Measurement Mechanics

resolves QM measurement discrepancies

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Abstract: *Measurement Mechanics proposes a major modification to the current understanding of measurement. This modification adds a unit standard as axiomatic to measurement theory. Currently, measurement theory (representational) defines the result of a measurement comparison to be a ratio of quantities (numerical value and unit) where the unit terms are equal and cancel. When the units cancel such a ratio is local to one measurement system, i.e., probabilistic. Thus all QM measurement results, based upon current measurement theory are local, as J. S. Bell confirmed. However, the units of each metrology measurement result are correlated by calibration to an independent unit standard. Then measurement result comparisons are a ratio of quantities each correlated to a common unit standard, which allows independent measurement result comparisons that are non-local and deterministic. When unit standards are included in measurement theory all the apparent discrepancies between QM and classic measurements are resolved.*

Keywords: measurement problem, metrology, calibration, uncertainty, precision, accuracy.

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This paper develops and applies more rigorous definitions for existing measurement terms than The International Vocabulary of Metrology **(**VIM) [1]. To improve understanding, please read the definition in the Annex when a footnote and italics indicates the initial use of a redefined term.

1. Introduction

In current representational theory [2][3], the result of a measurement is a *quantity*[1] consisting of a numerical value and a *unit*[2](*u*). A comparison of two measurement result quantities with the same *u* is a ratio where each *u* cancels. In quantum mechanics (QM) this ratio of numerical values is invariant to changes in *u* [4], i.e., *u* is arbitrary. This form of *u* does not address Heisenberg's uncertainty, where *u* has a *precision*[3] equal to or greater than a Planck (*h*) and at larger scales does not address measurement precision. Given this uncertainty or precision, the *measurement apparatus*'[4] intervals, representing units or a factor thereof, are not exactly equal and, in a ratio of measurement result quantities, the intervals do not cancel.

The underlying assumption in representational theory and its application in QM is that the uncertainty or precision is small enough to be ignored in empirical measurements. Such an assumption is not appropriate in a rigorous theory. MM proposes that uncertainty and precision are different scales of the same effect.

When the units cancel, they are without formal effect, and a ratio of quantities is reduced to a ratio of numerical values. Measurement mechanics (MM) proposes that such ratios are local[5] and appear as probability distributions. In physics probabilities are a means of comparing local numerical values that are not relative to an independently defined unit *standard*.[6]

L. Euler (1765) also explained that measurement comparisons require quantities: “Now, we cannot measure or determine any quantity, except by considering some other quantity of the same kind as known and pointing out their mutual relation” [5].

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[1] A2.12

[2] A1.1

[3] A2.10

[4] A2.7

[5] Local in MM identifies that the numerical value of a measurement result without independently defined units is relative to undefined (local) units.

[6] A2.15

A unit standard is proposed as the 'known mutual relation' that supports independent measurement comparisons in physics. However, *calibration*[7] to a unit standard, as defined in representational theory, understood in QM and practiced in metrology (see Figs. 1 & 2), is empirical. Then the requirement, explained above, for an independent unit standard to support independent comparisons is not recognized. MM proposes that calibration, as defined (see Fig. 3) is required for independent (non-local) measurement comparisons.

Repeatable measurement experiments are applied in metrology to understand measurement result deviation [6]. Three such experiments (A., B., C. below), are proposed to analyze independently three variables: *property*[8], *accuracy*[9] and precision of a measurement result:

A. The *observables*[10] are changed and the measurement apparatus fixed. The deviation of a common property of a set of observables may be determined. The QM neutron (observable) spin (property) experiments [7] are an example of A.

B. The observable and the measurement apparatus are both fixed. Then the accuracy of a distribution of the numerical values of the measurement result quantities may be determined. The effects of noise and distortion, which are not considered in this theoretical paper, dominate accuracy.

C. The observable is fixed and the fixed measurement apparatus is recalibrated to a fixed unit standard each time. Then the precision of the measurement apparatus intervals relative to a unit standard may be determined. Calibration effects dominate precision and are the focus of this paper.

MM implements Relative Measurement Theory (RMT) [8] to explain and verify that the QM measurement discrepancies [9] do not require different interpretations of QM theory as has been proposed [10], but rather require that measurements are calibrated to a unit standard. However, a unit standard is not treated in mathematics[11] therefore QM does not distinguish between a local ratio, and an independently comparable measurement result quantity which is calibrated to a unit standard. MM proposes that the BIPM standards [11], as independent unit standards, are axiomatic in physics.

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[7] A2.2

[8] A2.11

[9] A2.1

[10] A2.9

[11] The axiomatic nature of a standard suggests changes to various mathematical disciplines. These changes are not addressed in this paper.

