Measurement Mechanics

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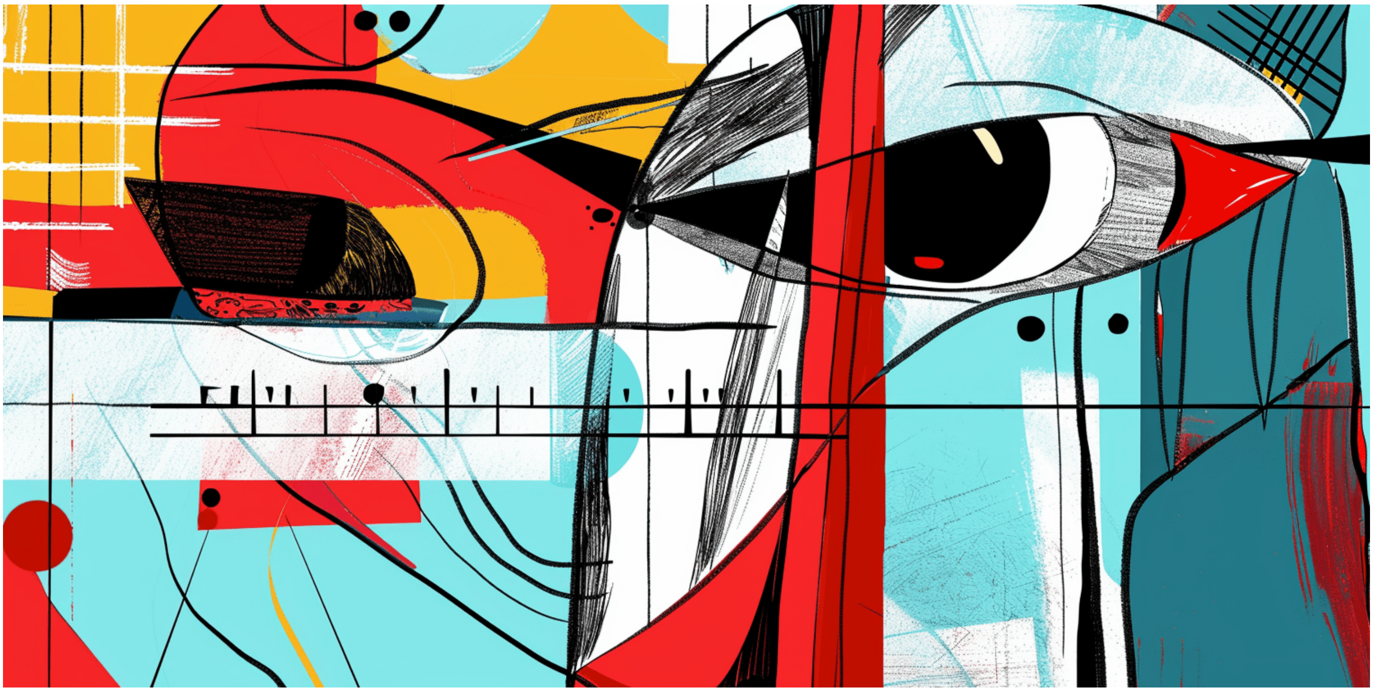

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Abstract: *"Measurement Mechanics" (MM) applies statistical mechanics to reframe representational measurement theory (probabilistic) and metrology (deterministic). This is shown to unify the different measurement theories and correlate the different formulations of measurement result deviation (i.e., uncertainty, standard deviation, variance, precision and accuracy) across the sciences.*

Following Euler (1765), MM identifies that repetitive measurements of an unchanged observable produce measurement result distributions relative to a reference or standard. Such repetitive measurement results (without noise or distortion) appear as a Gaussian distribution of probabilistic measurement result quantities (numerical values and units), not a single numerical value with an error distribution as current representational measurement theory indicates. MM treats these distributions not as errors, but as all the statistically possible sums of a measurement apparatus's interval values relative to a reference or standard.

Fig. 1 identifies the frame of reference; Fig. 2 diagrams representational measurement theory; Fig. 3 diagrams how metrology relates to representational theory; and Fig. 4 proposes measurement mechanics.

The result of the paper's development (eq. (8) on page 12) identifies that the proposed statistical measurement function converges to the commonly applied metrology measurement function when the numerical value is large. The ramifications to measurement theories and experiments are summarized in the three paragraphs below it.

Keywords: measurement theory, metrology, calibration, uncertainty, precision, quantization.

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Measurement Mechanics[1]

1. Introduction

Measurement theories and results differ across the scientific disciplines. Fig. 1 presents the relationships between different measurement theories and results. The differences in the vertical dimensions represents changes that occur in the scale of the measured quantities or sizes. The differences in the horizontal dimension are differences in the presentation of measurement results due to unexplained differences in the theories. These differences are loosely grouped as probabilistic, mixed, deterministic and metrology, which is set apart.

