presence early on of one of isomer-zero and isomer-three, one of isomer-one and isomer-four, and one of isomer-two and isomer-five. Such early presences might associate with a later lack of nearby stuff for the galaxy to accrue.

5.3.3. Aspects regarding stellar stream GD-1 in the Milky Way galaxy

Data regarding stellar stream GD-1 suggest the possibility of effects from a yet-to-be-detected non-OM clump - in the Milky Way galaxy - with a mass of 10^6 to 10^8 solar masses. (References [129] and [130] provide data and discussion regarding the undetected object. Reference [130] cites reference [131] and reference [132].) SUPP suggests that the undetected object might be a clump of DM.

5.4. Some possibilities for detecting non-isomer-zero dark matter

This unit discusses some possibilities for detecting dark matter that does not associate with isomer-zero stuff.

Table 8 points to electromagnetic phenomena that associate with reaches of two and, thereby, suggests that OM equipment might be able to catalyze or detect transitions within isomer-three atoms. Discussion related to table 15a suggests that data point to detection, by OM equipment, of light emitted by transition events that associate with isomer-three atoms. Presumably, some isomer-three atoms pass (essentially unimpeded by isomer-zero stuff) through isomer-zero stuff that is near to and includes the Earth. SUPP suggests that experiments - based on light produced by OM stuff and light detected by OM stuff - might be able to detect (via transition events that associate with isomer-three atoms) isomer-three atomic stuff. This paper does not discuss notions regarding whether techniques are now or when techniques might become sufficiently sensitive that such experiments would be feasible.

5.5. Some information that gravitational waves might convey

This unit suggests that adequately detailed analyses of the gravitational signatures that associate with collisions of objects - such as black holes - might enable the development of data that associate with the extents to which the colliding objects include stuff that associates with more than one isomer or more than one isomer-pair.

Reference [133] discusses opportunities for research regarding gravitational waves.

Extending notions that associate with table 8 suggests intrinsic uses of 2g" solution-pairs that might associate with the producing of gravitational waves.

SUPP suggests that intrinsic use of the 2g1'3'4 solution-pair might be relevant. Intrinsic use of the 2g1'3'4 solution-pair associates with a reach of six. SUPP suggests that intrinsic use of 2g1'3'4 associates with significant aspects of the production of gravitational waves.

Intrinsic use of 2g4'5'7 associates with a reach of one. Intrinsic use of any one of 2g1'4'5, 2g3'4'5, and 2g1'4'7 associates with a reach of two. To the extent that intrinsic use of at least one such 2g" solution-pair is relevant regarding the producing of gravitational waves, SUPP suggests that adequately detailed analyses of the gravitational signatures that associate with collisions of objects - such as black holes - might enable the development of data that associate with the extents to which the colliding objects include stuff that associates with more than one isomer or more than one isomer-pair.

5.6. Modeling regarding gravity

This unit discusses some notions regarding the accuracy of using modeling based on GR.

Present GR is not necessarily adequately compatible with data.

Present GR is not necessarily adequately compatible with SUPP notions of DM. SUPP suggests that POST modeling based on GR can be less than adequately accurate.

Tests of GR have featured phenomena that associate with the isomer-pair that includes isomer-zero and isomer-three. Each one of the Sun, the planet Mercury, and the Earth associates with isomer-zero. Relevant radiation from distant stars and galaxies associates essentially just with isomer-zero stuff and isomer-three stuff.

For cases in which POST suggests that uses of general relativity adequately (or nearly adequately) comport with data, SUPP suggests that the following notions - about uses of SUPP solution-pairs and about the GR stress-energy tensor - might help bridge from SUPP to GR or from GR to SUPP. (The GR stress-energy tensor is symmetric.) Intrinsic 2g2 associates with the one stress-energy-tensor component (T^{00}) that associates with energy density. Extrinsic 2g2'4 associates with the three components (T^{01}, T^{02}, and T^{03}) that associate with momentum density and also associates with the three components (T^{10}, T^{20}, and T^{30}) that associate with energy flux. Intrinsic 2g2'4 associates with the three components (T^{11}, T^{22}, and T^{33}) that associate with pressure. Each one of extrinsic 2g2'4'8 and intrinsic 2g1'2'3 associates with the three components (T^{21}, T^{31}, and T^{32}) that associate with momentum flux and also associates with the three components (T^{12}, T^{13}, and T^{23}) that associate with shear stress.

