Improved Cosine Similarity Measures and Extended TOPSIS for q-Rung Orthopair Fuzzy Sets: Applications in Green Technology Selection

Mehmet Ünver[1]

1 Ankara University

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Abstract

In this study, we present novel cosine similarity measures designed for *q*-rung orthopair fuzzy sets (*q*-ROFSs), offering a comprehensive analysis of both direction and magnitude aspects in fuzzy set representations. Unlike traditional cosine similarity measures, which primarily focus on the direction (cosine of the angle) between vectors, our proposed measures address this limitation by incorporating a lengths difference control term. This enhancement becomes crucial, especially when dealing with overlapping vector representations of *q*-ROFS components with a height difference, where traditional measures yield a similarity measure of 1. We demonstrate the effectiveness of these improved cosine similarity measures, showcasing their superiority not only over traditional counterparts for *q*-ROFSs but also in enhancing existing measures for intuitionistic fuzzy sets and Pythagorean fuzzy sets. The proposed measures consist of an average or Choquet integral of two components. The first component quantifies the cosine similarity between two *q*-ROFSs at each element, while the second component captures the difference in lengths between the vector representations of these *q*-ROFSs at the same element. This innovative length-difference term ensures sensitivity to variations in both direction and magnitude, making the measures well-suited for applications where both aspects are crucial. The Choquet integral-based measure further considers interactions among elements, enhancing sensitivity in diverse applications. In addition to introducing these cosine similarity measures, we extend our contributions to the realm of multi-criteria group decision making (MCGDM) through an extended The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) methodology. The proposed TOPSIS methodology is applied to a real-world problem in green technology selection, providing a comprehensive evaluation framework. Our comparative analysis with some other MCGDM methods further highlights the effectiveness of our proposed approach.

Mehmet Ünver

Ankara University, Faculty of Science,

Department of Mathematics, 06100 Ankara Türkiye

munver@ankara.edu.tr

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1. Introduction

Fuzzy sets, as introduced by Zadeh [1], have proven to be highly effective in handling uncertainty and representing partial membership within a set. These sets are characterized by their membership functions. Extending this concept, Atanassov [2] introduced the theory of intuitionistic fuzzy set (IFS), which includes a membership function $\mu_A : X \to [0,1]$ and a non-membership function $\nu_A : X \to [0,1]$, subject to the constraint $\mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$. Subsequently, Yager [3] introduced Pythagorean fuzzy sets (PFS), where the membership and non-membership functions satisfy the condition $\mu_A^2(x) + \nu_A^2(x) \leq 1$. When an element exhibits a membership degree of 0.6 alongside a non-membership degree of 0.5, this situation is valid within the context of PFSs, whereas it does not conform to the criteria for IFSs. However, when the membership degree is 0.7 and the non-membership degree is 0.9, such a case is valid for neither IFSs nor PFSs. This observation underscores the need for a further extension of the PFS concept. To address this need, Yager [4] extended the concept of PFS to the concept of q-rung orthopair fuzzy set (value) (q-ROFS(V)), introducing the condition $\mu_A^q(x) + \nu_A^q(x) \leq 1$ for each $x \in X$, where $q \geq 1$. When q takes the value of 3, this specific category of fuzzy sets is referred to as Fermatean fuzzy sets (FFSs) [5]. The q-ROFSs have proven to be highly useful in multi-criteria (group) decision making (MC(G)DM) due to their versatile structure, as evidenced by their frequent application in various contexts. For instrance, Liu and Wang [6] contributed to the field with the introduction of various q-rung orthopair fuzzy aggregation operators, providing valuable tools for MCDM processes. Similarly, Wang et al. [7] presented a set of q-rung orthopair fuzzy Muirhead means and demonstrated their applicability in MCGDM situations. In a more recent work, Ünver and Olgun [8] studied in the realm of continuous function-valued q-rung orthopair fuzzy sets (CFVq-ROFSs), extending the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) to accommodate this novel fuzzy set type. This continuous function-valued approach enhances the versatility of TOPSIS in handling real-world decision-making problems.

TOPSIS is a widely employed method in solving MCDM and MCGDM problems, aiming to identify an alternative that maximizes the distance from the ideal worst solution while minimizing the distance from the ideal best solution [9]. Various extensions and adaptations of TOPSIS have been proposed to address MC(G)DM challenges in fuzzy environments. For instance, Ashtiani et al. [10] introduced an interval-valued fuzzy TOPSIS for scenarios with unequal criterion weights. Gündoğdu and Kahraman [11] presented an interval-valued spherical fuzzy TOPSIS to tackle specific MCDM problems. Akram et al. [12] extended TOPSIS to handle complex spherical fuzzy information in decision-making contexts. Garg et al. [13] proposed a novel TOPSIS method based on the complex interval-valued q-rung orthopair fuzzy sets. Wang et al. [14] introduced a TOPSIS approach for interval-valued q-rung dual hesitant fuzzy sets. Additionally, Ünver and Aydoğan[15] introduced information measures for CFVq-ROFSs and explored an extended version of the TOPSIS. In this paper, our focus extends beyond the development of enhanced cosine similarity measures for q-ROFSs. We seamlessly integrate our theoretical advancements into the realm of practical decision-making by introducing an extended TOPSIS methodology. Our innovative approach incorporates the newly proposed cosine similarity measures, providing a robust foundation for assessing similarity between alternatives in a more nuanced manner. By integrating these advanced measures into the TOPSIS framework, we aim to enhance the precision and applicability of the decision-making process. The extended TOPSIS methodology is then applied to a real-world problem in green technology selection, offering practical insights into the effectiveness of our proposed approach. This integration of theoretical advancements with practical applications underscores the holistic nature of our contributions in the field of fuzzy set theory and MCGDM.

A similarity measure serves as a valuable tool for assessing the resemblance between two mathematical objects. Among these similarity measures, cosine similarity measures stand out as a specific variant. Cosine similarity measures are employed to quantify the similarity between two fuzzy sets by considering the cosine of the angle between the vector representations of their reciprocal components, and this concept has found successful applications in fuzzy set theory as well. In a recent work, Kirişci [16] introduced novel cosine similarity and distance measures for FFSs. The study also employed the TOPSIS approach to showcase the practical applicability of the proposed measures. Lahitani et al. [17] explored the use of cosine similarity in online essay assessment, employing it as a key component in determining the similarity measure. Gupta and Tiwari [18] contributed to the field by devising measures of cosine similarity specifically designed for fuzzy sets, IFSs, and interval-valued IFSs. Ye [19] presented cosine similarity measures for IFSs and explored their applications in various contexts. Ünver et al. [20] extended the concept of cosine and cotangent similarity measures based on the Choquet integral to spherical fuzzy sets. Furthermore, Garg et al. [21] introduced a Choquet integral-based cosine similarity measure specifically crafted for interval-valued IFSs.

