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Potential competing interests: No potential competing interests to declare.

[1] Review of the paper:Reviewer<u>Harish Parthasarathy</u>. The paper gives a very interesting and fine analysis of the philosophy and meaning of measurement in the mathematical and physical sciences. Some of the interesting issues discussed here are as follows:[1] Measurement without a reference scale has no meaning, in other words, as Euler had said, measurement is always relative, <u>ie</u>, relative to a fixed scale or standard. The notion of absolute measurement should therefore carry no meaning.

[2] If a standard scale is available, then repeated measurements of a physical quantity relative to such a scale would invariably produce random errors, the sum of which will, according to the celebrated De-<u>Moivre</u>-Laplace central limit theorem, after an appropriate scaling, have a normal distribution in the limit as the number of measurements tends to infinity. However, as pointed out by the author, measurement errors can arise due to faulty fluctuations in the measurement apparatus, due to faulty fluctuations in the standard scale, or due to faulty fluctuations in the comparison process made by the person making the measurement relative to the standard scale. In short, the outcomes of repeated measurements are specified not by a deterministic value but rather by a probability distribution over the outcomes of the measurement.

[3] The situation of measurement in quantum mechanics, as noted by the author, is more subtle owing to Heisenberg uncertainty, entanglement of the states of two or more observers, the state collapse postulate, and the inequalities of John Bell, which ensure that the measurement model in quantum mechanics cannot be enlarged to describe the measurement process using a classical probability space. I think the author should also discuss the collapse postulate here, which states that if a person carries out a measurement on his state, then the overall state of himself and the others would collapse to a state dependent upon the outcome of his measurement. This manifests itself in the <u>EPR</u> paradox and, in particular, prevents successive measurements from having identical probability laws because the next measurement is made on the collapsed state following the first measurement. The notion of identically distributed independent random variables used in the proof of the classical central limit theorem therefore carries no meaning in quantum mechanics. However, it should be noted that the work of R.L. Hudson has shown that one can still talk about a central limit theorem in quantum mechanics by replacing the notion of <u>iid</u> random variables with <u>arbitrary</u> tensor powers of a given quantum state. As K.R. <u>Parthasarathy</u> had noted, such a quantum central limit theorem in quantum mechanics can be proved by using the idea of the quantum characteristic function or the quantum Fourier transform of a state in terms of the <u>Weyl</u> translation operators. Moreover, the author should mention here that in quantum mechanics, we cannot generally talk about the joint probability distribution of two observables in a given state when the observables do not commute because the two of them

cannot be simultaneously diagonalized when they do not commute.

[4] The situation of quantum statistical mechanics may also be mentioned in this paper, wherein we have a classical <u>Gibbsian</u> distribution over a set of pure states according to the energy levels of these states, so that the overall probability distribution of an observable is calculated using a combination of classical and quantum probabilistic methods: The probability distribution of the observable given that the system is in a state of definite energy is calculated using quantum mechanics, while the probability that a given definite energy state occurs is specified by the classical <u>Gibbsian</u> probability distribution.

[5] In the context of quantum measurements, the author very rightly has cited the classic double slit experiment and Richard <u>Feynman's</u> famous statement that the moment the observer notes which slit the electron has passed through, the probability distribution of the electrons on the screen changes. This is precisely a special case of the state collapse postulate.

[6] The author has presented the subject very nicely, but perhaps he should throw some more light on the fact that without an observer, the notion of measurement in quantum mechanics is void, and with the presence of the observer, repeated <u>iid</u> quantum measurements on the state of the system cannot be made unless the signal used to measure the state does not disturb it (non-demolition measurements in the language of V.P.<u>Belavkin</u>).

[7] I recommend publication of the paper after these modifications.