To the extent that octupole components - such as extrinsic 2g1'2'3'4x - have significant roles, GR might be less than adequately accurate.

This paper does not further explore the usefulness of such notions.

5.7. Applications of a series of formulas for lengths

This unit suggests a series of formulas for lengths (including the Schwarzschild radius and the Planck length) and discusses phenomena that might associate with lengths that the formulas suggest.

Equations (71), (72), and (73) define a set of lengths r_l. m denotes a mass. r_0 has dimensions of length. POST associates the two-word term Planck length with r_0. b has the dimensions of a number.

$$r_0 = (G_N)^{1/2} m^0 \hbar^{1/2} c^{-3/2} 2^0 \tag{71}$$

$$b = (G_N)^{-1/2} m^{-1} \hbar^{1/2} c^{1/2} 2^{-1} \tag{72}$$

$$r_l = r_0 b^l \tag{73}$$

POST associates the two-word term Schwarzschild radius with r_1. Equation (74) pertains.

$$r_1 = (G_N)^1 m^1 \hbar^0 c^{-2} 2^1 \tag{74}$$

For a charged pion, equation (73) yields $r_{-1} \sim 0.7 \times 10^{-15}$ meters. Reference [134] states a value of $(0.640 \pm 0.007) \times 10^{-15}$ meters for a measured charge radius of a charged pion.

For a Z boson, equation (73) yields $r_{-1} \sim 2 \times 10^{-18}$ meters. (The r_{-1} for a W boson, is about 1.1 times the r_{-1} for a Z boson.) Reference [135] suggests that, for a separation of approximately 10^{-18} meters between two interacting particles, the weak interaction and the electromagnetic interaction have similar magnitudes. For a separation of approximately 3×10^{-17} meters, the magnitude of the weak interaction is less than the magnitude of the electromagnetic interaction by approximately a factor of 10^4.

SUPP suggests that the notion of r_{-1} has significance regarding some sizes of objects and some ranges of interactions.

5.8. Harmonic oscillator mathematics, gauge symmetries, and the Higgs mechanism

This unit discusses possible similarities between gauge symmetries that POST SM features and STS DOF symmetries that SUPP suggests. This unit discusses possible associations between the POST SM notion of the Higgs mechanism and notions that SUPP suggests.

5.8.1. Some harmonic oscillator mathematics

Modeling for a j-dimensional isotropic harmonic oscillator can feature j linear coordinates $x_{k'}$ - each with a domain $-\infty < x_{k'} < \infty$ - and an operator that is the sum - over k' - of j operators of the form that equation (75) shows. The number C is positive and is common to all j uses of equation (75). The word isotropic associates with the commonality - across all j uses of equation (75) - of the number C.

$$-\frac{\partial^2}{\partial(x_{k'})^2} + C \cdot (x_{k'})^2 \tag{75}$$

For $j \geq 2$, one can split the overall operator into pieces. Equation (76) associates with a split into two pieces. Here, each of j_1 and j_2 is a positive integer.

$$j = j_1 + j_2 \tag{76}$$

In discussion below, the symbol D might be any one of j, j_1, and j_2.