Existing cosine similarity measures in the literature focus solely on the cosine value of the angle between vector representations of fuzzy values. However, in numerous scenarios, the magnitude or length of a fuzzy set representation can be equally essential to its orientation. For example, in the context of MCDM, the lengths of vector representations of fuzzy set components can provide vital information about the intensity or strength of membership. The primary motivation of this study is to address this issue by incorporating length considerations into the definition of the cosine similarity measure. Our measures combine the traditional cosine similarity with a lengths difference control term to provide a more comprehensive assessment of similarity. The q-ROFSs under consideration here are a versatile and expressive extension of classical fuzzy sets that can model complex and imprecise relationships more accurately. In addition to its applicability to q-ROFSs, it is worth noting that our novel cosine similarity measures extend their benefits to IFSs and PFSs, which share similarities with q-ROFSs in terms of the need to capture both direction and magnitude aspects. By introducing a length control term these novel similarity measures enhance the assessment of similarity not only for q-ROFSs but also for IFSs and PFSs, making them a versatile and robust measure for various fuzzy set representations. Furthermore, the cosine similarity provided by the Choquet integral takes into account the interactions among elements, enhancing its sensitivity to variations and nuances in the data.

The selection of green technologies is a critical decision-making process that plays a pivotal role in sustainable development and environmental conservation. With the increasing awareness of climate change and environmental concerns, organizations and decision-makers are faced with the challenge of identifying and adopting eco-friendly technologies. This process involves evaluating various alternatives based on criteria such as energy efficiency, environmental impact, cost-effectiveness, ease of integration, and sustainability. The complexity of these decisions necessitates robust methodologies that can provide comprehensive assessments, making it an ideal domain for the application of advanced decision-making techniques such as the extended TOPSIS method proposed in this paper. For instance, Kumari and Mishra [22] proposed a MCDM for green supplier selection based on parametric measures for IFSs. Onar et al. [23] explored the multi-expert wind energy technology selection using interval-valued IFSs. Rani et al. [24] introduced a novel MCDM approach incorporating entropy and divergence measures of PFSs to evaluate renewable energy technologies. Keshavarz-Ghorabaee et al. [25] presented a decision-making approach based on FFSs for green construction supplier evaluation. Çalık [26] contributed a Pythagorean fuzzy analytic hierarchy process and fuzzy TOPSIS methodology for green supplier selection in the Industry 4.0 era. Additionally, Bansal et al. [27], Bao [28], and others have contributed the literature with various fuzzy decision approaches and algorithms aimed at enhancing the evaluation and selection of green technologies. In this context, our paper introduces an extended TOPSIS method utilizing advanced cosine similarity measures for q-ROFSs, offering a novel perspective to address the complexities associated with green technology selection. Additional investigations on the application of fuzzy sets to environmental technology issues are illustrated in Table 1 [Table Refs.:[29][30][31][32][33][34][35][36]].

Authors	Environment	Application	Decision Method
Yin and Li [29]	Intuitionistic fuzzy	Green building technologies	Matching model
Sang and Liu [30]	Interval type-2 fuzzy	Green supplier selection	TODIM
Narayanamoorthy et al. [31]	Hesitant fuzzy	Wind turbine technology selection	Intercriteria correlation
Kumar et al. [32]	Grey-fuzzy	Green technology thermal drilling process	Multi-objective optimization
Xiong et al. [33]	Intuitionistic fuzzy	Resilient-green supplier selection	WASPAS, BWM, TOPSIS
Rouyendegh et al. [34]	Intuitionistic fuzzy	Green supplier selection	TOPSIS
Pınar et al. [35]	q-rung orthopair fuzzy	Green supplier selection	TOPSIS
Soni et al. [36]	q-rung orthopair fuzzy	Waste treatment technology selection	SWARA-COPRAS
Present Study	**q-rung orthopair fuzzy**	**Green technology selection**	**TOPSIS**
BWM: Best Worst Method COPRAS: Complex Proportional Assessment	SWARA: Stepwise Weight Assessment Ratio Analysis TODIM: Interactive and Multi-criteria Decision Making	TOPSIS: Technique for Order Preference by Similarity to Ideal Solution WASPAS: Weighted Aggregated Sum Product Assessment	

Table 1. Literature Review of Applications of Fuzzy Decision Making Methods on Environmental Technologies

Some main contributions of the paper are listed below:

- The paper introduces some novel cosine similarity measures designed specifically for$q$-ROFSs. This measures address the limitations of traditional cosine similarity by incorporating a length difference control term.
- These measures offer a balanced assessment of similarity by simultaneously capturing both the direction and the magnitude aspects of q-ROFSs. This enhancement is crucial in applications where both factors play a significant role. The paper establishes that if the similarity measure between two q-ROFSs equals 1, then they are identical.
- The paper emphasizes that these measures are not limited to$q$-ROFSs but also improve similarity assessment for IFSs and PFSs. This broadens the scope of their utility across different fuzzy set representations.
- The utilization of the Choquet integral is demonstrated in the introduction of a novel cosine similarity measure, allowing for a more comprehensive assessment of similarity that accounts for element interactions, particularly in scenarios where interdependence among elements is of paramount importance.
- The paper introduces an extended TOPSIS method in the q-rung orthopair fuzzy environment, incorporating the newly developed cosine similarity measure to address the limitations of traditional methods.
- The novel TOPSIS methodology is employed to tackle a contemporary and widely recognized challenge – the green technology selection problem. This application showcases the practical relevance and effectiveness of the proposed

approach in addressing real-world decision-making scenarios.

2. Preliminaries

In this section, we revisit fundamental definitions employed within the scope of this paper. Following that, we proceed to introduce the anticipated enhanced cosine similarity measures for q-ROFSs. Unless mentioned otherwise, throughout the paper, we maintain the assumption that $X = \left\{x_1, \ldots, x_n\right\}$ represents a finite set and $w = (w_1, \ldots, w_n)$ is a weight vector, where $w_j \in [0, 1]$ for all $j = 1, \ldots, n$ and $\sum_{j=1}^{n} w_j = 1$.

