Shapiro Time Delay Using Newtonian Gravitation

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Abstract: This paper derives the Shapiro time delay for light travelling close to a massive object using Newtonian gravitation. Using the fact that in black body radiation, the flux density of photons is proportional to the square of the frequency of the associated light, a relationship between frequency and light speed was derived. This, along with the basic gravitational redshift equation, was used to determine the speed of light entering a gravitational field. This light speed equation enabled the derivation of the complete Shapiro time delay formula, inclusive of the factor of 2 missing from a previous Newtonian derivation.

Key words: Shapiro Time delay, gravitational redshift, Newtonian gravitation

1. Introduction

In 1964 Shapiro developed a test of general relativity in which light travelling close to a massive body experiences a time delay [1, 2]. Possel has discussed the derivation of this time delay using Newtonian gravitation in an attempt to avoid the complexities of general relativity [3]. Unfortunately, he was only able to derive half of the delay and attributed this to not being able to take into account the curvature of space in his derivation. In this paper we revisit the Newtonian method. Using a different approach in the determination of the speed of light in a gravitational field, we are able to fully account for the missing factor of two such that the complete Shapiro delay is obtained.

2. Light Speed in a Gravitational Field

In order to derive the time delay using Newtonian gravitation, we need to determine the effect of a gravitational field on the speed of light. Towards this end, we note that the flux density of photons is proportional to the square of the frequency of the associated light. This relation arises from the number of standing wave modes per unit frequency interval as shown in the derivation of Planck's black body radiation formula as given for example by Blatt [4, p69]. From Wesley [5, p83], since the number of photons remains the same as the photons traverse a gravitational field, it follows that the net flow rate of photons into a tube of flow is equal to the net flow rate out of that tube. This yields the relation between frequency and light speed in and out of the tube as

$$f^2 c = f'^2 c' \quad (1)$$

where the photons enter with frequency f and velocity c and leave with frequency f' and velocity c'. This equation is used in deriving the relationship between light speed and gravitational field intensity.

Now it has been experimentally observed that a light beam can change the momentum of an object upon which the light is incident. The corresponding pressure experienced by the object is referred to as radiation or light pressure. If the light energy E_p is absorbed by the object, the momentum change experienced by the object is E_p / c. From the principle of conservation of momentum, this means that a photon of energy E_p behaves as if it has a mass m_p given by [6]

$$m_p = \frac{E_p}{c^2} \quad (2)$$

Using the approach by Narlikar [7], consider a photon of mass m_p leaving the surface of a massive object of mass M and escaping to infinity. At a distance r from the center of the mass M, the force of attraction toward M is

$$F_{m_p} = \frac{GMm_p}{r^2} \quad (3)$$

The work done in raising the photon through height dr is given by

$$F_{m_p} dr = \frac{GMm_p}{r^2} dr \quad (4)$$

Hence the work done in raising the photon from a distance R from the center of the massive object to an infinite distance is given by

$$W = \int_R^\infty F_{m_p} dr = \int_R^\infty \frac{GMm_p}{r^2} dr = \frac{GMm_p}{R} \quad (5)$$

This work is done at the expense of the photon's energy. This loss of energy by the photon results in a reduction of its frequency from f to f' given by

$$hf - hf' = \frac{GMm_p}{R} \quad (6)$$

Using (2) and $E_p = hf$, (6) becomes

$$hf - hf' = \frac{GMhf}{Rc^2} \quad (7)$$

Hence the fall in frequency is given by

$$f - f' = \frac{GM}{Rc^2} f \qquad (8)$$

Equation (8) represents gravitational redshift as light leaves the surface of a massive body.

Now for a photon with frequency f and velocity c moving from a great distance away to a distance r from the surface of a mass M, it follows that the photon experiences an increase in frequency from f to f' given by

$$f' - f = \frac{GM}{rc^2} f \qquad (9)$$

This yields

$$f' = f(1 + \frac{GM}{rc^2}) \qquad (10)$$

Substituting for f' in (1) gives

$$c' = \frac{cf^2}{f'^2} = c(1 + \frac{GM}{rc^2})^{-2} \qquad (11)$$

This when expanded gives

$$c' = c(1 - \frac{2GM}{rc^2}) \qquad (12)$$

Thus, the speed of light decreases as it enters the gravitational field of a massive object and this introduces a time delay.