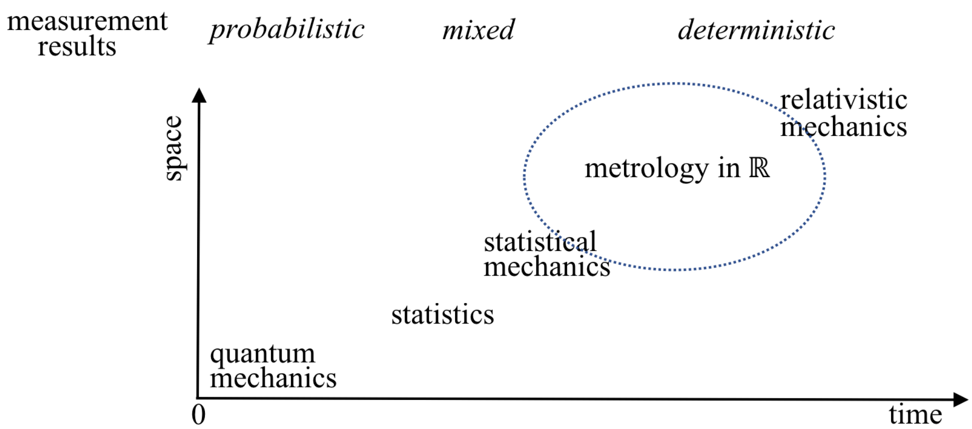

Figure 1. Theoretical measurements in spacetime ($\mathbb{R}^n$)

Mixed and deterministic measurement theories appear reconciled. However, probabilistic and deterministic measurement theories are closely related, but are not reconciled and metrology is set apart as empirical. One formal measurement function consistent across the physical sciences would be invaluable. This is what Measurement Mechanics proposes.

2. Measurement inconsistencies

Measurement results at the quantum scale (Fig. 1 bottom left) are probabilistic (has numerical values). Statistics and statistical mechanics support measurement results as probability distributions or deterministic results (mixed). Relativistic mechanics and metrology treat a

[1] Measurement Mechanics refines or adds the definitions of 21 measurement terms in the Annex. The first text instance of a word defined in the Annex is identified in a footnote with its Annex location (Ax.x).

measurement result[2] as a quantity (e.g., 5 kilograms, where 5 is the numerical value and kilogram is the unit[3], reference[4] or standard[5]) which is deterministic (has quantities).

Currently there are two measurement paradigms that appear consistent, but are not:

- Representational theory - a measurement result is a distribution of numerical values on a scale (probabilistic). How a physical measurement result occurs is empirical.
- Metrology - an empirical measurement result is a quantity with deviation (deterministic).

The basic book on representational measurements is *Foundations of Measurement* [1]. In this theory a measurement result is a numerical value on one of three basic scales[6] (ordinal, counted/linear and ordered).

In metrology [2], a measurement result is a quantity which is the product of a numerical value and a reference. In metrology practice, a scale is calibrated relative to a reference or standard[7].

A quantity is foundational to a measurement result. A quantity with a unit, equal or relative to a reference or standard, is required for any independent comparisons[8] of measurement results. When a measurement result is only a numerical value, only probabilities can appear. L. Euler (1765) made this clear: “Now, we cannot measure or determine any quantity, except by considering some other quantity of the same kind as known and pointing out their mutual relation” [3].

The lack of a 'known' (i.e., a unit, reference or standard), which supports 'mutual relations', explains the probabilistic measurement results in representational theory and in quantum mechanics, statistics and statistical mechanics, disciplines that apply representational theory.

Other inconsistencies between representational theory and statistics also appear. In statistics the central limit theorem identifies that when applying a scale (without noise or distortion), repetitive measurement results of unchanged observables[9] will have a distribution that converges on a normal distribution [4]. Such distributions are treated in representational theory as distributions of errors due to noise and distortion in measurement processes [5]. In experimental measurement systems where noise and distortion are closely controlled, a normal distribution

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2 A2.8

3 A1.0

4 A2.13

5 A2.20

6 A2.15

7 Metrology comingles the terms reference and standard (VIM 5.6).

8 A2.3

9 A2.9

still appears [6], therefore a normal distribution must be caused by something other than noise and distortion.

Measurement Mechanics (MM) identifies that these normal distributions occur because of random sampling or quantification effects, currently assumed to cancel, which sum into deviations that can be significant. These unexplored deviations have been masked by the inconsistent usage of the term unit between the disciplines (see A1.1), as well as the mathematics necessary to sum random quantization effects.

3. Representational theory

A linear scale (equal intervals in $\mathbb{R}$) is a focus in Foundations of Measurement [7] and is illustrated in Fig. 2 with no noise or distortion. The representational theory of measures[10] establishes a mutual relation between the observable and the scale which determines the numerical value of the observable. In Fig. 2, representational theory treats a standard as arbitrary [8] and the calibration to a standard as empirical, therefore not shown. This is inconsistent, as metrology requires calibration and requirements are imposed by theory.

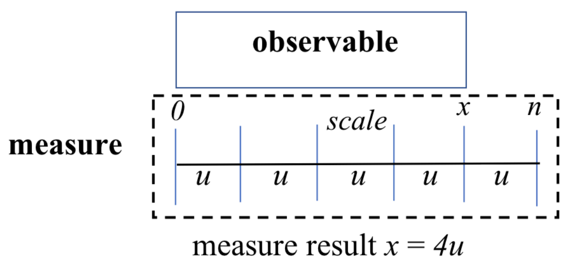

Figure 2. Representational theory

Applying representational theory, measures with a scale undefined to a reference or standard can only be represented independently as a probability distribution. Then metrology converts this probability distribution into a deterministic result by applying a standard. This appears consistent, but on closer examination is not.

4. Metrology

In Fig. 3 (without noise or distortion), an observable which has a preexisting [9] property[11], shown as a set of u, is termed a measurand[12]. A measure then establishes a mutual relation between the measurand and the scale which quantifies the property of the observable.

[10] From Section 3 to the end, the terms measure and measurement are applied as defined in the Annex.