Definition 1. [4] *Let $q \geq 1$. A q-ROFS A in X is given by*

$$A = \left\{\left\langle x_j, \mu_A\left(x_j\right), \nu_A\left(x_j\right)\right\rangle : j = 1, \ldots, n\right\}$$

where $\mu_A, \nu_A : X \to [0, 1]$ are membership and non-membership functions, respectively, satisfying

$$\mu_A^q\left(x_j\right) + \nu_A^q\left(x_j\right) \leq 1.$$

The pair $\langle \mu, \nu \rangle$ consisting of non-negative real numbers is termed a q-rung orthopair fuzzy value (q-ROFV) if $\mu^q + \nu^q \leq 1$. Observe that q-ROFS is conceptualized as a collection of q-ROFVs.

Remark 1. *When q takes on the value of 1, the fuzzy set is referred to as a IFS*[2]*, when q equals 2, it is denoted as an PFS* [3]*, and when q is equal to 3, it is called a FFS* [5]*.*

Gerstenkorn and Mańko [37] introduced a correlation coefficient for IFSs. This concept serves as a foundation and motivation for Ye's cosine similarity measure [19].

Definition 2. [37] *Let A and B be two IFSs. A correlation coefficient between A and B is defined by*

$$k(A, B) := \frac{\sum_{j=1}^{n}\left(\mu_A(x_j)\mu_B(x_j) + \nu_A(x_j)\nu_B(x_j)\right)}{\sqrt{\sum_{j=1}^{n}\left(\mu_A^2(x_j) + \nu_A^2(x_j)\right)\sum_{j=1}^{n}\left(\mu_B^2(x_j) + \nu_B^2(x_j)\right)}}.$$

In the work by Ye [19], a cosine similarity measure and a weighted cosine similarity measure for IFSs were introduced as follows.

Definition 3. [19] *Let A and B be two IFSs. A cosine similarity measure between A and B is defined by*

$$C_{IFS}(A,B) := \frac{1}{n}\sum_{j=1}^{n}\frac{\mu_A(x_j)\mu_B(x_j)+\nu_A(x_j)\nu_B(x_j)}{\sqrt{\mu_A^2(x_j)+\nu_A^2(x_j)}\sqrt{\mu_B^2(x_j)+\nu_B^2(x_j)}}.$$

For $n=1$ the cosine similarity measure C_{IFS} equivalent the correlation coefficient k.

Definition 4. [19] *Let A and B be two IFSs. A weighted cosine similarity measure between A and B is defined by*

$$W_{IFS}(A,B) := \sum_{j=1}^{n} w_j \frac{\mu_A(x_j)\mu_B(x_j)+\nu_A(x_j)\nu_B(x_j)}{\sqrt{\mu_A^2(x_j)+\nu_A^2(x_j)}\sqrt{\mu_B^2(x_j)+\nu_B^2(x_j)}}.$$

In particular, if $w=(1/n, 1/n, \ldots, 1/n)$, then W_{IFS} is reduced to C_{IFS}.

Liu et al. [38] *extended the concepts of C_{IFS} and W_{IFS} to encompass q-ROFSs. We now revisit the weighted one.*

Definition 5. [38] Let A and B be two q-ROFSs. A weighted cosine similarity measure between A and B is defined by

$$WC_{qROF}(A,B) := \sum_{j=1}^{n} w_j \frac{\mu_A^q(x_j)\mu_B^q(x_j)+\nu_A^q(x_j)\nu_B^q(x_j)}{\sqrt{\mu_A^{2q}(x_j)+\nu_A^{2q}(x_j)}\sqrt{\mu_B^{2q}(x_j)+\nu_B^{2q}(x_j)}}.$$

3. Improved cosine similarity measures

While the existing cosine similarity measures have been valuable in the assessment of similarity among fuzzy sets, they often fall short in capturing the complete picture. These traditional measures primarily focus on the cosine of the angle between vector representations, overlooking the significance of vector magnitudes. This limitation can lead to inaccurate similarity assessments, especially when the lengths of vector representations vary significantly. In light of these shortcomings, we present our enhanced cosine similarity measures for q-ROFSs. These measures not only address the deficiencies of traditional cosine similarity but also introduce a length difference control term to provide a more comprehensive and accurate evaluation of similarity. Our approach aims to bridge the gap left by previous measures, ensuring a robust and versatile tool for similarity assessment in various applications.

Definition 6. *Let A and B be two q-ROFSs. An improved cosine similarity measure between A and B is defined by*

$$ICSM_q(A,B) := \frac{1}{2n}\sum_{j=1}^{n}\left(\mathrm{Cos}_{A,B}x_j + L_{A,B}x_j\right)$$

where

$$\mathrm{Cos}_{A,B}x_j := \frac{\mu_A^q\left(x_j\right)\mu_B^q\left(x_j\right)+\nu_A^q\left(x_j\right)\nu_B^q\left(x_j\right)}{\sqrt{\mu_A^{2q}\left(x_j\right)+\nu_A^{2q}\left(x_j\right)}\sqrt{\mu_B^{2q}\left(x_j\right)+\nu_B^{2q}\left(x_j\right)}}$$

and

$$L_{A,B}x_j := 1 - \left| \sqrt{\mu_A^{2q}\left(x_j\right) + \nu_A^{2q}\left(x_j\right)} - \sqrt{\mu_B^{2q}\left(x_j\right) + \nu_B^{2q}\left(x_j\right)} \right|.$$

We now introduce a weighted variant of $ICSM_q$, denoted as $IWCSM_q$.

Definition 7. *Let A and B be two q-ROFSs. An improved weighted cosine similarity measure between A and B is defined by*

$$IWCSM_q(A,B) := \frac{1}{2} \sum_{j=1}^{n} w_j \left(\mathrm{Cos}_{A,B}x_j + L_{A,B}x_j \right).$$

This newly introduced measure combines the benefits of $ICSM_q$ with weighted considerations, providing a more versatile similarity assessment for q-ROFSs.

The subsequent theorem outlines the properties of the improved cosine similarity measure $ICSM_q$. Of particular significance within this theorem is the assertion that distinct q-ROFSs cannot exhibit similarity of 1, a departure from conventional cosine similarity measures found in the literature.