2. Measurement theories

Currently there are two measurement theories that appear consistent, but are not:

- Representational theory - a *measurement*[12] is a numerical value on a local scale. When the scale is normalized, the numerical value is a probability. How a physical measurement result occurs is empirical.
- Metrology - an empirical measurement result is a quantity which consists of a numerical value and a unit which is correlated by calibration to a unit standard or factor thereof.

MM, a new measurement theory, is shown to be consistent and to resolve the measurement discrepancies between QM and metrology.

2.1. Representational theory

The basic text on representational theory is *Foundations of Measurement* [12]. In this theory a measurement is a numerical value on one of three basic *scales*[13] ordinal, counted/linear and ordered.

Inconsistently, representational theory defines the measurement processes differently from the empirical measurement practice defined in metrology (VIM). Metrology requires the correlation and equalization of the measurement apparatus' scale of *intervals*[14] to a unit standard (termed calibration), while representational theory assumes the measurement apparatus' scale of units has been equalized by the practice of calibration and therefore treats calibration as empirical. This difference is known [13] but not recognized as the beginning of the measurement discrepancies.

Representational theory, in the most common form [14], determines the numerical value of the observable (x) assuming a scale of equal units. When equal units are assumed a quantity is the product of a numerical value (x) times a unit (u), which is xu. In Fig. 1, representational theory recognizes that equal unit ratios cancel, thus a comparison of two quantities is a ratio of numerical values. Since the units cancel, the standard which defines the units is arbitrary and the calibration of the intervals of a measurement apparatus to a standard is only empirical [15], therefore not shown in Fig. 1. This is not consistant as interval equality cannot be assumed because of Heisenberg's uncertainty at small scales and ± interval precision at larger scales. Metrology applies calibration to a unit standard to achieve measurement apparatus interval equality.

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[12] A2.6

[13] A2.14

[14] A2.5

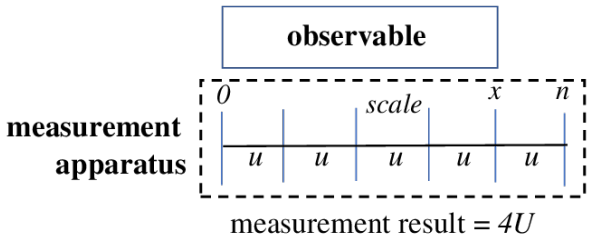

Figure 1. Representational theory

In representational theory, metrology practice converts the representational result, *4u* into *4U* a measurement result. This is inconsistent, as metrology calibration generally corrects $\overline{u}_i$ to *U*, not *u* to *U* (see Fig. 2).

2.2. Metrology

In current theory and practice [16], measurements that apply a scale of *n* intervals assume that any small non-linearity of the u_i will likely cancel in a summation (1) producing one true value *measurement result*[15], the quantity *xu*. (1) is shown to be incorrect at small scales.

$$q = \sum_{i=1}^{i=x} u_i = xu \quad \text{where } i = 1 \text{ - } n \tag{1}$$

In Fig. 2 (without noise or distortion), an observable with a property, shown as a set of *u*, is termed a measurand in VIM. Then the ratio of this preexisting property [17] of the measurand to the scale is the numerical value of that property of the observable.

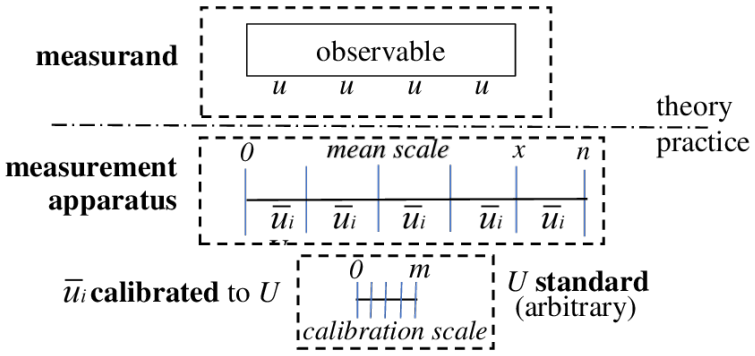

Figure 2**.** Representational theory and metrology practice.

[15] A2.8

In Fig. 2, the calibration process is: The numerical value of the mean interval $\overline{u}_i$ is calculated (not equalized to a standard) using (1). $\overline{u}_i$ is divided into *1/m* calibration states and compared to *U* establishing the numerical value of $\overline{u}_i$ relative to U in calibration states which generates an arbitrary corrected *mean scale*.[16]

Inconsistencies: Calibration to the mean is likely to be valid when $x >> 1/m$ (e.g., classic measurements), but may not be when the $x \sim 1/m$ (e.g., quantum scale measurements). Correcting $\overline{u}_i$, which is calculated to have equal intervals, means that *U* is arbitrary, even though it is required in metrology.

