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WHY A UNIFORMLY ACCELERATED CLASSICAL CHARGE MUST RADIATE.

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Abstract. It is generally accepted that all quantities of electromagnetic origin are either relativistic scalars or components of relativistic four-vectors and tensors. This opinion is based on the Lorentz invariance of the Maxwell equations and this opinion is so commonly established that no one tries to verify this symmetry of electromagnetic fields. The idea that any EM field is covariant and, therefore, its components are changed in accordance with the Lorentz transformations in a new inertial reference frame, is used in studying some electrodynamical systems. One such application of this concept is to apply the Lorentz transformation of the EM field created by a uniformly accelerated charge to show that this charge does not radiate.

However, am analysis of the EM fields created by this charge and calculated in two inertial frames shows that the required Lorentz covariance of these fields is absent.

The explanation of violation of such a Lorentz covariance is given in this work.

1. Introduction

It is stated in most of modern textbooks on the classical electrodynamics that all quantities of electromagnetic origin are either relativistic scalars or components of relativistic four-vectors and tensors. But only few examples with demonstration of the mentioned properties of the EM quantities are given in these textbooks. A demonstration of Lorentz covariance of the potentials created by uniformly moving charge is made by Feynman [1]. In the general case, the conclusion on the Lorentz invariance of the EM quantities is made from corresponding invariance of the Maxwell equations. Actually, if some system of the differential equations has a certain symmetry (in our case it is a translational symmetry with respect to the Lorentz transformations of the variables of the system), it is reasonable to accept that the solutions of this system have the same symmetry. However, it does not follow with necessity a presence of some symmetry of the solutions of the differential equations if the differential equations have the same property.

For example, Lorentz's theory of the absolute ether, supplemented by the statement of contraction of moving charges, provides a perfect explanation for the null results of experiments to detect the motion of the Earth relatively to the static ether. Thus, solutions of the Maxwell equations may have a different translational symmetry, and these solutions describe the EM fields which are realized in experiments.

The latter means that in order to be sure that the EM quantities are Lorentz invariant, it is necessary to check this property by analyzing the expressions for the EM fields. This is important because, based on the opinion that all electromagnetic quantities must be transformed in accordance with the special relativity, some assumptions are made about the behavior of EM fields. However, the use of such an opinion without proper verification of the correctness of the Lorentz covariance can lead to incorrect conclusions. Such an example of an (incorrect) conclusion about the physical properties of some electrodynamic system, made on the basis of the above opinion, will be considered in this work.

2. Lorentz covariance of the expressions for EM field created by a classical charge moving with variable velocity.

Shortly after the appearance of Einstein's famous paper on the basis of the special relativity, the scientists realized that the Lorentz invariance of the Maxwell equations is not sufficient for mathematical foundation of this theory. So Poincaré, as a true mathematician, attempted to prove that

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the expressions for the EM field created by a single charge being in arbitrary motion are Lorentz–covariant [2]. To do it, Poincaré used Langevin's [3] concept of representation of the EM field of the charge as a composition of the velocity wave and the acceleration wave. It allows him to state:
'We can therefore reduce the computation of the two waves to the case where the electron velocity is zero. Let's start with the velocity wave, we first note that this wave is the same as if the electron motion was uniform.

If the electron velocity is zero, then (in modern notation):

$$\mathbf{A} = 0, \quad \mathbf{H} = 0, \quad \Phi = \frac{q}{4\pi r}\,, \tag{1}$$

where Φ and $\mathbf{A}$ and the potentials and $\mathbf{H}$ the magnetic field created by the charge q'.

However, for a uniformly accelerated charge, when its velocity is equal to zero, the expressions for the potentials, created by a single charge moving in the x axis in accordance to $\xi(t) = \sqrt{k^2 + c^2t^2}$, are [4]

$$\Phi = \frac{q(k^2 + x^2 + \omega^2)}{x\sqrt{(-k^2 + x^2 + \omega^2)^2 + 4k^2\omega^2}}\,, \quad A_x = -\frac{q}{x}\,, \tag{2}$$

but $\mathbf{H} = 0$ in all space. In Eq. (2), notation of Schott [4] is used. So ω is the radial coordinate.