Theorem 1. The similarity measure $ICSM_q$ satisfies the following properties:
i) $0 \le ICSM_q(A,B) \le 1$ for any q-ROFS A and
B.
ii) $ICSM_q(A,B) = ICSM_q(B,A)$ for any q-ROFS A and B.
iii) $ICSM_q(A,B) = 1$ if and only if $A = B$.

Proof. **i)** It is clear that $0 \le \mathrm{Cos}_{A,B}x_j \le 1$ for any $j = 1, \ldots, n$. On the other hand since the vectors $\left(\mu_A^q\left(x_j\right), \nu_A^q\left(x_j\right)\right)$ and $\left(\mu_B^q\left(x_j\right), \nu_B^q\left(x_j\right)\right)$ have length in $[0,1]$, we have $0 \le L_{A,B}x_j \le 1$ for any $j = 1, \ldots, n$. Hence we obtain

$$0 \le \frac{\mathrm{Cos}_{A,B}x_j + L_{A,B}x_j}{2} \le 1$$

which yields that $0 \le ICSM_q(A,B) \le 1$.
ii) The proof is straightforward.
iii) It is clear that if $A = B$, then $ICSM_q(A,B) = 1$. Conversely, assume that $ICSM_q(A,B) = 1$. Then for any $j = 1, \ldots, n$ we get

$$\frac{\mathrm{Cos}_{A,B}x_j + L_{A,B}x_j}{2} = 1$$

which implies that $\mathrm{Cos}_{A,B}x_j = 1$ and $L_{A,B}x_j = 1$. Then the measure of the angle between $\left(\mu_A^q\left(x_j\right), \nu_A^q\left(x_j\right)\right)$ and

$\left(\mu_B^q\left(x_j\right), v_B^q\left(x_j\right)\right)$ is zero and and they have equal length. So

$$\left(\mu_A^q\left(x_j\right), v_A^q\left(x_j\right)\right) = \left(\mu_A^q\left(x_j\right), v_A^q\left(x_j\right)\right)$$

for any $j = 1, \ldots, n$. Hence $A = B$. □

Remark 2. *All the properties stated in Theorem 1 are applicable to* $IWCSM_q$ *as well.*

In conventional cosine similarity measures, Property (iii) of Theorem 1 is met without a necessity; namely, the similarity measure being equal to 1 does not necessarily imply that the sets are identical. In order to illustrate this deficiency and effectiveness of our novel similarity measure, let us consider a practical example involving FFSs. We compare the similarity results obtained using both the traditional cosine similarity measure WC_{qROF} and our enhanced measure $ICSM_q$. It is important to note that, in the example, the FFSs considered are distinct but exhibit a similarity measure of one with the existing cosine similarity measure recalled in Definition 5. This arises due to the overlapping vector representations of the components of these q-ROFSs, characterized by a length difference.

Example 1. *Let* $X = \{x_1, x_2, x_3\}$. *Consider the FFSs A and B defined as follows:*

$$A = \left\{\left\langle x_1, 0.3, 0.2\right\rangle, \left\langle x_2, 0.9, 0.6\right\rangle, \left\langle x_3, 0.6, 0.9\right\rangle\right\}$$

and

$$B = \left\{\left\langle x_1, 0.9, 0.6\right\rangle, \left\langle x_2, 0.15, 0.1\right\rangle, \left\langle x_3, 0.2, 0.3\right\rangle\right\}.$$

Then we have:

$$\begin{aligned} WC_{3ROF}(A, B) &= w_1 \frac{0.3^3 0.9^3 + 0.2^3 0.6^3}{\sqrt{0.3^6 + 0.2^6}\sqrt{0.9^6 + 0.6^6}} + w_2 \frac{0.9^3 0.15^3 + 0.6^3 0.1^3}{\sqrt{0.9^6 + 0.6^6}\sqrt{0.15^6 + 0.1^6}} \\ &+ w_3 \frac{0.6^3 0.2^3 + 0.9^3 0.3^3}{\sqrt{0.6^6 + 0.9^6}\sqrt{0.2^6 + 0.3^6}} \\ &= w_1 + w_2 + w_3 \\ &= 1 \end{aligned}$$

for any weight vector $\left(w_1, w_2, w_3\right)$ *such that* $\sum_{j=1}^{3} w_j = 1$. *It is evident that the cosine similarity measure* WC_{3ROF} *yields a similarity of* 1 *for A and B despite their substantial differences. Now, let's calculate the similarity between A and B using* $ICSM_3$*:*

$$ICSM_3(A,B) = \frac{1}{2} + \frac{1}{6}\left(\begin{array}{l} 3 - \left|\sqrt{0.3^4+0.2^4} - \sqrt{0.9^4+0.6^4}\right| - \left|\sqrt{0.9^4+0.6^4} - \sqrt{0.15^4+0.1^4}\right| \\ - \left|\sqrt{0.6^4+0.9^4} - \sqrt{0.2^4+0.3^4}\right| \end{array}\right)$$
$$= 0.59373$$

Now, considering the result obtained using our novel similarity measure, $ICSM_3$, we find that the similarity between A and B is approximately 0.59373*. It's worth noting that the improved cosine similarity measure provides a more rational and meaningful result.*

Remark 3. *In Example 1, the results of the similarity measures highlight an interesting observation. While WC_{3ROF} assigns a perfect similarity score of* 1 *to A and B, indicating a high degree of similarity, $ICSM_3$ yields a value of* 0.59373*, suggesting a more nuanced assessment. This discrepancy arises from $ICSM_3$'s consideration of both direction and magnitude aspects. In this case, the lengths of vector representations play a significant role in determining the similarity, leading to a more comprehensive evaluation.*

Next, we present an additional enhanced cosine similarity measure employing fuzzy measure theory. The classical versions of cosine similarity measures within the framework of fuzzy measure theory and fuzzy integrals are elaborated in [39]. Let us recall some basic definitions.

Definition 8. [40] *Let $P(X)$ be the power set of X. If*

i.) $\sigma(\emptyset) = 0$,

ii.) $\sigma(X) = 1$,

iii.) $\sigma(U) \leq \sigma(V)$ *for any* $U, V \subset X$ *such that* $U \subseteq V$,

then the set function $\sigma: P(X) \to [0,1]$ *is called a fuzzy measure on* X.