For $D \geq 2$, mathematics related to isotropic harmonic oscillators can feature partial differential equations, a radial coordinate, and $D-1$ angular coordinates. Equation (77) defines a radial coordinate.

$$x = (\sum_{k'} (x_{k'})^2)^{1/2} \tag{77}$$

SUPP suggests replacing x via the expression that equation (78) shows. Here, r_{HO} denotes the radial coordinate and has dimensions of length. The parameter η has dimensions of length. The parameter η is a nonzero real number. The magnitude $|\eta|$ associates with a scale length. (Here, r_{HO} associates with mathematics for HO - or, harmonic oscillators - and does not necessarily associate with uses of r elsewhere - for example, in equation (1) - in this paper.)

$$x = r_{HO}/\eta \tag{78}$$

Applications of equations (79) and (80) can associate with POST. Each of ξ and ξ' is an as-yet unspecified constant. The symbol $\phi_R(r_{HO})$ denotes a function of r_{HO}. The symbol ${\nabla_{r_{HO}}}^2$ denotes a Laplacian operator. Ω associates with aspects that associate with angular coordinates. (For $D = 3$, reference [136] shows a representation for Ω in terms of an operator that is a function of spherical coordinates.)

$$\xi \phi_R(r_{HO}) = (\xi'/2)(-\eta^2 {\nabla_{r_{HO}}}^2 + (\eta)^{-2} r_{HO}^2)\phi_R(r_{HO}) \tag{79}$$

$${\nabla_{r_{HO}}}^2 = r_{HO}^{-(D-1)}(\partial/\partial r_{HO})(r_{HO}^{D-1})(\partial/\partial r_{HO}) - \Omega r_{HO}^{-2} \tag{80}$$

SUPP applications assume that the symbol Ω is a constant. SUPP applications do not necessarily require that D is a positive integer for which $D \geq 2$. SUPP applications include solutions that pertain for the domain that equation (81) shows. With respect to the domain $0 \leq r_{HO} < \infty$, ϕ_R associates with the mathematics notion of having a definition almost everywhere. (Some aspects of POST applications associate with the following notions. D is a nonnegative integer. ϕ_R associates with a radial factor that is part of a representation of a wave function. For $D = 1$, equation (80) might not be appropriate. For

$D > 1$, a representation of a wave function may need to include a factor for which angular coordinates play roles. The domain for a representation of such a wave function needs to include $r_{HO} = 0$. For SUPP applications, ϕ_R does not necessarily associate with the notion of a factor in a representation for a wave function and does not necessarily need to have a definition that associates with $r_{HO} = 0$.)

$$0 < r_{HO} < \infty \tag{81}$$

In discussion below, the symbol D might be any real number.

SUPP considers solutions of the form that equation (82) shows. (In POST, solutions that associate with equation (75) and with $D = 1$ have the form $H(x)\exp(-x^2)$, in which $H(x)$ is a Hermite polynomial. Mathematics that SUPP suggests can allow for a SUPP-adequately-useful set of solutions for which each solution associates with - in effect - a one-term polynomial.)

$$\phi_R(r_{HO}) \propto (r_{HO}/\eta)^\nu \exp(-r_{HO}^2/(2\eta^2)), \text{ with } \eta^2 > 0 \tag{82}$$

Equations (83) and (84) characterize solutions. The parameter η does not appear in these equations.

$$\xi = (D + 2\nu)(\xi'/2) \tag{83}$$

$$\Omega = \nu(\nu + D - 2) \tag{84}$$

$\phi_R(r_{HO})$ normalizes if and only if equation (85) pertains. The symbol $(\phi_R(r_{HO}))^*$ denotes the complex conjugate of $\phi_R(r_{HO})$.

$$\int_0^\infty (\phi_R(r_{HO}))^* \phi_R(r_{HO}) r_{HO}^{D-1} dr_{HO} < \infty \tag{85}$$

Equation (86) associates with the domains of D and ν for which normalization pertains for $\phi_R(r_{HO})$. For $D + 2\nu = 0$, normalization pertains in the limit $\eta^2 \to 0^+$. Regarding mathematics relevant to normalization for $D + 2\nu = 0$, the delta function that equation (87) shows pertains. Here, $(x')^2$ associates with r_{HO}^2 and 4ϵ associates with η^2. (Reference [137] provides equation (87).) The difference in domains, between $-\infty < x' < \infty$ and equation (81), is not material here.