Obviously, because of discrepancy between the expressions for the potentials, Eqs. (1) and (2) one can conclude that Poincaré's proof of the Lorentz covariance of the expressions for the EM fields is incorrect and, at least, incomplete.

If verification of the Lorentz covariance of these expressions cannot be made in the general case, it is possible to do in some specific cases for which the expressions are explicitly given, *i.e.* written not via the retarded time but via the present time variables.

Unfortunately, only in two cases of motion of the charge creating these fields corresponding expressions of the EM field can be obtained, *i.e.* when the charge moves with constant velocity and when it moves with constant acceleration in accordance to the law $\xi(t) = \sqrt{k^2 + c^2t^2}$.

The problem of the EM fields created by uniformly moving charge can be reduced to static case where the charge is at rest and the EM field is static (contracted Coulomb field) which transformations coincide in both special relativity and Lorentz's theory.

Consideration of the second case, *i.e.* transformation of the EM field created by a uniformly accelerated charge is of certain interest since one physical property of this system is assumed to obtain from the Lorentz covariance of the EM fields – if this covariance takes place.

As it was firstly obtained by Born [5], at the instant when the accelerated charge is at rest the magnetic field is equal to zero in all space. This result was confirmed by Schott [4] and then used by Pauli [6] to state that the absence of the magnetic field means that the EM field does not form the wave zone, *i.e.* a process of radiation is absent at this instant.

Since for a charge having velocity v , one can always choose a co-inertial frame in which this charge is instantaneously at rest. In this frame, the magnetic field is also equal to zero and the charge does not radiate. Moreover, one can present the motion of an accelerated charge as its being in an infinite number of co-directed inertial frames, and in all these frames the charge does not radiate. Therefore Pauli concludes that the charge at such motion does not radiate.

But this consideration contains one weak point, namely, the Lorentz transformation of the EM fields of the charge having the velocity v to the EM fields in the frame, where that charge is instantaneously at rest, are not verified. Let us fulfill this procedure.

The magnetic field of the charge at rest $\mathbf{H}(v = 0)$ can be found using the Lorentz transformations of the magnetic field of a charge moving with the velocity v, ı.e. i.e. $\mathbf{H}(v)$ and the electric field $\mathbf{E}_\perp(v)$ that is transverse to the direction of motion,

$$H_\phi(v = 0) = \frac{1}{\sqrt{1 - (v/c)^2}}\left[H_\phi(v) - \frac{v}{c}E_\perp(v)\right] = 0\,, \tag{3}$$

since for the charge moving in the x axis the only nonzero component is the angular one, H_ϕ. The components in the above equation are given by Eq. (85) of [4],

$$E_\perp = \frac{8k^2 x\omega}{s^{3/2}}\,;\ H_\phi = \frac{8k^2 ct\omega}{s^{3/2}}\,;\ s = \sqrt{(x^2+\omega^2-k^2-c^2t^2)^2+4k^2\omega^2}\,. \qquad (4)$$

For the accelerated charge its velocity at the instant t is $v = c^2t/\sqrt{k^2+c^2t^2}$, and this value of v is used as a velocity between the inertial frames. Then the Lorentz factor is

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{\sqrt{k^2+c^2t^2}}{k}\,.$$

Inserting the expressions of Eq. (4) to (3), one obtains (Eq. (15) of [7])

$$H_\phi(v=0) = \frac{\sqrt{k^2+c^2t^2}}{k}\left[\frac{8k^2ct\omega}{s^3} - \frac{ct}{\sqrt{k^2+c^2t^2}}\frac{8k^2x\omega}{s^3}\right] = \frac{8kct\omega}{s^3}\left[\xi(t)-x\right]\,. \qquad (5)$$

where x and t are the coordinate and time variables in the frame where the charge moves.

The obtained result, Eq. (5), means that, in opposite to Pauli's assumption, the Lorentz transformations of the EM field created by a moving charge do not give a zero value of the magnetic field in co-moving frames. Therefore, Pauli's claim, that a uniformly accelerated classical charge does not radiate, is incorrect.