Exploring the inconsistences in Figs. 1 & 2. All empirical measurements in physics are based upon BIPM standards. Consider, a physical measurement of mass: a measurement apparatus' scale, to support the measurement of mass, is calibrated to a kilogram standard, *U*. The scale is then is brought to a brick (observable). First the scale's *zero point*[17] and the brick's zero mass are aligned, then the brick's mass property is measured. When the same measurement apparatus is defined by calibration to a pound standard it determines the same brick's weight property.

Here *U*, the kilogram standard via a measurement apparatus with a scale, determines one property to be measured of a brick's many properties (e.g., mass, weight, American spatial dimension, metric spatial dimension, number of molecules, color, etc.). A standard defines an observable's property via a local scale (see Fig. 3). The local scale establishes the numerical value of that property of the observable.

When the equality of $\overline{u}_i$ is given, calibration to a unit standard is solely empirical. Then precision is perfect in repeated measurement experiments A. and B described in Section 1. Perfect precision results in the possibility of a true value measurement result. Heisenberg's uncertainty theory [18] as well as ± interval precision[18] makes a true value measurement result not only unattainable, but nonexistent. Not treating calibration in theory also ignores ± interval precision as one cause of the *Gaussian distributions*[19] of measurement results and ignores the definitional and equalizing functions of an independent unit standard. Thus, not treating calibration to a unit standard in representational theory is not valid.

2.3. Measurement mechanics

RMT identified that one cause of Gaussian measurement result distributions is the calibration effects on the intervals of a scale [19]. These effects, currently assumed to cancel, can sum into significant deviations. These unexplored interval deviations have been masked by the inconsistent usage of the term unit between different disciplines (see A1.2), as well as the lack of a formal measurement function (4) which statistically sums (see Section 3) the random calibration effects.

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[16] A2.14.2

[17] A2.16

[18] Classic ± interval precision is explained in Section 3.

[19] A2.4

A quantity (product) is expedient for many experimental measurement results. A quantity (capital Q when summed), see (2), is a proper superset of a quantity, $Q \supseteq q$. Eq. (1) is proposed as the first step towards a formal measurement function based upon Fig. 3.

$$\text{quantity distribution } Q = \sum_{i=1}^{i=x} u_i \tag{2}$$

In (2), u_i represents each of the intervals of the relative scale in Fig. 3. However, each u_i is not treated as equal, whereas all the u on the local scale are assumed to be equal. RMT identified and verified that the u_i are not theoretically or empirically equal, especially at quantum scales [20]. The notation applied in RMT for the random deviation of u_i is Δ. In MM this notation is changed to ± precision.

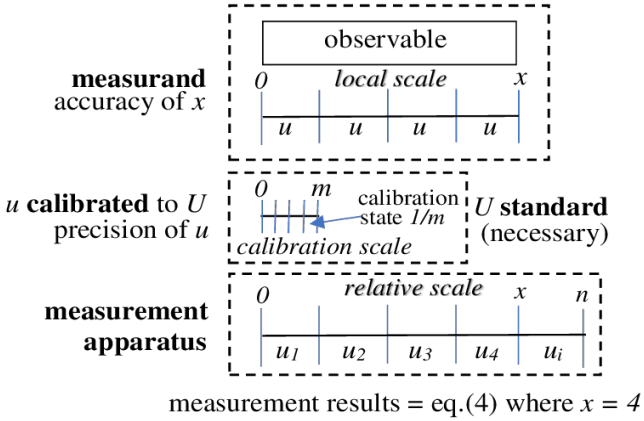

Figure 3. Measurement mechanics

In Fig. 3, the calibration scale generates a relative scale by determining the numerical values in $1/m$ calibration states of each u on the local scale relative to U determining u_i. Then the numerical values of each u_i ± precision are statistically summed (explained in Section 3) establishing a distribution of measurement results, the same as appear in repeated experimental measurements.

U defines an observable's property and determines the precision of that property. The u_i of the relative scale have a random component (± a calibration state) relative to U, even in theory, and at the minimum limit each interval will deviate ± a Planck. When each u_i has this random component, a measurement function in theory must be a sum over each u_i, not nu or nU. This summation is required even when the random result component is very small, because statistical summing produces a distribution (examples in Sections 4.1 and 4.2 below).

A physical measurement apparatus may include transducers (not shown in Fig. 3) which convert a quantity on a local scale to a quantity on a relative scale. Additionally calibration stages which include intermediate standards often occur in practice, and are also not shown. With these provisos, MM proposes that Fig. 3 diagrams the theory of a physical measurement to a standard without noise or distortion.