$$D + 2\nu \geq 0 \tag{86}$$

$$\delta(x') = \lim{}_{\epsilon \to 0^+} (1/(2\sqrt{\pi\epsilon})) e^{-(x')^2/(4\epsilon)} \tag{87}$$

5.8.2. Possible associations that might link SUPP notions with SM gauge symmetries and the Higgs mechanism

POST makes the following associations between some interactions and gauge groups. The electroweak interaction associates with $SU(2) \times U(1)$ symmetry. The electromagnetic interaction associates with $U(1)$ symmetry. The strong interaction associates with $SU(3)$ symmetry. POST points to difficulties regarding developing a so-called Grand Unified Theory, which would unite - at high energies - into one force the electromagnetic interaction, the weak interaction, and the strong interaction. (Reference [138] provides an overview of grand unification.)

Regarding elementary particles, SUPP suggests that one-step cascades that table 9 shows associate with aspects of interactions in which elementary particles participate.

Aspects related to table 9 and to equation (11) suggest that the following SUPP notions pertain regarding relationships between intrinsic solution-pairs and the related one-step cascades.

- For each case, one intrinsic solution-pair pertains. For some cases, a second intrinsic solution-pair pertains. SUPP suggests that the monad that associates with one solution-pair associates with whether the relevant elementary particles have nonzero mass or zero mass. SUPP suggests that, if a second solution-pair pertains, the monad associates with placing the second intrinsic solution-pair at the same temporal location as the first solution-pair.
- Each one-step cascade associates with a set Z, as per equation (3).
  - For each case that does not associate with the strong interaction, each one-step cascade associates with a set Z that differs from the set Z that associates with each other one-step cascade. SUPP suggests that three triad-related STS DOF pertain.

  - For each case that associates with the strong interaction, there are exactly two one-step cascades and the two one-step cascades share one set Z. SUPP suggests that two sets of three triad-related STS DOF pertain.

- For each case that associates with nonzero mass and nonzero spin, the following notions pertain.
  - For each case that does not associate with the strong interaction, SUPP suggests that one triad-related STS DOF associates with excitation and the other two triad-related STS DOF associate with $SU(2)$ symmetry. (Regarding excitation for fermions, discussion related to equation (53) pertains. Regarding excitation for bosons, discussion related to equation (39) pertains.) For a case that associates with fermion elementary particles, SUPP suggests that the three generators that associate with the $SU(2)$ symmetry associate with three flavours. For a case that associates with boson elementary particles, SUPP suggests that the $SU(2)$ symmetry might associate with the $SU(2)$ symmetry that the SM considers to be part of the $SU(2) \times U(1)$ gauge symmetry that the SM associates with the electroweak interaction.
  - For each case that associates with the strong interaction, quark elementary particles pertain. This paper does not discuss these cases.

- For each case that associates with zero mass (and, thus, with the notion of boson) and nonzero spin, the following notions pertain.
  - For each case that does not associate with the strong interaction, SUPP suggests that two triad-related STS DOF associate with excitation and that the other one triad-related STS DOF associates with $U(1)$ symmetry. (Regarding excitation for bosons, discussion related to equation (39) pertains.) One of the first two STS DOF associates with excitation for the left-circular polarization mode. The other one of the first two STS DOF associates with excitation for the right-circular polarization mode. For the case that associates with photon elementary particles, SUPP suggests that the $U(1)$ symmetry might associate with the $U(1)$ symmetry that the SM associates with the electromagnetic interaction.
  - For each case that associates with the strong interaction, SUPP suggests that notions - regarding excitations - discussed just above pertain regarding one set of three STS DOF and that three new (compared to notions discussed just above) STS DOF pertain. SUPP suggests that one triad-related STS DOF that above associates with $U(1)$ associates with aligning the three new STS DOF with the three previous STS DOF. SUPP suggests that the three new STS DOF associate with $SU(3)$ symmetry. SUPP suggests that the $SU(3)$ symmetry might associate with the $SU(3)$ symmetry that the SM associates with the strong interaction.