[11] A2.11

[12] A2.5

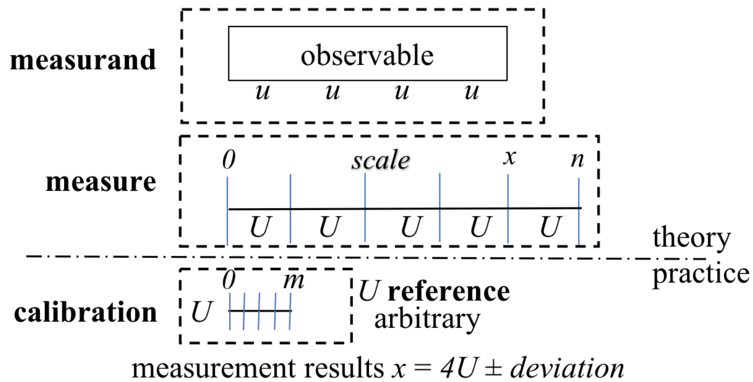

Figure 3. Metrology practice with representational theory

Representational measures are shown slightly differently in Fig. 3 to conform to metrology practice. Metrology practice calibrates the scale to a standard, establishing deterministic measurement results with deviation. The usage of the terms 'reference' and '*U*' in Fig. 3 (following metrology) are comingled.

In metrology, calibration[13] is empirical and compares measurement[14] instrument intervals[15] to the standard *U* or a factor of *U*. This determines the precision of the measurement instrument, which is presented (slightly inconsistently) as the precision[16] of a measurement result.

The division of representational theory results (numerical values) from metrology results (quantities) is not consistent. In metrology the *u* of the measurand, being preexisting, cannot be adjusted by calibration to *U*. And in representational theory, the *u* of the observable and the *U* are defined as equal, even though this requires calibration, which representational theory does not include. These are significant inconsistencies in representational theory and metrology.

All physical measurements standards are based upon BIPM [10]. As example, a scale (a physical measurement instrument) to make a measurement of mass is calibrated to a kilogram standard, *U*. The scale is then is brought to a brick (observable). First the scale's zero point[17] and the brick's zero point are aligned, then the brick's mass property is measured. Here *U*, the kilogram standard via the measurement instrument, defines one property to be measured of a brick's multiple properties (e.g., mass, length, volume, number of molecules, etc.).

A standard or reference defines the property measured as well as its numerical value. A standard or reference is required for comparable measurements (as Euler noted) in theory and practice, and its numerical value is arbitrary in first use only.

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13 A2.2

14 A2.7

15 A2.18

16 A2.10

17 A2.21

When calibration in representational theory is solely empirical, the precision of the unit of a measurement instrument is also solely empirical. The precision of u of a measurement instrument relative to a standard then becomes perfect in a measurement system theory where noise and distortion are removed. This results in one single measurement result termed a true value in VIM (para 2.11). Heisenberg's uncertainty theory [11] makes the possibility of a true value measurement result erroneous (even though a very small error) and also ignores the ubiquitous central limit theorem; therefore calibration demonstrates the fallacy of representational theory.

5. Measurement mechanics

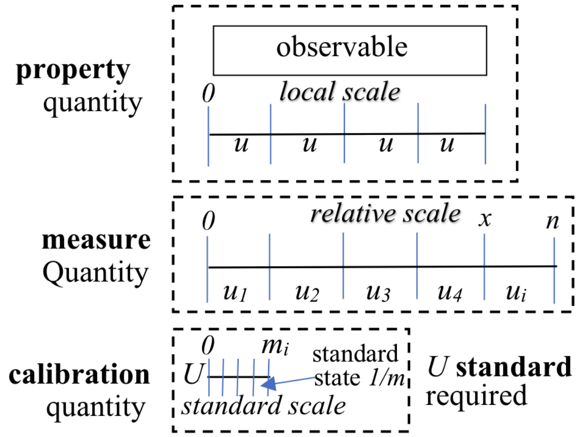
In 1891, J.C. Maxwell [12] proposed that a measurement result in theory and experiment is a quantity (see Section 11.1) which includes two variables: a numerical value (n) and a unit (u). His proposal has been widely applied, but not in representational theory. Following Maxwell's proposal a measurement process must measure both variables to determine a quantity and its deviation. However this form of quantity still has limitations.

When the unit is defined as exact (i.e., relative to a standard), which is the representational theory assumption that $u = U$, then a scale of n u or U intervals is linear and each $u = 1/n$. Such perfection is not possible in a quantized system. A measurement result applying the intervals of any physical measurement scale will deviate ± the minimum quantization. That is, the intervals have a random component relative to U even in theory and at the limit will deviate by a Planck (i.e., ± Planck). When each interval has a non-linear component, a measurement function in theory must be a sum over each u_i, not nu or nU. This may be required even if the non-linear component is very small, because of statistical summing (developed in Sections 6 and 8 with examples in Section 7.1 and 7.2 below).

A quantity (product) is expedient for many experimental measurement results. A Quantity (capitalized to identify the summation), see (1), is a proper superset of a quantity, $Q \supseteq q$. Eq. (1) is proposed as the first step towards a formal measurement function based upon Fig. 4.

Quantity distribution $$Q = \sum_{i=1}^{i=n} u_i \qquad (1)$$

In (1), u_i represents each of the intervals of the relative scale in Fig. 4. However, each u_i is not treated as equal, whereas all the u on the local scale are defined as equal. Relative Measurement Theory [13] (RMT) verified that the u_i are not theoretically or empirically equal, especially at quantum scales. The notation applied in RMT for a random component of u_i is Δ. This notation is changed to ± precision in MM. The random ± precision of each u_i converges to a normal distribution of quantities when it is summed [14].



measurement results of x = eq. (4)

Figure 4. Measurement mechanics

A physical measurement instrument often includes transducers (not shown in Fig. 4) which convert a quantity on a local scale to a Quantity on a relative scale. Additionally calibration stages which include intermediate standards often occur in practice, and are also not shown. With these provisos this paper proposes that Fig. 4 applies to all physical measurements in theory or practice and is definitional for physical measurements in theory without noise or distortion.