In Fig. 3, there is no arbitrary boundary between theory and practice. When the observable's variation is not considered, any deviation of a repeated measurement result as evaluated in B. or C. (Section 1.) has three possible causes: noise (external to a measurement system), distortion (internal to a measurement system) and the precision of the *calibration states*[20] of a *calibration scale*[21](±*1/m*).

Each interval of the relative scale in Fig. 3 may be calibrated (more rigorous and less practical) to a standard, or (most common practice) the mean interval of the scale is determined by calibration to a standard. MM, as a theory, treats the more rigorous calibration of each u_i.

1/m is the smallest identifiable change of *U* in a measurement system. The assumption here is that *U* is quantified into even smaller states independent of this measurement system. Integers *n* and *m* represent counts when *1/n* and *1/m* represent the smallest interval/state of their respective scales.

3. The measurement processes

In Fig. 3, there are four measurement processes. The first process correlates an observable to a *local scale*.[22] The second process determines a numerical value (i.e., count) of local states with an accuracy, but without precision. To implement these two process:

1. Align the observable's beginning to the local scale's zero point.
2. Count *x* of the observable's *u* on the local scale.

Process 2 identifies that the observable is ordered and additive (countable) therefore it has a property with a numerical value, but not an independently defined property.

As example, a wooden stick (observable) is determined to have a length property by counting regular notches (a local scale) on the wooden stick. Comparisons only occur by moving this wooden stick to other observables that appear to have the same length property. That is, this wooden stick verifies that other observables have its length property, but the wooden stick's counts of other observables do not have precision.

The third and fourth processes (calibration to *U*) transform the local scale into a *relative scale*[23] and establishes a measurement result. These processes define both the property being measured and equalize each u_i to *U* which determines the numerical value of each u_i with a precision. To implement these processes:

3. Determine the precision of each *u* relative to *U* establishing the numerical value of each u_i.

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[20] A2.3

[21] A2.14.4

[22] A2.14.1

[23] A2.14.3

4. Statistically sum (see below) all x of the $u_i = (u \pm 1/m)_i$ which includes their precision, calculating a distribution of measurement results.

The above four processes are combined into eq. (3), measurement results with precision distribution:

$$Q = \sum_{i=1}^{i=x} \left(u \pm 1/m\right)_i \quad \text{which converges to a Gaussian distribution as} \quad m \to \infty \tag{3}$$

Eq. (3) is a statistical sum measurement version of the central limit theorem [21].

Statistical summing example (without noise or distortion) of repeated measurement results C. (Section 1): The numerical value of a measurement result (e.g., *4*) is the sum of four contiguous intervals (i = *1 - 4*) on the relative scale of Fig. 3: $u_1+u_2+u_3+u_4$. Each u_i has a precision, which is one of *2* random ±(*1/m*) calibration states, relative to the U standard or factor thereof. The two random states of each of the four u_i are statistically summed, producing $2^4 = 16$ measurement result quantities that approximate a Gaussian distribution of u_i.

This deviation of an observable relative to U is termed precision ±(*1/m*) in metrology and is termed uncertainty (at the limit ±Planck) in QM. This precision/uncertainty measurement deviation is fundamental in any calibrated or QM state space [22].

Whenever a scale is applied to an observable, alignment is an issue. In the worst case it cannot be determined which interval an end of an observable is closest to, of a contiguous pair of intervals on the scale. This is ± interval precision which also appears in accuracy. This requires that the worst case accuracy is also ± an interval (*1/n*). Including this accuracy produces the final measurement results distribution equation:

$$Q = \sum_{i=1}^{i=x\pm(1/n)} \left(u \pm (1/m)\right)_i \tag{4}$$

Eq. (4), based upon Fig. 3, applies to all formal and empirical measurement systems without noise or distortion and with defined-equal calibration states. The accuracy term (±*1/n*) means that (4) is a near Gaussian distribution.

4. Empirical measurement examples

Eq. (4) is a paradigm shift from the inconsistenties in theory and practice upon which measurement results are based today. The following examples of repeated measurement result experiments C. (Section 1) are offered in support of this paradigm shift.

4.1 Additive relative scale

An example of an additive relative scale is a thermometer which measures a quantity of the property temperature. This example demonstrates how additive imperfect intervals statistically increase the distribution of measurement results, producing a Gaussian measurement result distribution.