Definition 9. [40] *Let σ be a fuzzy measure on X. The Choquet integral of a function $f: X \to [0,1]$ with respect to σ is defined by*

$$(C)\int_X f d\sigma := \sum_{k=1}^{n} \left(f(x_{(k)}) - f(x_{(k-1)})\right)\sigma(E_k),$$

where the sequence $\left\{x_{(k)}\right\}_{k=0}^{n}$ *is a permutation of the sequence* $\left\{x_k\right\}_{k=0}^{n}$ *such that* $0 := f(x_{(0)}) \leq f(x_{(1)}) \leq f(x_{(2)}) \leq \ldots \leq f(x_{(n)})$ *and* $E_k := \left\{x_{(k)}, x_{(k+1)}, \ldots, x_{(n)}\right\}$.

Now, we are prepared to introduce the ultimate enhanced cosine similarity measure.

Definition 10. *Let σ be a fuzzy measure on X and let A and B be two q-ROFSs. An improved Choquet cosine similarity measure between A and B is defined by*

$${}_{Cqt}ICSM_q(A,B) := (C)\int_X f_{A,B} d\sigma$$

where $f_{A,B}\left(x_j\right) = \frac{1}{2}\left(\mathrm{Cos}_{A,B}x_j + L_{A,B}x_j\right)$ *for any* $j = 1, \ldots, n$.

The subsequent proposition presents an expected property of the Choquet integral.

Proposition 1. *Consider a function* $f: X \to [0, 1]$ *and a fuzzy measure* σ *on* X *such that* $\sigma(U) < 1$ *whenever* $U \neq X$*. Then,* $(C)\int_X f d\sigma = 1$ *if and only if* f *is identically equal to* 1.

Proof. If f is identically equal to 1, then it is evident that its Choquet integral equals 1. Conversely, assume that $(C)\int_X f d\sigma = 1$, and there exists some $x_{k_j} \in X$ such that $f\left(x_{k_j}\right) < 1$ for $j = 1, \ldots, m$ and $m \leq n$. Without loss of generality, assume that $f\left(x_{k_j}\right) \leq f\left(x_{k_{j+1}}\right)$ for any $j = 1, \ldots, m$. Then we have

$$0 = f\left(x_{(0)}\right) \leq f\left(x_{k_1}\right) \leq \ldots \leq f\left(x_{k_m}\right) \leq 1 = f\left(x_{(m+1)}\right) = \ldots = f\left(x_{(n)}\right).$$

Thus, we obtain

$$\begin{aligned}(C)\int_X f d\sigma &= f\left(x_{k_1}\right)\sigma(X) + \left(f\left(x_{k_2}\right) - f\left(x_{k_1}\right)\right)\sigma\left(E_2\right) \\ &\quad + \ldots + \left(1 - f\left(x_{k_m}\right)\right)\sigma\left(E_{m+1}\right) \\ &< f\left(x_{k_1}\right) + f\left(x_{k_2}\right) - f\left(x_{k_1}\right) + \ldots + 1 - f\left(x_{k_m}\right) \\ &= 1\end{aligned}$$

which leads to a contradiction. □

Following theorem presents some properties of the cosine similarity measure ${}_{Cqt}ICSM_q$.

Theorem 2. The similarity measure ${}_{Cqt}ICSM_q$ satisfies the following properties:

i) $0 \leq {}_{Cqt}ICSM_q(A, B) \leq 1$ for any q-ROFS A and B.

ii) ${}_{Cqt}ICSM_q(A, B) = ICSM_q(B, A)$ for any q-ROFS A and B.

iii) ${}_{Cqt}ICSM_q(A, B) = 1$ if and only if $A = B$.

Proof. The proof of (i) and (ii) are can be made similar to proof (i) and (ii) of Theorem 1. By considering Proposition 1 (iii) can be proved as (iii) of Theorem 1. □

4. Extended TOPSIS via $ICSM_q$

In this section, we introduce an extension to the TOPSIS by incorporating the $ICSM_q$ similarity measure. Subsequently, we apply this extended methodology to a real-life scenario involving the selection of green technologies.

4.1. **TOPSIS Methodology.** The proposed technique follows these steps:

- **Step 1:** Establish a MCGDM problem comprising m alternatives $A = \left\{A_1, \ldots, A_m\right\}$, n criteria $C = \left\{C_1, \ldots, C_n\right\}$, and involvement of l decision makers.
- **Step 2:** Develop a q-rung orthopair fuzzy decision matrix $D^{(i)}$ for the i^{th} decision maker, defined as

$$D^{(i)} = \begin{bmatrix} d_{11}^{(i)} & \cdots & d_{1n}^{(i)} \\ \vdots & \ddots & \vdots \\ d_{m1}^{(i)} & \cdots & d_{mn}^{(i)} \end{bmatrix}$$

for $i = 1, \ldots, l$.

- **Step 3:** Aggregate l decision matrices into a unified decision matrix AD:

$$AD = \begin{bmatrix} ad_{11} & \cdots & ad_{1n} \\ \vdots & \ddots & \vdots \\ ad_{m1} & \cdots & ad_{mn} \end{bmatrix}$$

where $ad_{kj} = AGG\left\{d_{kj}^{(1)}, \ldots, d_{kj}^{(l)}\right\}$, and AGG denotes an aggregation operator for q-ROFVs.

- **Step 4:** Employ a defuzzification process on the decision matrix AD using a score function to compute the criteria weights. This involves solving the following optimization model:

$$\begin{aligned} \max f & = \sum_{j=1}^{n} \sum_{k=1}^{m} w_j ad_{kj} \\ \text{such that} \quad & w_j \in W \\ & \sum_{j=1}^{n} w_j = 1. \end{aligned}$$

Here, W denotes a set of partial information regarding the weights.

- **Step 5:** Compute the weighted decision matrix WAD using a q-rung orthopair fuzzy multiplication formula. Each element of this weighted matrix is a q-rung orthopair fuzzy value, denoted as $\left\langle \mu_{kj}, \nu_{kj} \right\rangle$.
- **Step 6:** Calculate the positive and negative ideal solutions as follows: Positive ideal solution

$$PIS = \left\{\gamma_j^+ : j = 1, \ldots, n\right\}$$

and negative ideal solution

$$NIS = \left\{\gamma_j^- : j = 1, \ldots, n\right\}$$

where

$$\gamma_j^+ = \left\langle \max_{k=1,\ldots,m} \mu_{kj}, \min_{k=1,\ldots,m} \nu_{kj} \right\rangle$$

and

$$\gamma_j^- = \left\langle \min_{k=1,\ldots,m} \mu_{kj}, \max_{k=1,\ldots,m} \nu_{kj} \right\rangle.$$

In the case of C_j being a benefit criterion, and for C_j being a cost criterion, we interchange the roles of γ_j^+ and γ_j^-.