In MM, there is no arbitrary boundary between theory and practice. The first measure is probabilistic, and calibration (second measures) is relative to a standard, establishing deterministic measurement results with accuracy and precision. When operator errors are not considered, any deviation of a measurement result quantity has three possible causes: quantization (standard state[18], $1/m$), noise (external to the measurement system), and distortion (internal to the measurement system). Fig. 4 focuses on quantization.

In Fig. 4 each interval of the scale may be calibrated (more rigorous and less practical) to a standard or (most common practice) the mean interval of the scale is calibrated to a standard. $1/m$ is the smallest identifiable change of U (resolution in VIM) in the measurement mechanics system. The assumption here is that U is known to an even smaller resolution. Integers n and m represent counts when $1/n$ and $1/m$ represent the smallest resolution of their respective scales.

A precise measurement result (i.e., the numerical value of the measurement result has a precision smaller than one unit), in theory or experiment, can only occur when a quantity is divided into yet smaller states than its unit. Therefore, calibration to a standard is required in any precise measurement theory as well as in any precise experimental measurement.

[18] A2.14

6. The four mutual relations in measurement mechanics

An MM measurement system correlates an observable by two measures to a standard scale[19]. In the first measure, a local scale[20] provides the correlation between an observable and a relative scale[21] which establishes a numerical value of local states. This measure is implemented with two mutual relations:

1. A count of the x states establishes a numerical value (first mutual relation).
2. Determination of a common zero point (a mutual relation to the relative scale).

Fig. 4 identifies that when an observable's property is ordered, additive, and has a zero point (modeled by a local scale), the observable's property can appear on the relative scale as a count. The observable's property is not preexisting as is shown in Fig. 3, but established by the local scale as shown in Fig. 4. Then a count - which is a measure, but not an independent comparable measurement - can occur.

As example, a wooden stick (observable) is determined to have a length quantity by counting regular notches (a local scale) on the wooden stick. But there is no precise means (precision better than than the distance between two contiguous notches) to compare the length of this wooden stick with others.

The second measures, often termed calibration to U, determine each u_i relative to U. These measures are implemented with two more mutual relations:

3. The measures of each u_i.
4. Determination of the precision of each u_i (a mutual relation to U). Where the minimum quantity $1/m$ = one standard state of $U = 1$. This results in two forms (quantized or normalized) of u_i precision:

$$u_i = u \pm (1/m)_i = (U \pm 1)_i \tag{2}$$

When these four mutual relations are in place, a statistical summation (3) of each of the x $(U \pm 1)_i$, establishes a distribution of quantities - the measurement results:

$$\sum_{i=1}^{i=x} (U \pm 1)_i \text{ which converges to a normal distribution} \tag{3}$$

Statistical summing (without noise or distortion) example: The numerical value of a measurement result (e.g., *4*) is the sum of four contiguous intervals ($i = 1 - 4$) on a relative scale: $u_1+u_2+u_3+u_4 = 4u_i$. Each u_i has a ± precision (which is *2* random $1/m$ calibration states) due to the quantization ($1/m$ or *resolution* in VIM) of the measurement instrument relative to the standard or factor thereof, which when each of the two possible states of four u_i are summed, produces $2^4 = 16$ measurement result quantities that converge to a normal distribution of u_i. Such normal distributions of u_i occur in all repetitive physical measurement results of unchanged

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[19] A2.19

[20] A2.16

[21] A2.17

observables, verifying statistical summing. The statistical variation of an observable relative to a known is fundamental in a quantized state space [15].

Converting from $\mathbb{R}$ (the basis applied in this paper) to $\mathbb{R}^n$ (where vectors sum), a measurement result quantity in $\mathbb{R}$ must be squared (e.g., $(nu)^2$) in $\mathbb{R}^n$ and is not squared (e.g., nu) when treated in $\mathbb{R}$. This effect is seen in the standard deviation (σ) in $\mathbb{R}$, which is the square root of the sum of the squares (in $\mathbb{R}^n$) and variance (in $\mathbb{R}^n$), which is the square of precision in $\mathbb{R}$.

$\pm$ *1* in (2) may be seen as a normalized form of the standard deviation (σ)of u_i in $\mathbb{R}$. This standard deviation of $u_i = \sqrt{\tfrac{1}{2}\left[(U+1)^2 + (U-1)^2\right]} = \sqrt{U^2+1} = 1$ (normalized) when $U = 0$ which creates a unit normal distribution. When the measurement system is without noise or distortion, the standard deviation of each u_i is ± 1 relative to U, which is termed precision $\pm(1/m)$ in metrology and is termed uncertainty (at the limit ±Planck) in quantum mechanics.

Then (3), another unit normal distribution, may be seen as all the statistical sums of $U + \sigma$. (3) also completes a derivation of the central limit theorem in $\mathbb{R}$ [16].

In (3), a normal quantity distribution, x varies either because of a population change or due to accuracy. In order to treat accuracy the $u \pm (1/m)_i$ form (quantized) of (2) must be used. Applying the same standard state ($1/m$) to x (which determines the accuracy of x) produces the final form:

$$\text{measurement result Quantities} = \sum_{i=1}^{i=x\pm(1/m)} \left(u \pm (1/m)\right)_i \tag{4}$$

Eq. (4) is the measurement mechanics measurement result function that applies to all formal and experimental measurement systems without noise or distortion.