In this connection a question arises: how to explain the absence of Lorentz covariance in the expressions (4)?

The explanation of this 'phenomenon' is as follows. A configuration of the EM field, which gives the value $H = 0$ at $t = 0$ and therefore, $v = 0$, is created not by the charge located at the point P_0 with the coordinates $\omega = 0$, $x = k$ (Fig. 1a). The EM field detected at $t = 0$ at a point Q_1 is created by this charge when it was located at an instant t_P at point P_1. The coordinate of this point is determined from the condition: when the charge moves from x_P to $x = k$, the EM wave emitted by the charge from the point P_1 should approach the point Q_1, or

$$\frac{\sqrt{(x_P-x_Q)^2+\omega_Q^2}}{c} = (0-t_P)\,, \qquad (6)$$

where t_P is determined from $x_P = \sqrt{k^2+c^2t_P^2}$. Thus, Eq. (6) becomes,

$$\frac{\sqrt{(x_P-x_Q)^2+\omega_Q^2}}{c} = \frac{\sqrt{x_P^2-k^2}}{c}\,.$$

Solution of this equation is

$$x_P = \frac{x_Q^2+k^2+\omega_Q^2}{2x_Q} \ \to\ t_P = -\frac{\sqrt{(x_Q^2+k^2+\omega_Q^2)^2-4k^2x_Q^2}}{2x_Q}\,.$$

Actually, the charge being at the points P_1 emits the spherical wave so Φ_Q as scalar quantity has the same value on the sphere $R = \sqrt{(x_P-x_Q)^2+\omega_Q^2}$. The same consideration is correct for emitting the spherical wave by the charge when it is at the point P_2 (fig 1.a). Thus, the configuration of the EM field at $t = 0$ is formed by spherical waves emitted by the charge at previous times $t_P < 0$.

When one makes the Lorentz transformations of the charge and related EM field, one should make corresponding transformation of all expanding spherical waves mentioned above. Such a transformation can be made as follows:

the shape of the every EM wave is determined as a product of the speed of propagation of the wave in new frame and the time $T = 0 - t_P$, Eq. (6). This time is equal for any direction of propagation of the wave. But it is not so for the speed which is found from the formula for relativistic transformation of the velocities.

The speeds of propagation in the normal (along the x axis) and transverse directions in new frame moving with velocity u with respect to the frame where the charge is at rest,

$$c_{\parallel} = \frac{(c-u)}{1 - \dfrac{u}{c^2}\cdot c} = c\,,$$

$$c_{\perp} = \frac{\sqrt{1-(u/c)^2}}{1 - \dfrac{u}{c^2}\cdot c} = c\sqrt{\frac{c+u}{c-u}} > c\,.$$

By means of two 'speeds of light' in new frame, the transformed surface of the EM wave should be

$$R_{wave} = c|t_P| \cdot \sqrt{\cos^2\theta + \frac{c+u}{c-u}\sin^2\theta}\,, \tag{7}$$

where θ is the angle between the x axis and the direction of propagation of the wave vector ($\angle P_0 P_1 Q_1$, fig 1.b). One may transform the time t_P but because it is the scalar, such a transfoormation cannot change the shape of the wave in new frame.

Meanwhile, it is known from the special relativity (its second postulate) that in any inertial frame, including new frame where the charge has the velocity u, the EM wave emitted by a charge at any time should have the spherical shape. However, the shape of the transformed wave, Eq. (7), is not a sphere but ellipsoid. Therefore, we should conclude that the Lorentz transformation of the EM fields propagating with the speed of light give incorrect result. This fact explains the result (5).