- For the two cases that associate with zero spin (and, thus, with the notion of boson), the following notions pertain.
  - For the case that associates with nonzero mass, the following notions pertain. POST associates the ground state for the Higgs field with a notion of three spatial dimensions. SUPP suggests that the three triad-related STS DOF associate with modeling that associates excitations with a three-dimensional harmonic oscillator. With respect to three spatial dimensions and per equation (83), SUPP suggests that the ground state of the Higgs field associates with $D = 3$ and $\nu = 0$. Per equation (86), SUPP suggests that the state $D = 3$ and $\nu = -1$ can have relevance. SUPP suggests that the ground state of the Higgs boson associates with $D = 3$ and $\nu = -1$. Per equation (83), the ground state ($D = 3$ and $\nu = 0$) for the Higgs field would associate with one more unit of energy than does the three-dimensional ground state ($D = 3$ and $\nu = -1$) for the Higgs boson. (SUPP suggests that, regarding POST notions of excitations of the Higgs boson, modeling associates with a ground state that associates with $D = 1$ and $\nu = 0$. SUPP suggests that discussion related to equation (39) pertains.) SUPP suggests that these notions might associate with SM notions of the Higgs mechanism.
  - This paper does not discuss the case that associates with zero mass (and the inflaton).

Thus, SUPP might include notions that associate with the SM gauge symmetries and the Higgs mechanism. This paper does not discuss such notions further. (Discussion above suggests possibilities for symmetries that might pertain regarding other interactions, including interactions that would associate with gravitons. This paper does not discuss such notions further.)

This paper does not discuss further the notion that SUPP might provide further insight regarding the POST SM notion of grand unification.

5.9. Possible bases for insight regarding the three-body problem

This unit suggests a possible extrapolation - from SUPP notions regarding the spin states of two-component systems - that might lead to insight regarding the spin states of three-component systems.

SUPP associates internal states of elementary particles with 0g solution-pairs for which $\{1,3\} \subset Z_\Gamma$ and - for each positive odd number k for which $k \geq 5$ - $k \notin Z_\Gamma$. SUPP associates internal states of some two-component systems with 0g solution-pairs for which $\{5,7\} \cap Z_\Gamma \neq \emptyset$ and - for each positive odd number k for which $k \geq 9$ - $k \notin Z_\Gamma$.

SUPP suggests - and this paper does not further discuss - the notion that internal states of some three-component systems might associate with 0g solution-pairs for which $\{9,11\} \cap Z_\Gamma \neq \emptyset$.

5.10. Connections between classical modeling and quantum modeling

This unit suggests the possibility that SUPP might provide bases for insight about connections between continuous CM modeling and discrete QM modeling.

POST explores connections between continuous CM modeling and discrete QM modeling. (Reference [139] discusses aspects of this exploration and provides further references. Reference [140] discusses aspects regarding electromagnetism.) Such explorations tend to explore notions of developing discrete modeling from continuous bases.

SUPP features discrete notions. SUPP suggests possible relationships between continuous and discrete.

POST notions of DOF associate with continuous. SUPP notions of DOF-related aspects associate with continuous and with discrete.

Equation (24) includes aspects that associate with continuous and aspects that associate with discrete.

Steps from notions that associate with table 5 to notions that associate with table 7a to notions that associate with table 7b might provide insight regarding transitions - regarding modeling - from discrete (including QM) to continuous (including CM).

This paper does not further address the extent to which SUPP might provide bases for insight about connections between continuous CM modeling and discrete QM modeling.

5.11. Possibilities regarding sub-elementary-particle physics and yet more quantum numbers

This unit suggests that SUPP might hint at aspects of nature that POST might associate with notions of extending - beyond elementary particle physics - a series that might include chemistry, chemical elements, atomic physics and nuclear physics, intra-hadron physics, elementary particle physics, and so forth.