7. Empirical measurement examples

The development presented above is a paradigm shift from the inconsistent representational theory and metrology practice which measurement are based upon today. The following three examples are offered in support of this paradigm shift.

7.1 Additive relative scale

An example of an additive relative scale is a thermometer which measures the quantity of thermodynamic temperature. This example demonstrates how additive imperfect intervals statistically increase the deviation of measurement results, producing a normal measurement result distribution.

The measurement instrument consists of a hollow glass tube with a reservoir filled with mercury at one end, which fits inside another hollow glass tube that slides over the first. The two glass tubes are held together and placed in an adjustable temperature oven which has a resolution of 0.1^0 (degree). Then the outside glass tube is marked at the level of mercury which appears and marked again with each 1.0^0 (u_i) increase in temperature of the oven. $n + 1$ marks or *101* marks are made to quantize the outside glass tube. Each of the $100\ u_i$ is correlated using the oven to $1/0.1 = 10 = u_i \pm 0.1^0$ precision.

After 101 marks are made, the instrument is removed from the oven and an ice water bath is applied to the tube with mercury reservoir. The outside glass tube is now slid over the inside glass tube until the top of the inside mercury column lines up with the first mark on the outside glass tube. Now this mark on the outside glass tube is the zero point for ice water (0^0C) (mutual relation 2).

Consider the water in a glass (observable) to be in contact with the reservoir of this measurement instrument. If the temperature (a property) of the water is 80^0, the *81st* mark on the outside glass tube represents $80^0 \pm 0.1^0$ nominal precision or $\pm 8^0$ which is the worst case (very very rarely possible). The $\pm 0.1^0$ nominal precision occurs when the $\pm 0.1^0$ precision of each $80\ u_i$ is uniformly distributed and cancels. The $\pm 8^0$ precision occurs when each set of the $80\ u_i$ has the same $+0.1^0$ or -0.1^0 precision, which sums.

In the proper design of experimental measurement systems, the statistical sum of the quantization effects are reduced to less (usually) than the noise or distortion and is ignored. But in this thought experiment without noise or distortion, when each mark's precision is specified to be $\pm 0.1^0$, $\pm 8^0$ is very very rare, but not impossible. The statistical sums of the precision from $\pm 0.1^0$ to $\pm 8^0$ establish a normal distribution of measurement results (see Section 7.3 below).

7.2 Length measurement instrument

A physical metre stick (a scale calibrated to a standard standard metre U) is divided into 100 intervals (smallest u_i). Consider the length whose numerical value is $x = 70$. In the standard measurement theory proposed here, the numerical value of each u_i is treated individually and then added to the next u_i (70 times).

When first calibrated to U (e.g., the standard metre), each $u_i = (U/100) \pm (1/m)$ precision where, as example, each $1/m$ is $1x10^{-6}$ metres (e.g., $m = 10^6$). The accuracy[22] of x is ignored in this example. In MM, the Quantity deviation is established by the random application of $\pm (1/m)$ to each u_i producing a normal distribution of measurement results. In the statistically rarest two cases, when n of the u_i, each with a precision of $+(1/m)$, are summed and in another measurement of n, all of the u_i, each with a precision of $-(1/m)$, are summed, the maximum Quantity deviation appears $2(70)10^{-6} = 1.4x10^{-4}$ metres, which is sufficient precision ($\pm 0.7x10^{-4}$) for a metre stick. When $m >> n$, the effect of quantization is usually and realistically ignored. However, when n and m are both small, e.g., quantum scale measurements, the statistical summing of each $\pm (1/m)$ causes deviations which can be significant (Section 7).

[22] A2.1

7.3 Normal measurement result distributions

Fig. 5 presents the characteristic Gaussian shape of a large distribution of repetitive linear physical measurement result comparisons of unchanged observables (normal distribution). This shape has been verified in many different forms of experimental measurement results where noise and distortion are minimized [17]. The ubiquitous nature of a normal distribution of repetitive measurement comparisons, caused by the summing of the quantized precision, strongly supports MM.

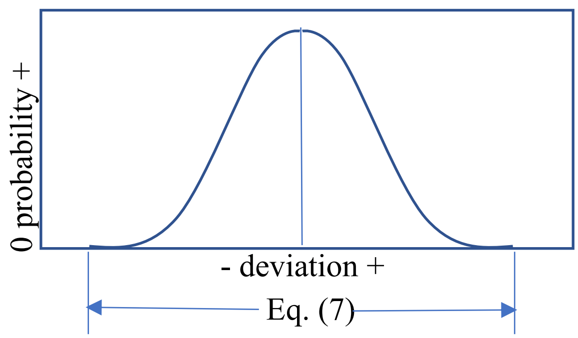

Figure 5. Gaussian distribution of quantized measurement results.

8. Deviation from a standard.

Over many repetitive measurement results, each statistical sum of (3) is one measurement result in a normal distribution (see Fig. 5 above). In statistically rarer cases (but still valid measurement results), the normal distribution of (3) becomes increasingly dispersed. For a measurement function to represent the normal distribution created by the statistical sums, a Quantity (summation) must be used. When a quantity (product) is used, the statistical sums are not treated.

Examining the range of deviation of (4):

$$\text{each } u_i \text{ precision } = \pm\left(1/m\right)_i \tag{5}$$

This precision, ± *(1/m)* relative to U in each of the x u_i in (4), varies randomly (in theory) and statistically sums into:

$$\text{the deviation distribution of (4) } = \pm \sum_{i=1}^{i=x\pm(1/m)} \pm(1/m)_i \tag{6}$$

$$\text{the worst case deviation of (6) } = (2/m) + (2x/m) \tag{7}$$

In (7) worst case deviation (not exactly Gaussian) produced by (4) is determined by *2/m* (accuracy) and *2x/m* (precision). In counted population distributions (precision is perfect) the accuracy term is obvious.