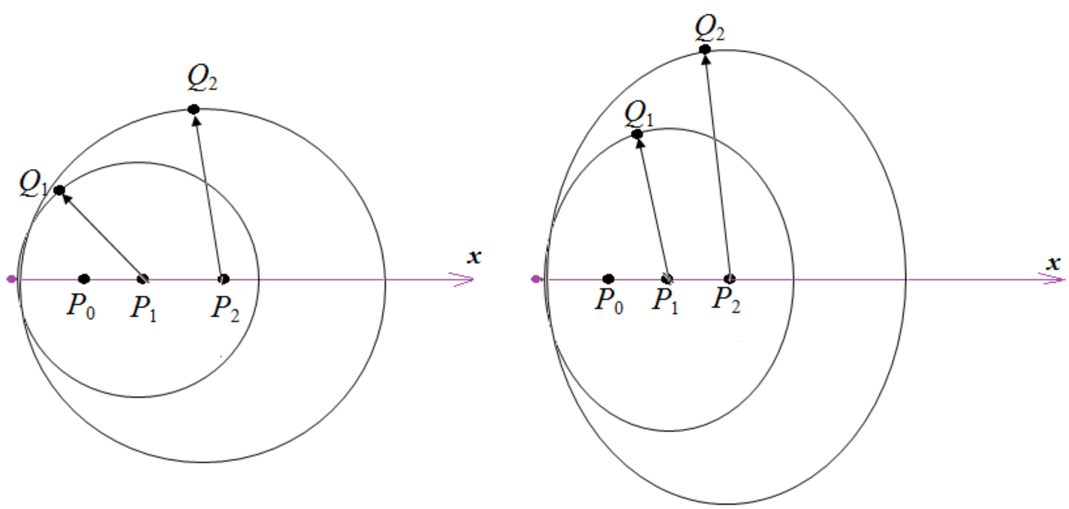

Figure 1. a (left). Distribution of the field created by expanding spherical EM waves emitted by the charge while it moves from $x = \infty$ to $x = k$.
FIGURE 1b (right). Distribution of the field in the case when the expanding EM waves are Lorentz–transformed.

3. Conclusions

In this work, a problem of the Lorentz covariance of the EM fields is considered. In contrast to commonly accepted opinion that this covariance directly follows from the Lorentz invariance of the Maxwell equations, such a problem exists. A system of differential equations can have a solution which obeys some symmetry (the Lorentz covariance is some kind of translational symmetry). But it does not follow with necessity that this solution describes the EM fields which are found within the Maxwell-Lorentz electrodynamics (or found as retarded integrals of the wave equations).

In order to determine that the expressions for the EM fields obtained as solutions of the wave equations are Lorentz covariant, one should check the covariance of these expressions but not the equations written for these expressions. To the author's knowledge, the only attempt to do this was made by Poincré [2]. Corresponding analysis of Poincaré's approach and direct verification of the Lorentz covariance of the expressions for the EM fields of a uniformly accelerated charge shows that the problem of verification of the EM fields covariance has not been resolved.

Based on the above analysis of this problem, three conclusions can be made

1. There is a significant logical gap in Pauli's concept of existence of infinite set of inertial frames chosen in such a way that in any frame the uniformly accelerated charge is instantaneously at rest, its magnetic field is zero in all space and, therefore, the charge does not radiate. This gap is as follows: the Lorentz transformations of the EM field of moving charge do not give the zero magnetic field in new frame where the charge will be at rest. Thus, the infinite set of inertial frames, in every of which there is no radiation, cannot be chosen. Therefore, a uniformly accelerated charge must radiate.
2. The example of violation of the Lorentz covariance of the EM fields and the absence of rigorous analysis of the expressions for the EM field of a charge in arbitrary motion give the reason to state that in every specific case of application of the Lorentz transformations to some EM fields, one should check whether these transformations can be applied.
3. When it is necessary to find the EM fields in new inertial frame from the EM fields of the charge moving with variable velocity in original frame, application of the Lorentz transformations for this is questionable. In the original frame, such fields are created by expanding spherical waves. Due to the rules of relativistic transformations of the velocities, in new frame the transformed expanding waves do not have the spherical shape. It is in contradiction with the first postulate, namely, that any physical process in all inertial frames pass in identical way. If the EM waves emitted by point-like source have the spherical shape in one frame, they must have the same shape in another frame.

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