- **Step 7:** Compute the distances between each A_k and the positive ideal solution using

$$u_k^+ = 1 - ICMS_q(A_k, PIS)$$

and the distances between A_k and the negative ideal solution using

$$u_k^- = 1 - ICMS_q(A_k, NIS).$$

- **Step 8:** Determine the closeness coefficients

$$c_k = \frac{u_k^-}{u^- + u_k^+}$$

and rank the alternatives based on these coefficients. A larger c_k value indicates a superior alternative.

The steps of the extended TOPSIS are visualized in Figure 1.

Figure 1. Illustrating Key Steps in The Extended TOPSIS Methodology

4.2. Green Technology Selection Problem

In this section, we apply the proposed extended TOPSIS methodology to address a green technology selection problem.

- **Step 1:** In this scenario, we consider the selection of green technologies to address various environmental concerns and promote sustainability. The available alternatives are Solar Power Generation System (A_1), Wind Turbine Energy System (A_2), Green Building Technologies (A_3), Smart Grid Technology (A_4), and Hybrid Electric Vehicles (A_5). The decision criteria encompass Energy Efficiency (C_1), Environmental Impact (C_2), Cost-Effectiveness (C_3), Ease of Integration (C_4), and Sustainability (C_5). Multiple decision-makers contribute their perspectives on the alternatives and criteria through 4-rung orthopair fuzzy decision matrices. The subsequent steps involve the application of the extended TOPSIS methodology, incorporating the improved cosine similarity measure $ICSM_q$, to rank the green technologies based on their overall suitability.
- Step 2: Employing the scale provided in Table 2, three decision makers offer their evaluations of the alternatives across the criteria. Subsequently, these opinions are transformed into 4-ROFVs, also referring to Table 2.

Importance	Scale	q-ROFV
Extremely high	7	$\langle 0.90^{1/q}, 0.10^{1/q} \rangle$
Very high	6	$\langle 0.75^{1/q}, 0.25^{1/q} \rangle$
High	5	$\langle 0.70^{1/q}, 0.30^{1/q} \rangle$
Moderate	4	$\langle 0.60^{1/q}, 0.40^{1/q} \rangle$
Low	3	$\langle 0.45^{1/q}, 0.55^{1/q} \rangle$
Very low	2	$\langle 0.20^{1/q}, 0.80^{1/q} \rangle$
Extremely low	1	$\langle 0.10^{1/q}, 0.90^{1/q} \rangle$

Table 2. Scaling Factors for q-ROFVs

	C1	C2	C3	C4	C5
A1	$\langle 0.93, 0.71 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.88, 0.80 \rangle$	$\langle 0.93, 0.71 \rangle$	$\langle 0.97, 0.56 \rangle$
A2	$\langle 0.91, 0.74 \rangle$	$\langle 0.93, 0.71 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.88, 0.80 \rangle$	$\langle 0.93, 0.71 \rangle$
A3	$\langle 0.97, 0.56 \rangle$	$\langle 0.97, 0.56 \rangle$	$\langle 0.82, 0.86 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.91, 0.74 \rangle$
A4	$\langle 0.88, 0.80 \rangle$	$\langle 0.88, 0.80 \rangle$	$\langle 0.93, 0.71 \rangle$	$\langle 0.97, 0.56 \rangle$	$\langle 0.88, 0.80 \rangle$
A5	$\langle 0.93, 0.71 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.88, 0.80 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.93, 0.71 \rangle$

Table 3. Decision Matrix of Decision Maker 1

	C1	C2	C3	C4	C5
A1	$\langle 0.91, 0.74 \rangle$	$\langle 0.88, 0.80 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.93, 0.71 \rangle$	$\langle 0.97, 0.56 \rangle$
A2	$\langle 0.93, 0.71 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.93, 0.71 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.93, 0.71 \rangle$
A3	$\langle 0.97, 0.56 \rangle$	$\langle 0.93, 0.71 \rangle$	$\langle 0.88, 0.80 \rangle$	$\langle 0.88, 0.80 \rangle$	$\langle 0.91, 0.74 \rangle$
A4	$\langle 0.88, 0.80 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.97, 0.56 \rangle$	$\langle 0.93, 0.71 \rangle$	$\langle 0.88, 0.80 \rangle$
A5	$\langle 0.91, 0.74 \rangle$	$\langle 0.88, 0.80 \rangle$	$\langle 0.93, 0.71 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.93, 0.71 \rangle$

Table 4. Decision Matrix of Decision Maker 2

	C1	C2	C3	C4	C5
A1	$\langle 0.88, 0.80 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.88, 0.80 \rangle$	$\langle 0.93, 0.71 \rangle$	$\langle 0.97, 0.56 \rangle$
A2	$\langle 0.91, 0.74 \rangle$	$\langle 0.93, 0.71 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.88, 0.80 \rangle$	$\langle 0.93, 0.71 \rangle$
A3	$\langle 0.97, 0.56 \rangle$	$\langle 0.97, 0.56 \rangle$	$\langle 0.82, 0.86 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.91, 0.74 \rangle$
A4	$\langle 0.88, 0.80 \rangle$	$\langle 0.88, 0.80 \rangle$	$\langle 0.93, 0.71 \rangle$	$\langle 0.97, 0.56 \rangle$	$\langle 0.88, 0.80 \rangle$
A5	$\langle 0.93, 0.71 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.88, 0.80 \rangle$	$\langle 0.91, 0.74 \rangle$	$\langle 0.93, 0.71 \rangle$

Table 5. Decision Matrix of Decision Maker 3

- **Step 3:** Utilizing the innovative exponential aggregation operator introduced by Peng et al.[41], we combine the decision matrices from Step 3 into an aggregated fuzzy decision matrix, as illustrated in Table 6.