9. Understanding measurement discrepancies

$$\sum_{i=1}^{i=n}\left(u \pm (1/m)\right)_i \supseteq n\overline{u}_i \qquad (8)$$

Eq. (8) compares measurement mechanics (4) with metrology. Without treating the accuracy of *n*, noise or distortion, the left side of (8) is the MM measurement function and the right side of (8) is a common metrology measurement function. Statistically, (8) may be seen as a comparison of the standard deviation (left side of $\supseteq$) to the mean deviation (right side). It is well know that when *n* is small the standard deviation diverges from the mean deviation [18].

The two sides of (8) produce equal measurement results with high probability when *m* and *n* have large numerical values (e.g. classic measurements). In (8) when *n* or *m* is large (a common experimental measurement, see Section 7.2), the two random calibration states $\pm(1/m)$ of each u_i likely have a very small effect on measurement results' deviation.

Conversely, when *n* and *m* both have small numerical values (e.g., in quantum spin experiments, $n = m = 2$) [19], two repetitive measurement result Quantities of the same observable will often be different because the precision of each u_i ($\pm (1/m)$) will likely be different. As example, applying (4), 50% of the repetitive measurements on equal observables will be different when $n = m = 2$. This measurement result difference appears in representational theory as varying numerical values (*n*), or non-commuting Fourier pairs (see Section 10.3), when the precision of each u_i is not statistically summed.

10. Explaining the discrepancies in quantum scale measurements

Irrespective of the above development, quantum mechanics (QM) has applied representational theory by applying the ratios of quantities with common units which have invariant numerical values. That is, in such a ratio of quantities, the numerical value ratios remain the same when the common units in the ratio change numerical values. Thus QM, based upon representational theory, successfully ignores the units and their quantities.

However, quantum scale experiments and thought experiments still evidence discrepancies because the four mutual relations are not recognized, causing experimental discrepancies. These discrepancies are currently recognized in QM and it is assumed they will be (or are) resolved by an interpretation of QM theory. This has not yet occurred [20].

In the following, the identified inconsistancies of Fig. 2 and Fig. 3 are shown to be the basis for the discrepencies that appear in quantum scale measurement experiments.

10.1 Remote entanglement

Remote entanglement is used here to describe the numerical value entanglement evidenced between two measures in the Stern-Gerlach experiments. There have been many attempts to understand remote entanglement. J.S. Bell's formalization (since verified experimentally [21]), which perhaps consolidates the earlier attempts, is addressed. J. S. Bell, in his paper [22] stated:

"...there must be a mechanism whereby the setting of one measurement[23] device can influence the reading of another instrument, however remote." In the Stern-Gerlach experiments Bell discusses, that mechanism is mutual relation 2 (described in Section 6).

N. D. Mermin in 1981 [23] statistically analyzed the results of Stern-Gerlach experiments [24] that identify remote entanglement. Remote entanglement is described by Mermin without QM formalism, which indicates it will occur in all measurement results. Mermin identifies that the measure results of the two remote entangled particles are relative which requires an unknown interaction between the two measure instruments. Not recognizing that a measurement requires the four mutual relations, Mermin identifies the unknown interaction - the correlation of the two measure instruments' zero points - without realizing it.

In Mermin's model of the experiments, the two measure instruments each have a three position selector which selects one of three 120^0 intervals, which represents the spin's numerical value. Mermin's two measure instruments require a common zero point among the three 120^0 units since the common unit (spin) and common scale (three 120^0 intervals, each a u) are provided by the experimental set-up.

There are nine possible combinations of the two, three position selectors: Three (Mermin's case a) when each selector (a scale) is in the same position and six (Mermin's case b) when the positions of the two selectors are different. When the two remote selectors are in different positions (the two measure instruments' zero points are uncorrelated), each n (0^0 or 180^0) of the two particles' spin vectors appears randomly over a large number of runs. Only in Mermin's case a is each n of the two particles correlated, because there is a common zero point (mutual relation 2) between the two measure instruments' selectors. When this necessary zero point correlation appears in experiments, but does not appear in representational theory, discrepancies occur.

10.2 Compton-Simon experiment

In the Compton-Simon cloud chamber experiment [25], the energy and momentum after collisions between light and electrons are measured by the positions or by the central line of the collision (two independent ways) and at different times in the same experiment. However, both measures always confirm equal results unlike repetitive measurements which always have a Gaussian distribution of measurement results.

It appears that the energy mutual relations 1 & 2 (a measure, confusingly considered a measurement) foretell the momentum mutual relations 1 & 2 (or the reverse), before they are known. In actuality, in the one experiment, which describes two repetitive measures without a measurement, mutual relations 3 & 4 (which cause a Gaussian distribution) do not occur for the measure of energy or the measure of momentum, therefore both measure results are equal, i.e., they do not have a Gaussian distribution.

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23 Bell's use of the term measurement does not follow the definition in A2.7.

10.3 Heisenberg's uncertainty

In Heisenberg's 1927 experiment [26], a single particle's two Fourier dual quantities are measured at different times. From his discussion: p (momentum) has units of mass (m) and distance (d) divided by time (t) or md/t, and q (position) has the units of d and t or dt. Then in quantity calculus notation [27] $p = \{n_p\}[md/t]$ and $q = \{n_q\}[dt]$.