SUPP notions regarding T2BNT1 and either (or both of) non-point-like magnetic moment or non-point-like intrinsic angular momentum might suggest that - regarding (at least) elementary fermions - at least one of magnetic moment or intrinsic angular momentum associates with non-point-like spatial distributions of properties. That notion hints at possibilities for modeling that associates with aspects of nature that people might consider as associating with notions of components of elementary particles.

SUPP notions regarding integers N' (as in equations (34) and (54)) - and regarding, in effect, components that add up to yield the integers N' - might suggest that modeling for something that people might consider to be smaller than elementary particles might prove useful.

Work related to equation (34) and equation (54) might suggest a notion that - in some as yet possibly not fully specified sense - N' functions as a quantum number. That notion hints at possibilities for finding more relationships regarding properties, for considering a new modeling space that associates with such relationships, and for developing principles that associate with the new modeling space and inter-property relationships.

6. Concluding remarks

This unit lists specific predictions that this paper makes. This unit lists specific aspects - that this paper discusses - regarding modeling. This unit suggests general perspective about notions that this paper discusses.

6.1. Specific predictions and specific aspects regarding modeling

This unit lists specific predictions that this paper makes. This unit lists specific aspects - that this paper discusses - regarding modeling.

6.1.1. Quantitative predictions

This paper makes the following quantitative predictions. It might be possible to verify or refute most of the predictions soon or within not very many years.

- New elementary particles. (Table 9 lists suggested new elementary particles and all known elementary particles.) Popular modeling anticipates some of the suggested new elementary particles, such as an inflaton and a graviton. Popular modeling seems not to anticipate some of the suggested new elementary particles, such as a so-called jay boson.
- The following properties of elementary particles, plus the following relationships between properties of specific elementary particles.
    - More (than currently known) accurate masses for the W boson and the Higgs boson, plus integer ratios regarding the squares of the masses of the W boson, the Z boson, and the Higgs boson. (Equation (33) pertains.)
    - Relationships between the properties of all known and some suggested elementary bosons. (Table 10 pertains.)
    - A more (than currently known) accurate mass for the tau elementary fermion. (Equation (43) shows the predicted mass.)
    - Various ratios of masses of charged elementary fermions. (Equation (44), table 11, and equation (54) pertain.)
    - A more (than currently known) accurate anomalous magnetic moment for the tau elementary fermion. (Equation (43) shows the predicted anomalous magnetic moment.)
    - Well-specified masses for two neutrinos and a choice among three masses for the other neutrino. (Discussion related to equation (63) shows the suggested masses.)
    - Relationships between properties of all known charged elementary fermions. (Discussion related to equation (67) pertains.)
- A well-specified description of dark matter. (Discussion that leads to and includes equation (21) pertains. Table 12 pertains.)
- Seemingly preferred ratios of dark matter effects to ordinary matter effects. (Table 15 pertains.) Observations seem to suggest some of these ratios.
    - Ratios regarding the production and absorption of cosmic optical background and CMB (as in cosmic microwave background radiation). (Table 15a pertains.)
    - Ratios regarding the composition of some galaxies. (Table 15b pertains.)
    - Ratios regarding the composition of galaxy clusters (that have not collided with other galaxy clusters) and ratios regarding the composition of the universe. (Table 15c pertains.)
    - Ratios regarding galaxy clusters that have collided with other galaxy clusters. (Discussion that cites reference [71] pertains.)

6.1.2. Qualitative predictions

This paper makes the following predictions. It might be possible to verify or refute most of the predictions soon or within not very many years.

- Specific aspects of galaxy formation and evolution. (Table 14 pertains.)

6.1.3. Suggestions regarding how to reduce seeming discrepancies between popular modeling and data

This paper makes the following suggestions regarding how to reduce seeming discrepancies between popular modeling and data. It might be possible to test and use most of the suggestions now.

- The new modeling points (at least qualitatively) to how to reduce so-called tensions - between data and popular modeling - regarding the following.
    - The rate of expansion of the universe. (Discussion that cites reference [43] pertains.)
    - Large-scale clumping of matter. (Discussion that cites reference [86] pertains.)