3. Shapiro Time Delay

Using (12) and following the approach employed by Possel [3], we now calculate the time delay for light travelling from planet P, passing close to the sun S, and travelling on to the Earth E as shown in Fig.1. Assigning the centre of the sun as the origin, the planet P is at $x = -x_P$, $y = \eta$ and the Earth is at $x = x_E$, $y = \eta$. Assuming that the signal travels in a straight line, then $r_E = \sqrt{{x_E}^2 + \eta^2}$ and $r_P = \sqrt{{x_P}^2 + \eta^2}$. The distance η represents the light's closest approach to the sun. The light speed between P and E is governed by equation (5) where $r = \sqrt{x^2 + \eta^2}$. Noting that $c' = dx/dt$ we have

$$cdt = dx/(1 - \frac{2GM}{rc^2}) \simeq dx(1 + \frac{2GM}{c^2\sqrt{x^2+\eta^2}}) \qquad (13)$$

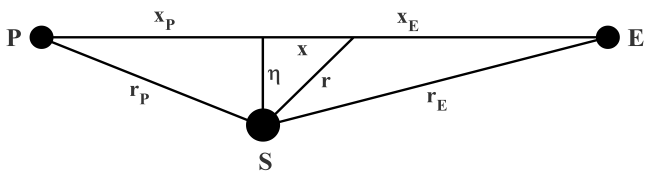

Fig.1 Planetary Arrangement for Time Delay Calculation

Integrating this equation gives

$$ct = x + \frac{2GM}{c^2} ln(\sqrt{x^2 + \eta^2} + x) + k$$

(14)

where $k$is the constant of integration. For light leaving Pat time $t_P$and arriving at Eat time$t_E$we have

$$ct_P = -x_P + \frac{2GM}{c^2} ln(\sqrt{x_P{}^2 + \eta^2} - x_P) + k \quad (15)$$

and

$$ct_E = x_E + \frac{2GM}{c^2} ln(\sqrt{x_E{}^2 + \eta^2} + x_E) + k \quad (16)$$

Subtracting equation (15) from (16) we get

$$t_E - t_P = \frac{x_E + x_P}{c} + \frac{2GM}{c^3} ln(\frac{r_E + x_E}{r_P - x_P}) \quad (17)$$

The first term on the right is the time taken for light travelling at constant speed c. The second term is the additional time due to the slowing of the light as it traverses the gravitational field of the mass M. It corresponds to the general relativistic correction confirmed by Shapiro in 1971 [2] and is referred to as Shapiro Time Delay or gravitational time delay. From (17) we can write this time delayΔt_{sh}as

$$\Delta t_{sh} = \frac{2GM}{c^3} ln(\frac{r_E + x_E}{r_P - x_P}) \quad (18)$$

The two-way time delay value would of course be twice the one-way value given in equation (18).

Using data extracted from Shapiro's 1971 paper, Possel [3] showed that the predicted time delay (18) was confirmed as shown in Fig.2. The formula derived by Shapiro in his 1964 paper [1] is based on Schwarzschild coordinates while that presented in his 1971 paper [2] has one less term and is based on isotropic coordinates. Possel notes that the differences between the two formulas is significant, the later version fitting the data better. In our derivation using Newtonian gravitation, there was no coordinate adjustment in arriving at the formula (18) which achieves an excellent fit with the data as shown by Possel in Fig.2. This derivation is much simpler than that based on general relativity.

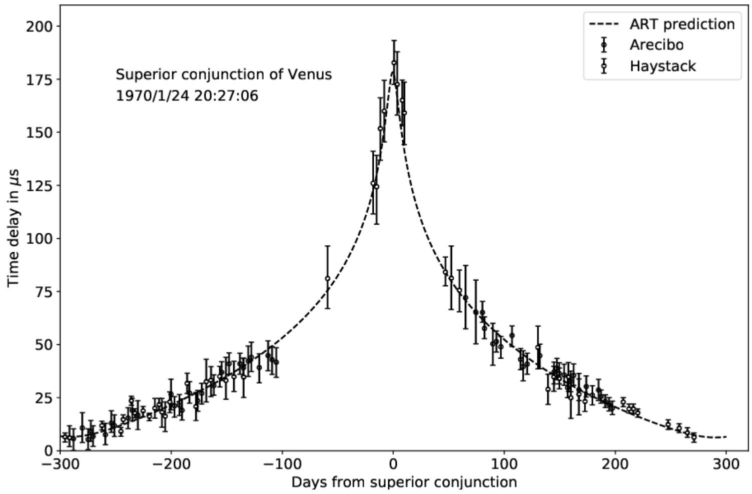

Fig.2 Plot of Time delay based on (18) and Radar echo data for Venus [3]

4. Conclusion

In this paper, the Shapiro time delay for light travelling close to a massive object was derived using Newtonian gravitation. Noting from black body radiation that the flux density of photons is proportional to the square of the frequency of the associated light, a relationship between frequency and light speed was determined. This in turn was employed in the basic gravitational redshift equation to determine the light speed change for light entering a gravitational field. This equation enabled the derivation of the correct Shapiro time delay formula, inclusive of the factor of 2 missing from the derivation by Possel. We believe that this approach involving reduced light speed for light entering a gravitational field, would enable a full understanding by undergraduate students of the delay effect, as compared with an explanation based on curved space-time of general relativity.

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