His experiment identifies that the precision of each quantity, $p_1 = \{ n_{p1}\}[md_1/t_1]$ - $\{n_{p2}\}[md_2/t_2]$ and $q_1 = \{n_{q1}\}[d_1t_1]$ - $\{n_{q2}\}[d_2t_2]$, varies inversely with time. This occurs because the time unit change t_1-t_2 varies inversely in p relative to q over the same distance unit change d_1-d_2. Therefore p_1 varies inversely relative to q_1 as time changes since the time units are not common and therefore are not invariant. Heisenberg certainly understood the inverse time relationship between p and q. However, since QM does not treat quantities, only their numerical values (n), the inverse variation of the time units of p relative to the time units of q is unclear in QM. Inverse time variation will always appears in Fourier duals and is not unique to QM.

Separately, the precision of the repetitive measurement results between p and q varies randomly and statistically adds, thus the product of p and q when measured twice may be different.

In QM, u is unitary and is a factor of everything. u then is without import, and a ratio of two measurement results is relative to each other, rather than each relative to a standard. Without including a standard, quantum uncertainty appears as relations between Fourier duals, rather than the precision of any quantized measurement result Quantity relative to a standard in a quantized space. A more rigorous analysis of the correlation between the different forms of uncertainty and precision is presented in the Relative Measurement Theory paper.

10.4 Double slit experiments

Feynman's [28] explanation of the double slit experiments offers a good example of how a standard correlated to a measurement instrument defines which property of an observable is measured. Feynman concludes, "...when we look at the electrons the distribution of them on the screen is different than when we do not look." What he meant is, applying a standard ("when we look") defines a property of an observable.

In these experiments the plate with slits is a transducer and the sensing screen is a scale for identifying patterns. The set of slits pass two different properties (frequency or mass), and the sensing screen presents two standard patterns. When the slits in the plate are replaced with small holes only the mass property appears on the sensing screen. Other cutouts or movements of the plate will produce different patterns.

Physical observables have both frequency and mass properties. The operator, by looking for a standard pattern on the sensing screen, identifies the wavelength of a particle's frequency property or identifies the point of impact of a particle's mass property.

Consider the brick example: the selection of a mass measurement instrument (defined by a kilogram standard) selects the brick's mass property. If the measurement instrument was a ruler (defined by a metre standard) a brick's dimension properties would be measured. When representational theory does not recognize how a standard defines a property, discrepancies occur.

11. Relating measurement mechanics to other theories

11.1 History of a quantity

J. C. Maxwell [12], proposed that a measurement result quantity is:

$$\text{measurement result quantity } q = nu \qquad (9)$$

Maxwell proposed that the unit of a quantity is "taken as a standard of reference" [29]. This wording (comingling reference and standard) strongly suggests the unit is equal to the standard. Then (9) implies representational theory: that perfect precision is possible in theory, which makes all units equal and the standard arbitrary in the same theory. However, representational theory is not consistent: a standard defines an observable's property, a standard's numerical value is only arbitrary in the first use, and in 1927 Heisenberg proved that perfect precision (the reverse of his uncertainty theory) is not possible in a quantum space.

Perhaps Maxwell assumed that a theoretical measurement result could be exact, whereby calibration is empirical and u or U are equal and arbitrary. In any event, Maxwell's usage instigated what is now representational theory (ref [26] describes this history).

11.2 The Einstein, Podolsky, Rosen (EPR) paper

Defining measurements relative to a standard, as MM proposes, identifies a standard as the hidden variable [30] which the EPR paper [31] suggested is missing. The EPR paper, which is based upon representational theory, does not recognize that all measurements are relative to a known reference or standard. Einstein explained (similar to Euler) that everything physical is relative [32], but didn't recognize that measurement result comparisons require mutual relations to knowns that representational theory does not identify.

Measurement mechanics is based upon understanding the significance of knowns. Isology (Iso = same, logy = science of) is the proposed name for the very broad scientific discipline that studies all forms of knowns and their creation, i.e., references, standards and standardization. When all else is relative, knowns are invaluable.

11.3 The representational theory

Representational theory does not recognize a quantity [33]; assumes measurement result comparisons occur without a calibrated scale or standard; assumes units are equal [34], which requires any calibration to be empirical [35]; and indicates that all measurement result quantity deviation is due to noise and distortion in the measurement system [36]. Each these inconsistencies has been shown to cause discrepancies.

11.4 Other measurement discrepancies

The inconsistencies in representational theory and metrology also create other measurement discrepancies. The Relative Measurement Theory paper described the entropy change (*log m*) caused by calibration in a measurement which is currently thought of as collapse or decoherence in different interpretations of QM measurements [37]. In Measurement Unification, 2021 [38], explanations based upon RMT are given of quantum teleportation experiments and Mach-Zehnder interferometer experiments. The Schrödinger's Cat thought experiment is explained in a short preprint [39]. These papers, together with the explanations in this paper, strongly support applying MM to all physical measurements in theory and practice.

12. Conclusion

Metrology and representational theory have been applied successfully for a long time. Measurements of population distributions, often to a reference, are very useful in the social sciences. Existing representational theory and metrology support such analyses. However, as Euler explained in 1765, the EPR paper formally developed in 1935, and Bell refined in 1989, the inconsistencies across measurement theories and experimental measurement results beg to be resolved.

In the physical sciences, measurement results are relative to a physical reference or standard in order to be precisely comparable. By recognizing that physical references and standards are invaluable, Measurement Mechanics establishes one formal measurement function consistent across the physical sciences. When this is applied, the inconsistencies across all the measurement theories and experimental measurement results can be resolved.