  - Effects on galaxies of gravity associated with nearby galaxies. (Discussion that cites reference [61] pertains.)

- The new modeling points - regarding gravity and the Einstein field equations - to the following notions. (Discussion that mentions specific components - including T^{21} - of the stress-energy tensor pertains.)
  - The new modeling comports with the notion that - regarding circumstances for which physics has tested the Einstein field equations - using the Einstein field equations is appropriate.
  - The new modeling suggests notions that might improve the effectiveness of the Einstein field equations regarding some circumstances for which notions of tensions - between modeling and data - pertain.
  - The new modeling comports with the notion that - regarding some circumstances - the Einstein field equations would not be adequately accurate.

6.1.4. Relationships between suggested new modeling and popular modeling

This paper suggests the following notions regarding the new modeling that this paper discusses.

- The new modeling aligns itself with popular modeling, including by the following notions.
  - The new modeling associates with (and extends from) a straightforward notion - that uses Diophantine equations - for adding insight regarding Newtonian modeling regarding forces. (Discussion that includes equation (1) and table 2 pertains.)
  - The new modeling associates - via a new modeling principle - with popular modeling notions of degrees of freedom. (Equation (11) pertains.)
  - The new modeling suggests reuses for aspects that associate with popular modeling. (The notion of isomers of a set that includes all known elementary particles, except the photon, provides an example. Discussion related to equation (21) and table 12 pertain.)
  - The new modeling shows possible associations with elementary particle Standard Model gauge symmetries and with the Standard Model notion of the Higgs mechanism. (Discussion that cites reference [138] pertains.)
- The new modeling augments popular modeling by doing the following.
  - Showing and using principles that organize a set of physics properties (including charge, magnetic moment, mass, energy, momentum, and angular momentum). Suggesting new (compared to popular modeling) aspects regarding properties. (Table 6 pertains.)
  - Including modeling that outputs a list of all known and some suggested new elementary particles and that suggests organizing principles for cataloging elementary particles. (Table 9 and related discussion pertain.)
  - Adding a new elementary particle internal quantum number - isomer. (Discussion related to equation (21) and table 12 pertains.)
  - Including modeling that outputs a list of properties that associate with elementary particles. (Table 5 pertains.)
  - Including modeling for spin-states of two-component objects (Discussion related to equation (26) pertains.) and applying that modeling to spin-states of atoms. (Table 8 pertains.)
  - Including modeling that outputs a list of conservation laws. (Table 7 pertains.)
  - Suggesting possibly new insight about relationships among observations of properties, aspects of fields (such as the electromagnetic field) that convey information about the properties of objects, and Lorentz invariance. (Discussion regarding equation (18) pertains. Table 6 pertains.)
  - Identifying components - of gravitational fields - that associate with attraction and repulsion between objects. (Discussion related to equation (19) and equation (20) pertains.)
  - Suggesting a mechanism that led to baryon asymmetry. (Discussion related to equation (69) pertains.)

  - Suggesting a mechanism that might convert some energy associated with one of ordinary matter and dark matter to energy that would associate with the other one of ordinary matter and dark matter. (Discussion related to equation (68) pertains.)
  - Suggesting eras (regarding increasing and decreasing rates of expansion of the universe) and mechanisms that drive those eras. The eras include two known eras, inflation, and some possible earlier eras. (Table 13 pertains.)
  - Suggesting a new way to estimate - given data about two charged leptons - the anomalous magnetic moment of the third charged lepton. (Discussion related to equation (61) pertains and yields a result for the tau - that comports with data and seems to comport with results from the Standard Model. Equation (43) pertains.)

6.1.5. Suggestions regarding possible experiments and observations

This paper makes the following suggestions regarding possible experiments and observations. This paper does not make suggestions as to when the experiments or observations might be feasible.