The measurement apparatus consists of a hollow glass tube with a reservoir filled with mercury at one end, which fits inside another hollow glass tube that slides over the first. The two glass tubes are held together and placed in an adjustable temperature oven which is calibrated to a temperature standard within a deviation of $\pm 0.1^0$ (degree). Then the outside glass tube is marked at the level of mercury which appears and marked again with each 1.0^0 (u_i) increase in temperature of the oven. $n + 1$ marks or 101 marks are made on the outside glass tube. Each of the 100 u_i is correlated by the oven to $1/0.1 = 10 = u_i \pm 0.1^0$ precision.

After 101 marks are made, the apparatus is removed from the oven and an ice water bath is applied to the tube with the mercury reservoir. The outside glass tube is now slid over the inside glass tube until the top of the inside mercury column lines up with the first mark on the outside glass tube. Now this mark on the outside glass tube is the zero point for ice water (0^0C) and the zero point of the thermometer has been calibrated to a *reference*[24] (i.e., ice water).

Consider the water in a glass (observable) to be in contact with the reservoir of the thermometer calibrated as described above. If the numerical value of the temperature (a property) of the water is $x = 80$, the $81st$ mark on the outside glass tube represents 80 numerical value in units of degrees ^{0}. Each degree is $\pm 0.1^0$ nominal precision or $\pm 8^0$ which is the worst case implementation of this thermometer (very very rarely possible). The $\pm 0.1^0$ nominal precision occurs when the $\pm 0.1^0$ precision of each 80 u_i is uniformly distributed and cancels. The $\pm 8^0$ precision occurs when each u_i of two different thermometers has the same $+0.1^0$ and -0.1^0 precision, which sums.

In the design of experimental measurement apparatus, the ± interval precision of each u_i is expected to cancel and is ignored. But in MM, without noise or distortion, when each mark's precision is defined to be $\pm 0.1^0$, a deviation of $\pm 8^0$ for the measurement results from two thermometer implementations is very, very rare, but not impossible. The statistical sums of the precision from $\pm 0.1^0$ to $\pm 8^0$ establish a Gaussian distribution of measurement results (see Fig. 4)

4.2 Length measurement apparatus

A physical metre stick (i.e., a scale calibrated to the standard metre U) is divided into 100 intervals (u_i). Consider an observable whose spatial dimension property has the numerical value

[24] A2.13

$x = 70$. In the relative measurement theory proposed, the precision of each u_i is treated individually and then added to the next u_i (70 times).

When first calibrated to U (e.g., the standard metre), each $u_i = (U/100) \pm (1/m)$ precision where, as example, $m = 10^6$. The accuracy of x is ignored in this example. In MM, the quantity deviation of each metre stick is caused by the random application of $\pm (1/m)$ to each u_i causing a Gaussian distribution of metre sticks which produce a Gaussian distribution of measurement results. In the statistically rarest two cases, when x number of u_i, each with a precision of $+(1/m)$, are summed and with a different metre stick a measurement of x number of u_i, each with a precision of $-(1/m)$, are summed, the maximum measurement result quantity deviation (across all possible metre sticks and their measurement results) appears as $2(70)10^{-6} = 1.4x10^{-4}$ metres, which is sufficient precision ($\pm 0.7x10^{-4}$) for a metre stick. When $m>>n$, the effect of calibration is usually and realistically ignored. However, when n and m are both small, e.g., quantum scale measurements, the statistical summing of each $\pm (1/m)$ cause significant measurement result discrepancies (see Section 5 below).

4.3 Measurement results distribution

Statistical summing produces a Gaussian distribution [23]. Fig. 4 represents a combined near Gaussian distribution of both accuracy (B. Section 1) and precision (C. Section 1) which are often Gaussian distributions. Actual experiments following B. and C. may be practical which measure the effects of both $\pm 1/n$ (accuracy, including experimental noise which may be Gaussian) and $\pm 1/m$ (precision, which is usually Gaussian) and identify the deviation changes closely enough to verify (4).

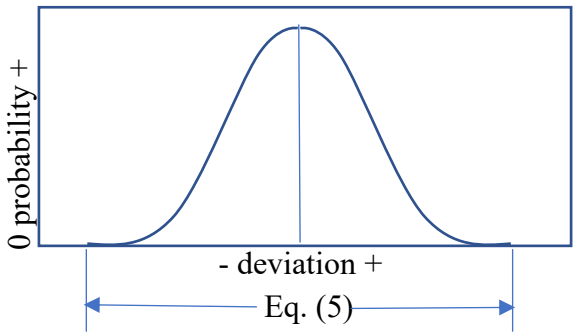

Figure 4. Measurement results with accuracy and precision distribution.