Annex Definitions

The definitions below have significant differences from the definitions in The International Vocabulary of Metrology [2]. This is caused by the paradigm shift to measurement mechanics proposed in this paper. The MM definitions are aligned with formal definitions where appropriate. This list of definitions is not exhaustive.

1.0 Units in different disciplines

1.1 Inconsistencies:

- In metrology practice, a unit (*u*) is empirically the mean *u* which is calibrated to *U* (known) which is relative. Also in metrology, *u* is a reference (known) which is representational.
- In statistics, a numerical value usually is relative to a mean unit or *U* (knowns).
- In statistical mechanics measurement results are a distribution of numerical values around an equilibrium [40], which may be a known.
- In QM (representational) bra-ket notation a ket vector is a vector sum of unit vectors [41] which are knowns.
- In relativity, mass and energy are formalized relative to a known (the velocity of light). However, the equality of concatenated rods (units) is assumed, not known [42].
- In representational theory (which professes to be foundational) quantities, units, references and standards (all forms of knowns) are not recognized.

1.2 In measurement mechanics:

- *u* is local and identifies each of the smallest intervals of a scale uncalibrated to *U*. Each uncalibrated *u* has a local numerical value (*1/n*, a probability) and unknown precision.
- u_i is quantified by the smallest intervals of a scale calibrated to *U*. Each calibrated u_i has a numerical value, quantity and precision all relative to *U* or a factor thereof.
- *U standard unit* (capitalized), is a defined property with a numerical value ± precision in the standard scale's states. A property of *U* may be defined in theory as an exact numerical value, even though any application (theory or practice) of *U* will have a ± precision in a state space.

2.0 Other definitions applied in this paper

2.1 *Accuracy* is the ± random change (50% + or 50% - when only quantization is considered) of the numerical value (*n*) of a measurement result Q/quantity relative to its mean numerical value over repetitive measurement result comparisons. Accuracy is not rigorously defined elsewhere.

2.2 *Calibration*, is the measure of one or more intervals of a relative scale to a reference or standard. Calibration may include multiple sequences of measures to intermediate references or standards. The more rigorous form of calibration presented in MM statistically sums the precision of each scale interval relative to the standard (*U*), establishing a mean interval, $\overline{u}_i$ relative to *U*.

2.3 *Comparison*, the similarities or differences between the numerical values of two measures (representational) with the same properties, or the similarities or differences between two measurements Q/quantities with the same units (relative or defined).

2.4 *Deviation* is all the combinations of the accuracy of a numerical value and the precision of a unit. The deviation of a measurement result distribution includes the precision to a reference (relative) or to a standard (defined).

2.5 *Measurand,* defined in VIM. Closely related to either a quantity or a property in MM.

2.6 *Measure* [43], a numerical value determined by applying a scale. When measures are compared to other measures this is representational. When a measure is compared to a standard this is termed calibration.

2.7 A *measurement* (a process) consists of two measures which compare an observable to a reference as shown in Fig. 4. The common use of the term measurement for one or two measures which may be representational, relative or defined is not rigorous and is abandoned starting in Section 3.

2.8 *Measurement result*, a quantity determined by a measurement. The distinctions between representational measure results (limited), metrology measurement results (have discrepancies) and measurement results to a standard (independent) is not currently recognized.

2.9 *Observable*, (common term in QM) that which is observed before a local scale is applied.

2.10 *Precision* is the statistical sum of the ± random change of each u_i of a Quantity relative to a U standard or factor thereof, as determined by calibration. When only quantization is treated, the notation ± represents the precision distribution 50% + and 50% -, which changes each u_i randomly. In statistics, precision in $\mathbb{R}$ with the same properties is $1/\sigma$ and in $\mathbb{R}^n$ is $1/\sigma^2,$ where σ^2 is termed *variance* in statistics. The metrology definition (VIM) of precision is not rigorous as it is in statistics.

2.11 A *property* of an observable is determined by applying a scale. In physics the term *variable* suggests a property. Euler used the words "same kind" for a property. The numerical value of physical properties is defined relative to BIPM standards. Properties not associated with BIPM standards or their derivations are measured often and may (metrology) or may not (representational) be relative to a reference or standard.

2.12 *Q/quantity* consists of a numerical value and a unit whose the numerical value is known relative to a reference or standard. Upper case Quantity indicates a statistical sum of u_i numerical values. Lower case quantity (closely related to the VIM term measurand) indicates a product of the numerical value and the $\overline{u}_i$ or u numerical value. Q/quantity identifies both the statistical sum or product functions. 2.12 is different from the definition of a quantity in VIM which requires that $u = U$.

2.13 *Reference* is a state or states of a scale that supports limited comparisons. In statistics, a reference can refer to a mean, median or mode.

2.14 *Standard state*, *1/m*, is the smallest defined-equal state of a standard. The smallest possible size of a standard state is a Planck. A set of states quantifies each interval, u_i.

2.15 *Scale* [44] is a contiguous set of common intervals or states which may represent a physical measurement instrument or a formal measurement function. A *local scale* (2.16) has defined-equal states (is linear) establishing an observable's order, additivity and zero point which transforms an observable into a numerical value of a property. A *relative scale* (2.17), has *intervals* (2.18) which are relative to a reference or standard. A relative scale's precision is determined by calibration to a *standard scale* (2.19) which has defined-equal states and is correlated to a standard.

2.20 *Standard* (established by standardization processes) is a more rigorous reference that supports independent comparisons. A standard supports comparisons that may have greater independence than the comparisons a reference supports.

2.21 *Zero point* is an infinitesimal between two intervals or states on a scale, which supports mutual relation 2.

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