- Possible means for directly detecting dark matter objects that have similarities to ordinary matter atoms. (Discussion related to table 8 and table 15a pertains.)
- Possible new types of information that gravity waves might convey. (Discussion that cites reference [133] pertains.)

6.1.6. Suggestions regarding possible new aspects of physics

This paper makes the following suggestions regarding possible new aspects of physics.

- New aspects of physics might arise from notions that (at least some) elementary particles might model as having structure. (Discussion related to equation (18), table 6, and equation (58) suggests that the nonzero values of anomalous magnetic moments for charged leptons associate with modeling that suggests oblate distributions of properties.)
- New aspects of physics might arise from relationships between electromagnetic properties and gravitational properties (Equation (42) suggests a relationship.), between strong-interaction properties and gravitational properties (Discussion regarding equation (73) suggests a relationship.), or between weak-interaction properties and gravitational properties (Discussion regarding equation (73) suggests a relationship.).
- New aspects of physics might arise from aspects that new modeling suggests regarding new quantum numbers that associate with elementary particles. (Discussion related to equations (34) and (62) pertains.)

6.2. General perspective

This unit suggests general perspective about notions that this paper discusses.

Each of the following sentences describes a physics challenge that has persisted for the most recent eighty or more years. Interrelate physics properties, properties of objects, and physics constants. Provide, for elementary particles, an analog to the periodic table for chemical elements. Describe bases for phenomena that POST (as in modeling that has bases in POpular modeling notions of Space-Time coordinates) associates with the two-word term dark matter. Describe bases for phenomena that POST associates with the two-word term dark energy. Explain the overall evolution of the universe. Interrelate physics models. Develop a list of principles that underlie physics or physics modeling.

Physics amasses data that people can use as bases for developing and evaluating modeling aimed at addressing the challenges.

SUPP (as in SUggested physics modeling based on notions of principles and modeling that associate with Particle Properties) addresses those physics challenges and has bases in notions of degrees-of-freedom-related aspects and in the following mathematics - integer arithmetic, multipole expansions, Diophantine equations, and multidimensional harmonic oscillators.

SUPP suggests a new principle - CODOFRA (as in Conservation Of Degrees-Of-Freedom-Related Aspects) - that links POST and SUPP.

SUPP suggests a new elementary-particle internal quantum number - isomer - that associates with notions that dark matter has similarities to ordinary matter.

SUPP unites and decomposes aspects of electromagnetism and gravity. For each of those two long-range interactions, the decomposition associates with properties - of objects - that people can measure and that POST features. For electromagnetism, the properties include charge and magnetic moment. For gravity, the properties include energy and intrinsic angular momentum.

SUPP points to all known elementary particles and to some might-be elementary particles. POST suggests some, but not all, of the might-be elementary particles. SUPP suggests relationships between properties of elementary particles. SUPP suggests more (compared to data or to POST modeling) accurate values for some properties of some elementary particles.

SUPP includes a notion of isomers of elementary particles that do not mediate long-range interactions. SUPP features a notion of instances of components of long-range interactions.

SUPP suggests a description of dark matter.

SUPP suggests explanations for data regarding dark matter. SUPP points to possible resolutions for tensions - between data and POST - regarding effects of dark energy. SUPP suggests insight regarding galaxy formation and evolution.

SUPP suggests explanations for data that POST seems not to explain, suggests results regarding data that people have yet to gather, and points to possible opportunities to develop models that unite aspects of physics and physics modeling. SUPP seems not to disturb aspects of POST that comport with data.

In summary, SUPP suggests augmentations - to POST - that might achieve the following results. Extend the list of elementary particles. Predict masses for at least two neutrinos. Predict masses - that would be more accurate than known masses - for some other elementary particles. Describe dark matter. Explain ratios of dark matter effects to ordinary matter effects. Provide insight regarding galaxy formation. Describe bases for phenomena that associate with the two-word term dark energy. Explain eras in the history of the universe. Link properties of objects. Interrelate physics models. Point to new principles regarding physics modeling. Point to physics phenomena that might underlie elementary particle phenomena.

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