5. Understanding measurement discrepancies

Examining the distribution of (4), precision, $\pm (1/m)$ in each of the x u_i in (4), varies randomly (in theory) and with $\pm 1/n$ accuracy produces:

$$\text{the worst case deviation of (4)} = (2/n) + (2x/m) \qquad (5)$$

Where $2/n$ represents worst case accuracy and $2x/m$ represents worst case precision. In distributions where $m>>n$, the accuracy distribution will dominate the precision distribution, but

precision begins to dominate when $n \sim m$ and has the most significant effect when n and m are small.

$$\sum_{i=1}^{i=x}\left(u \pm (1/m)\right)_i \supseteq x\overline{u}_i \qquad (6)$$

Eq. (6) compares measurement mechanics (4) with current metrology. Without treating the accuracy of x, noise, or distortion, the left side of (6) is the MM measurement function and the right side of (6) is a common metrology measurement function. Statistically, (6) may be seen as a comparison of the standard deviation (left side of $\supseteq$) to the mean deviation (right side). It is known that when x is small the standard deviation diverges from the mean deviation [24].

The two sides of (6) produce equal measurement results with high probability when x and m have large numerical values (e.g. classic measurements). In (6) when x or m is large (a common experimental measurement, see Section 4.2), the many statistical sums of the two random calibration states $\pm(1/m)$ of each u_i likely cancel, producing only very small effects on the distribution of measurement results.

Conversely, when x and m both have small numerical values (e.g., in quantum spin experiments, $x = m = 2$) [25], two repeated measurement result Quantities of the same observable will be different in 50% of the repeated measurement results on an unchanged observable because the precision of each u_i ($\pm (1/m)$) will likely be different. This measurement result difference appears in QM as uncertainty or non-commuting Fourier pairs (see Section 6.3).

6. Explaining the discrepancies in quantum scale measurements

Measurement discrepancies appear, not only because of the deviation caused by the statistical summing of u_i but because the four different measurement processes (Section 3) are not recognized.

6.1 Remote entanglement

Remote entanglement is used here to describe the numerical value entanglement evidenced between two separated measurements in the Stern-Gerlach experiments **[**26**]**. There have been many attempts to understand remote entanglement. J.S. Bell's local formalization (since verified experimentally [27]), which is also quite clear, is addressed. J. S. Bell stated [28]: "...there must be a mechanism whereby the setting of one measurement device can influence the reading of another instrument, however remote." In the Stern-Gerlach experiments Bell discusses, that mechanism (measurement process 1, Section 3) is explained below.

N. D. Mermin in 1981 statistically analyzed the results of Stern-Gerlach experiments that identify remote entanglement. [29] Remote entanglement is described by Mermin without QM notation, which indicates it will occur in all measurement results. Mermin identifies that the measurement results of the two remote entangled particles are relative. But he does not recognize that relative requires the two apparatus are calibrated to each other.

In Mermin's model of the experiments, the two apparatus each have a three position selector (a scale) on which one of three 120^0 intervals appear, which represents the measurement of the

spin direction's numerical value. The common property (spin) and common scale of spin direction (three 120^0 *u* intervals) are defined in the experimental set-up and not recognized as measurement process 2. Mermin's two measurement apparatus then require a common zero point (process 1) between the three 120^0 units to make comparisons.

There are nine possible combinations of the two three-position selectors: three (Mermin's case a) when each selector is in the same position and six (Mermin's case (b)) when the positions of the two selectors are different. When the two remote selectors are in different positions (the two measurement apparatus' zero points are uncorrelated), each numerical value (0^0 or 180^0) of the two particles' spin unit appears randomly over a large number of runs. Only in Mermin's case (a) is each numerical value of the two particles correlated, because there is a common zero point (process 1) between the two measurement apparatus' selectors (scales). When these necessary zero points are required in experiments, but are not recognized, discrepancies occur.

6.2 Compton-Simon experiments

In the Compton-Simon cloud chamber experiment [30], the energy and momentum after collisions between a light photon and electrons are measured by the positions or by the central line of the collision (two independent ways) and at different times in the same experiment. However, both measurements always confirm equal results unlike repeated measurements which always have a distribution of measurement results.

It appears that the energy of processes 1 & 2 foretells the momentum of processes 1 & 2 (or the reverse), before they are known. In actuality, in the one experiment, which describes two different repeated incomplete measurements, processes 3 & 4 (which cause a Gaussian distribution and complete a measurement) do not occur after the measurement of energy or the measurement of momentum, therefore both energy measures or the momentum measures are equal, i.e., they do not have a distribution.

These experiments also identify that light (an observable) has two distinct energy properties - wave and particle. These two energy properties are not contradictory. Observables present many properties, each of which is defined by a standard and selected via a scale. This is discussed further in Section 6.4.