	C1	C2	C3	C4	C5
A1	$\langle 0.5733, 0.5574\rangle$	$\langle 0.5531, 0.5636\rangle$	$\langle 0.5264, 0.5775\rangle$	$\langle 0.6472, 0.5271\rangle$	$\langle 0.8203, 0.4183\rangle$
A2	$\langle 0.6024, 0.5416\rangle$	$\langle 0.6244, 0.5345\rangle$	$\langle 0.6024, 0.5416\rangle$	$\langle 0.5264, 0.5775\rangle$	$\langle 0.6472, 0.5271\rangle$
A3	$\langle 0.8203, 0.4183\rangle$	$\langle 0.7580, 0.4641\rangle$	$\langle 0.4227, 0.6187\rangle$	$\langle 0.5531, 0.5636\rangle$	$\langle 0.5812, 0.5484\rangle$
A4	$\langle 0.5009, 0.5903\rangle$	$\langle 0.5264, 0.5775\rangle$	$\langle 0.7004, 0.4988\rangle$	$\langle 0.7580, 0.4641\rangle$	$\langle 0.5009, 0.5903\rangle$
A5	$\langle 0.6244, 0.5345\rangle$	$\langle 0.5531, 0.5636\rangle$	$\langle 0.5456, 0.5718\rangle$	$\langle 0.5812, 0.5484\rangle$	$\langle 0.6472, 0.5271\rangle$

Table 6. Aggregated Decision Matrix

	C1	C2	C3	C4	C5
A1	0.0138	−0.0088	−0.0415	0.1166	0.4759
A2	0.0546	0.0839	0.0546	−0.0415	0.1166
A3	0.4759	0.3276	−0.1381	−0.0088	0.0284
A4	−0.0704	−0.0415	0.2098	0.3276	−0.0704
A5	0.0839	−0.0088	−0.0220	0.0284	0.1166

Table 7. Defuzzified Decision Matrix

- **Step 4:** We employ the defuzzification process via the exponential score function introduced by Peng et al.[41] to obtain a crisp decision matrix, as depicted in Table 7.

To establish the weights, we solve the linear optimization problem presented in Table 8. The constraints reflect a relative importance of criterion C_1 over criterion C_3. The ratio of their weights is suggested to be at least0.5. Also we emphasize the importance of criterion C_2 compared to criterion C_5. The weight of criterion C_2 is suggested to be at least 80% of the weight of criterion C_5. Also the constraints control the relative importance of criterionC_4 compared to criterion C_3. The ratio is suggested to be less than or equal to 0.8. Upon solving the linear optimization problem, the resulting weights are $w_1 = 0.141$, $w_2 = 0.2819$, $w_3 = 0.125$, $w_4 = 0.1$, and $w_5 = 0.3523$.

$\max f$	$= \sum_{j=1}^{5} \sum_{k=1}^{5} w_j ad_{kj}$
such that	$w_i \geq 0.1, i = 1, ..., 5$ $\frac{w_1}{w_3} \geq 0.5$ $\frac{w_2}{w_5} \geq 0.8$ $\frac{w_4}{w_3} \leq 0.8$
	$\sum_{j=1}^{5} w_j = 1.$

Table 8. Linear Optimization Problem to Weight Criteria

- **Step 5:** Using the multiplication method presented in[4], we obtain the weighted decision matrix *WAD* as shown in

Table 9.

	C1	C2	C3	C4	C5
A1	$\langle 0.3556, 0.9209\rangle$	$\langle 0.4066, 0.8507\rangle$	$\langle 0.3157, 0.9337\rangle$	$\langle 0.3718, 0.9380\rangle$	$\langle 0.6614, 0.7356\rangle$
A2	$\langle 0.3747, 0.9172\rangle$	$\langle 0.4616, 0.8381\rangle$	$\langle 0.3637, 0.9262\rangle$	$\langle 0.2987, 0.9466\rangle$	$\langle 0.5063, 0.7980\rangle$
A3	$\langle 0.5343, 0.8844\rangle$	$\langle 0.5717, 0.8054\rangle$	$\langle 0.2522, 0.9417\rangle$	$\langle 0.3145, 0.9443\rangle$	$\langle 0.4521, 0.8093\rangle$
A4	$\langle 0.3091, 0.9284\rangle$	$\langle 0.3863, 0.8566\rangle$	$\langle 0.4289, 0.9167\rangle$	$\langle 0.4452, 0.9261\rangle$	$\langle 0.3879, 0.8305\rangle$
A5	$\langle 0.3893, 0.9155\rangle$	$\langle 0.4066, 0.8507\rangle$	$\langle 0.3277, 0.9325\rangle$	$\langle 0.3313, 0.9417\rangle$	$\langle 0.5063, 0.7980\rangle$

Table 9. Weighted Decision Matrix

- **Step 6:** From the weighted decision matrix *WAD*, we determine the PIS and NIS as follows:

$$PIS = \{\langle 0.5343, 0.8844\rangle, \langle 0.5717, 0.8054\rangle, \langle 0.4289, 0.9167\rangle, \langle 0.4452, 0.9261\rangle, \langle 0.6614, 0.7356\rangle\}$$
$$NIS = \{\langle 0.3091, 0.9284\rangle, \langle 0.3863, 0.8566\rangle, \langle 0.2522, 0.9417\rangle, \langle 0.2987, 0.9466\rangle, \langle 0.3897, 0.8305\rangle\}.$$

- **Step 7:** Utilizing the *PIS* and *NIS*, we compute the distances, as shown in Table 10.

u_1^+	u_2^+	u_3^+	u_4^+	u_5^+
0.0310	0.0414	0.0335	0.0524	0.0448
u_1^-	u_2^-	u_3^-	u_4^-	u_5^-
0.0357	0.0202	0.0314	0.0148	0.0173

Table 10. Distances

- **Step 8:** The resulting closeness coefficients are as follows: $c_1 = 0.5352$, $c_2 = 0.3279$, $c_3 = 0.4838$, $c_4 = 0.2202$, and $c_5 = 0.2786$. Consequently, the ranking is $A_1 \succ A_3 \succ A_2 \succ A_5 \succ A_4$.

4.3. Discussion

- **Discussion on the methodology:** The application of the proposed improved cosine similarity measure $ICSM_q$ in the context of the green technology selection problem has provided insightful results. The decision matrices derived from the opinions of three decision makers, scaled between 1 and 7, were aggregated using the exponential aggregation operator [41]. This process facilitated the creation of an aggregated decision matrix, which was subsequently defuzzified to obtain a crisp decision matrix (Table 7). To determine the weights for each criterion, a linear optimization problem was solved, resulting in weights $w_1 = 141$, $w_2 = 2819$, $w_3 = 125$, $w_4 = 0.1$, and $w_5 = 0.3523$. These weights were then utilized in the multiplication process, following the methodology proposed by Yager [4], resulting in the weighted decision matrix *WAD* (Table 9). Further analysis involved the calculation of *PIS* and *NIS* from *WAD*. Distances between each alternative and both *PIS* and *NIS* were computed (Table 10). Subsequently, closeness coefficients were determined, leading to the final ranking of alternatives: $A_1 \succ A_3 \succ A_2 \succ A_5 \succ A_4$.

The results indicate that Solar Power Generation System (A_1) is the most favorable option, followed by Green Building Technologies (A_3), Wind Turbine Energy System (A_2), Hybrid Electric Vehicles (A_5), and Smart Grid Technology (A_4). This ranking aligns with the preferences inferred from the opinions of the decision makers.

The application of $ICSM_q$ within the MCGDM framework has demonstrated its effectiveness in handling real-life problems. The proposed methodology enhances the decision-making process, providing a systematic and comprehensive approach to evaluating and ranking alternatives based on multiple criteria. The results underscore the potential of $ICSM_q$ in facilitating informed decision-making in diverse application domains, particularly in the selection of green technologies.

- **Discussion on the MCGDM problem:** The result of the Green Technology Selection Problem, obtained through the extended TOPSIS methodology utilizing the improved cosine similarity measure $ICSM_q$, reveals valuable insights. The ranking of alternatives reflects the preferences and priorities inferred from the opinions of the decision makers and the weighted criteria.

    - Solar Power Generation System (A_1) emerged as the top-ranked alternative, indicating its high suitability and desirability based on the specified criteria. This aligns with the growing emphasis on renewable energy sources, highlighting solar power as a promising and efficient option.
    - Green Building Technologies (A_3) secured the second position, emphasizing the significance of environmentally friendly construction practices and technologies. The third position is occupied by Wind Turbine Energy System (A_2), highlighting the continued importance of wind energy in the renewable energy landscape.
    - Hybrid Electric Vehicles (A_5) and Smart Grid Technology (A_4) complete the ranking, showcasing their potential contributions to sustainability but with comparatively lower preference in this specific context.

- Overall, the result reflects a well-balanced consideration of criteria, providing a meaningful and contextually relevant ranking of green technology alternatives. The application of $ICSM_q$ within the extended TOPSIS framework proves to be effective in guiding decision-making processes in the selection of environmentally conscious technologies.

4.4. Comparative Analysis:

In this section, we solve the same MCGDM problem using alternative MCDM methods. To facilitate this comparison, we begin by normalizing the decision matrix presented in Table 7. The resulting normalized decision matrix is provided in Table 11.

	C_1	C_2	C_3	C_4	C_5
A_1	0.5069	0.4956	0.4793	0.5583	0.7380
A_2	0.5273	0.5420	0.5273	0.4793	0.5583
A_3	0.7380	0.6638	0.4310	0.4956	0.5142
A_4	0.4648	0.4793	0.6049	0.6638	0.4648
A_5	0.5420	0.4956	0.4890	0.5142	0.5583

Table 11. Normalized Crisp Decision Matrix

- Ordinary TOPSIS: In applying the ordinary TOPSIS[9], the obtained closeness coefficients are as follows: $c_1 = 0.61$, $c_2 = 0.45$, $c_3 = 0.34$, $c_4 = 0.29$, and $c_5 = 0.21$. Consequently, the resulting ranking is $A_1 \succ A_3 \succ A_2 \succ A_5 \succ A_4$. The obtained ranking aligns precisely with the proposed method, suggesting that the complement of the introduced similarity measure could potentially function as the Euclidean metric within the q-rung fuzzy environment.
- ELECTRE: The Elimination and Choice Translating Reality (ELECTRE) method is a MCDM approach designed to handle complex decision problems involving multiple conflicting criteria [42]. It operates by comparing alternatives in terms of each criterion and determining outranking relationships. The method provides a systematic way to eliminate less desirable alternatives, leading to a final ranking of the remaining options. ELECTRE considers both positive and negative aspects, making it suitable for decision scenarios where trade-offs and compromises are essential. The outcomes are valuable for decision-makers seeking a clear understanding of the relative strengths and weaknesses of different alternatives in the context of specified criteria. Applying the ELECTRE method yields the following partial results: $A_1 \succ A_3$, $A_1 \succ A_4$, $A_2 \succ A_4$, $A_3 \succ A_2$, $A_3 \succ A_4$ and $A_5 \succ A_4$. This partial order indicates that A_4 is considered the least favorable alternative, aligning with the findings of the extended TOPSIS method presented in this paper. Importantly, none of these orderings contradicts the ranking produced by the proposed approach.

5. Conclusion

In this paper, we have introduced an array of enhanced cosine similarity measures for q-ROFSs. These novel measures address the limitations of traditional cosine similarity measures by simultaneously considering both the direction and magnitude aspects of q-ROFSs. Specifically; the arithmetic mean-based cosine similarity measure employs the traditional cosine similarity formula enhanced with a length difference control term, offering a more comprehensive assessment of similarity. The weighted arithmetic mean-based cosine similarity measure extends the concept further by introducing a weight vector, allowing for customized emphasis on individual components, making it adaptable to diverse applications. The Choquet integral-based cosine similarity measure uses the Choquet integral to consider element interactions, improving sensitivity and applicability in scenarios where interdependence among elements is critical. Furthermore, our contributions extend beyond the realm of cosine similarity measures. The introduction of an extended TOPSIS methodology, incorporating the novel similarity measures, provides a powerful framework for MCGDM. We applied this methodology to a real-world scenario, the Green Technology Selection Problem, involving alternatives such as Solar Power Generation System, Wind Turbine Energy System, Green Building Technologies, Smart Grid Technology, and Hybrid Electric Vehicles, evaluated against criteria like Energy Efficiency, Environmental Impact, Cost-Effectiveness, Ease of Integration, and Sustainability. Through theoretical analyses, examples, and practical applications, we demonstrated the effectiveness of our proposed measures and the extended TOPSIS method. The advantages include a comprehensive assessment of similarity, adaptability to diverse applications through customizable weight vectors, and enhanced sensitivity through the consideration of element interactions via the Choquet Integral. Our paper concludes with a comparative analysis, evaluating the proposed methodology against some other MCDM methods. The results underscore the superiority of our approach in providing coherent and meaningful rankings.

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