6.3 Heisenberg's uncertainty

In Heisenberg's 1927 analysis [31], when a single particle's two Fourier dual properties are measured at different times they have a fundamental uncertainty and seem to vary inversely. From his discussion: p (momentum) = mv, where m is mass and v velocity. Then q (position) = vt where t is the time interval of the wavelength of the light used for the observation.

In quantity calculus notation [32], curly brackets indicate the numerical value and square brackets indicate the unit(s). Then $p = \{x_p\}[mv]$ and $q = \{x_q\}[vt]$. And the product of the quantities of $pq = \{x_p x_q\}[mv^2t]$, which is the energy $[mv^2]$ during the $[t]$ time interval.

Heisenberg identifies the uncertainty of pq as $p_1 q_1 \sim h$, and that p_1 and q_1 change inversely as the observer's wavelength changes. This inverse change occurs because as $[t]$ changes, $[v]$ in

the p quantity [$v = q/t$] changes inversely to [t] in the q quantity which appears as [vt]. The numerical values $\{x_p x_q\}$ which are usually the dominant term, do not change inversely with [t].

Since QM does not apply quantity calculus, this is unclear in QM. Applying MM, the minimum numerical value of Heisenberg's uncertainty is the precision deviation of p_1 and of q_1 which is ± h for each and is statistically summed (see Section 3). The precision (but not the numerical value) of two Fourier dual quantities will always vary inversely and is not unique to QM.

Uncertainty, also ± precision, occurs in every u_i. Then each u_i varies randomly ± h at a minimum and statistically sums. Thus a comparison of repeated measurements of unchanged pq quantities (*4* units each with *2* states), will be different, in *12* of *16* comparisons on average and p and q appear not to commute. The RMT paper includes further analysis of the correlation between the different forms of uncertainty and precision.

6.4 Double slit experiments

Feynman's explanation of the double slit experiments offers a good example of how a standard via a measurement apparatus determines which property of an observable is measured. [33] Feynman concludes, "...when we look at the electrons the distribution of them on the screen is different than when we do not look." What he meant is ,"when we look" identifies a property of an observable.

In these experiments the plate with slits is a transducer and the sensing screen (measurement apparatus) presents patterns which the operator interprets as different energy properties. The slits pass two properties of energy (based upon defined standards for frequency and mass), and the sensing screen displays a different pattern for each property. When the slits in the plate are replaced with small holes only the mass property appears on the sensing screen.

Physical observables have many properties. The operator, by looking for one pattern on the sensing screen (a property), identifies the wavelength of a particle's frequency property or identifies the point of impact of a particle's mass property.

Properties are defined by a standard in physics and then established by the measurement apparatus. An observable has many properties, but they are defined independently in the relative universe, otherwise the properties could not be compared independently.

7. Relating measurement mechanics to other theories

7.1 History of a quantity

In 1891, J. C. Maxwell [34], proposed that a measurement result quantity is $q = xu$.

Maxwell proposed that the u of this quantity is "taken as a standard of reference" [35]. This wording suggests that u is equal to the standard and implies representational theory. However, in 1927 Heisenberg proved that perfect precision is not possible in QM.

Perhaps Maxwell assumed that a theoretical measurement result could be exact. In any event, Maxwell's usage appears to have instigated what is now representational theory. Reference [5] describes this history.

7.2 The Einstein, Podolsky, Rosen (EPR) paper

Defining measurements relative to a standard, as MM proposes, identifies a standard as a what the EPR paper [36] suggested is missing and was assumed to be a hidden by Bohm [37]. The EPR paper, which considers numerical values not quantities, does not recognize that all independent measurement results are relative to a standard. Einstein identified (similar to Euler) that everything physical is relative, but did not recognize that measuring-rods must be calibrated to standards to make independent measurement comparisons [38].

Measurement mechanics is based upon understanding the necessity for standards. Isology (Iso = same, logy = science of) is the name given to the very broad scientific discipline that studies references, standards and standardization. When all else is relative, standards are invaluable.

7.3 Other measurement discrepancies

The inconsistencies in representational theory and metrology also cause other measurement discrepancies. The RMT paper described the entropy change (*log m*) caused by calibration in a measurement process which is sometimes termed collapse or decoherence in different interpretations of QM measurements [39]. In Measurement Unification, 2021 [40], explanations based upon RMT are given of quantum teleportation experiments and Mach-Zehnder interferometer experiments. The Schrödinger's Cat thought experiment is explained in a short preprint [41]. These papers, together with the examples in this paper, strongly support applying standards to all physical measurements in theory as well as in practice.

8. Conclusion

As the EPR paper in 1935 formally identified, but did not recognize and Bell in 1989, refined, but did not recognize, representational theory, the basis of QM measurements is inherently local. Classic measurements as practiced in metrology are inherently independent. The difference is independent standards. The discrepancies that appear by not recognizing standards in mathematics and physics beg to be resolved.

In QM, all *u* are defined equal and the ratio of two quantities is relative to each other, rather than each quantity relative to an independent standard. Without including a standard, quantum uncertainty appears as relations between Fourier duals, rather than the precision of any measurement result quantity relative to a standard.

In physics, all measurement results, theory or experimental practice, must be quantities, defined and equalized relative to BIPM standards. By recognizing that unit standards are Euler's 'known mutual relation', MM establishes one formal measurement equation consistent across the physical sciences. When this equation is applied, the discrepancies between measurement theories and experimental measurement results are resolved.

Annex Theoretical definitions

These definitions have significant differences from the definitions in VIM [1]. This is caused by the change from the empirical (i.e., VIM) to the theoretical and empirical MM proposed.

1.0 Unit (general term)

1.1 In measurement mechanics:

- u is local (not relative to U) and identifies the assumed equal intervals of a local scale. u has a local numerical value which is $1/n$.
- u_i are one or more contiguous intervals which are calibrated to U. $\overline{u}_i$ is the mean u_i. After calibration each u_i has a numerical value and a precision relative to U or a factor thereof.
- *U unit standard* (capitalized) is one or a combination of defined properties of an axiomatic observable with a numerical value. U may be defined in theory as an exact numerical value, even though all empirical applications of U will have a ± precision.

1.2 Unit inconsistencies across disciplines:

- In common metrology practice, each interval (u_i) is empirically $\overline{u}_i$ which equals U after an empirical correction. This is not valid at very small scales.
- In statistics, a numerical value (not a quantity) may be relative to $\overline{u}_i$ or to U which may not be equal.
- In statistical mechanics, measurement results are a distribution of numerical values (not quantities) around an equilibrium [42], which may not be correlated to $\overline{u}_i$ or U.
- In QM (representational) bra-ket notation, a ket vector is a vector sum of unit vectors (u) [43] not correlated to $\overline{u}_i$ or U.
- In relativity (following representational theory), the requirement to calibrate measuring-rods (i.e., u_i intervals) relative to a standard to establish defined near equal measuring-rods, is not recognized [44].
- In representational theory calibration and standards are not included in the theory and u are assumed equal.

2.0 Other definitions applied in this paper

2.1 *Accuracy* is the change of the numerical value (x) of a measurement result relative to its mean over repeated measurement results B. (Section 1).

2.2 *Calibration* increases the precision of measurement result quantities by equalizing the u_i to a reference or standard or factor thereof. The common metrology and MM calibration processes (which are different) are described below Figs. 2 & 3 respectively.

2.3 *Calibration state*, *1/m*, is the smallest state of a *calibration scale* which is defined as equal to the other calibration states. The smallest possible size of a calibration state is a Planck.

2.4 *Gaussian distribution*s are created by statistical summing (Section 3). In this paper Gaussian distributions may include combinations of Gaussian distributions (see Section 4.3).

2.5 *Interval* (u_i) is the space between contiguous indications on a scale. Intervals may be calibrated to a standard to represent, but not be equal to *U.*

2.6 A *measurement* consists of the processes required to compare the properties of two observables, one of which may be a reference or standard.

2.7 A *measurement apparatus* measures an observable both in theory and practice.

2.8 *Measurement result* is a quantity determined by a measurement.

2.9 An *observable* in physics has one or more *properties*.

2.10 *Precision* is the statistical sum (Section 3) of the change of each u_i of a quantity relative to a *U* standard or factor thereof, as determined by calibration.

2.11, A *property* of an observable in physics is ordered and additive. An observable's property is only local before a BIPM standard defining the seven base properties, or some combination, is applied.

2.12 A *quantity* is a measured property consisting of a numerical value and a unit.

2.13 A *reference* is generally accepted, e.g., a mean. A standard is independently defined.

2.14 *Scale* is a set of contiguous intervals or states which quantify a property [45]. Four versions are presented:

- 2.14.1 *Local scale* has assumed equal states establishing an observable's zero point, order and additivity.
- 2.14.2 *Mean scale.* All intervals are $\overline{u}_i$.
- 2.14.3 *Relative scale* each interval is calibrated to *U*.
- 2.14.4 *Calibration scale* has defined-equal *calibration states* used to calibrate one or more intervals of a scale to *U*.

2.15 A *standard* (an axiomatic observable) is one or more sets of local scales (defined by processes independent of an observable and measurement apparatus) that support independent (non-local) repeatable comparisons.

2.16 *Zero point* is the beginning of an observable or an infinitesimal between two contiguous intervals on a scale, which supports measurement process 2 (Section 3).

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