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A Mathematical Characterisation of COVID-19 in Mauritius

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Contents

Abstract

Since the advent of COVID-19, a fair amount of work has been undertaken by researchers around the world to model its progression. It became clear from the start of pandemic that its progression is affected by numerous factors within different communities. Subsequently, the necessary means and the range of measures used to effectively control the virus would vary from place to place. And we have been witness to different approaches adopted around the world to maintain the virus under check both in the short term and the long term. Various metrics have been used to mathematically represent the effectiveness of the approaches used. In this work, an attempt is made at determining those metrics for Mauritius and comparing them with that of the rest of the world. We first develop mathematical models of the progression of COVID-19 in Mauritius and in numerous other countries primarily in Europe. An intriguing observation is made when the growth rate and the ceiling value of the mathematical models obtained for different countries are compared. We computed the reproduction number, which showed us how many subjects a contagious individual was infecting on average at the onset of the pandemic in Mauritius. This value in turn allowed the determination of the percentage of the population needing immunity to stop the spread of the virus. The case fatality rate as well as the crude mortality rate for different countries are also compared and contrasted.

1 Introduction

In Mauritius, the first COVID-19 wave started with three cases of COVID-19 detected on the 18th March 2020. ”Patient zero” is believed to be a Mauritian national who arrived from the United Kingdom a week earlier. Prompt and drastic measures were taken by the government six days later on the 24th March 2020 with a complete lockdown of all facilities to contain the virus, keeping only essential services and certain economic activities operational. The virus was still poorly understood at that stage, the first worldwide case being officially registered by the Chinese authorities on the 31st December 2019 [13]. The decision to close the air space ensured that the only action necessary to control the propagation of the virus was through effective social distancing and proper sanitary precautions taken at health care centres. Health authorities coped reasonably well at the time with the designation of dedicated COVID-19 areas. Sanitary restrictions imposed were relaxed on the 30th May 2020 after the number of new cases subsided considerably. This period corresponds to the first wave for Mauritius during which we shall fit an S-shaped logistic growth curve using the whole range of data released by the government (Ministry of Health) on a daily basis. The reason why we chose the S-shaped growth curves is because they are known to be suitable fits to numerous naturally and man-made processes [2]. Three parameters define the S-curves and these parameters are determined for a few countries during the first wave using data from [1]. The final value and the slope of the curve are then used to compare the progression of the virus in Section 3.

The second lockdown started on the 10th March 2021 and went on until the 30th April 2022, with the government imposing varying levels of restriction during this time (see the timeline in [14] for details). Mauritius peaked during the month of October 2021. We characterise the variation of the number of cases during the second COVID wave using logistic curves in Section 5, but this time assuming that only 80% of the data is available until the end of the second wave. In other words, the algorithm is required to determine an S-shaped model to predict the remaining 20% of the data. Furthermore, we allowed for a certain bound of uncertainty with 90% confidence interval in our determination of the parameters of the second-wave parameters.

An S-curve fit while being useful in analysing a pandemic is however not a tool that can usually be used by the authorities to control the propagation of a virus at the beginning of a pandemic. The model obtained using S-curve logistic fit tends to become more accurate as we gather more data or as we reach the end of a pandemic/wave(s). While we can cater for uncertainty in our model, it is usually much more helpful to use other figures of merit to have a clear idea of the level of restriction of social interactions that is needed at the beginning of a pandemic. In particular, we are interested in determining how many subjects a contagious person will infect on average, if no restrictions are imposed. This figure is known as the basic reproduction number and can be accurately estimated by using data available in the first few days, before any intervention by the authorities. The reproduction number is a function of time and will decrease as sanitary restrictions are imposed. We show how this figure varies with time in Mauritius during the first wave in Section 4. Furthermore, an accurate determination of the reproduction number allows for the calculation of the percentage of the population that needs to develop immunity to stop the exponential growth of the number of cases with the virus. Here, it must be emphasised that because of the small number of cases in Mauritius, particularly during the first wave, statistical errors arising through the analysis can tend to be high and if that is the case, the error bound of the computational results is clearly inferred in any conclusion made.

In Section 6.1, we determine the variation of the Case Fatality Rate (CFR) of Mauritius throughout the pandemic and compare it with those of the United States, United Kingdom, India and Fiji Islands. CFR is the ratio of the number of deaths and the number of infected cases. A few interesting observations are made here especially for the case of India showing the limitation of the metric. For good measure, we also investigate the performance of Mauritius using the Case Mortality Rate (CMR) in Section 6.2 and compare it with that of a few countries before finally concluding.

2 Modeling of the first COVID-19 wave

S-shaped logistic growth curves have been widely used to model and forecast evolutionary processes with applications in the areas of biology, ecology, epidemiology to the manufacturing industry, among numerous others [3]. The approach uses data obtained to fit an S-shaped curve with the aim of predicting the the final value at which it will settle, also called the ceiling value. As expected, the more of the S-curve range the measurements cover, the more accurate is the determination of the final value. It is often argued that the usefulness of the approach in *accurately* modelling a pandemic requires too much of it to have elapsed, by which time it would be difficult to plan and to take firm and balanced measures to control it. It becomes therefore necessary to include certain bounds of uncertainties in the determination of the parameters for the logistic growth curves.

The S-curve is mathematically represented by the logistic function:

$$C = \frac{M}{1 + e^{-\alpha(t-t_0)}} \tag{1}$$

where M is the final value, α is the slope of the curve (also called the rate of growth) and t_0 is the time-delay in a given process. These three parameters are determined using an optimization and an iterative approach, assuming that the fitting model is representative of a pattern of natural growth interacting with the environment. Time-series data collected on a daily basis as publicly released by the Ministry of Health in Mauritius was used for that purpose. For the first wave, the data used to determine the parameters of the logistic function represent the complete cycle of the pandemic, that is the function was fitted after the completion of the first wave and as such no bound of uncertainty was included to account for variations in the parameters and only the nominal S-curve is determined. The parameters are determined by minimising the following cost function:

$$L = \min_{M,\alpha,t_0} \|C(M,\alpha,t_0,t_d) - y_d\|_2^2 = \min_{M,\alpha,t_0} \sum_i [C(M,\alpha,t_0,t_{d_i}) - y_{d_i}]^2 \tag{2}$$

where t_d and y_d represents the time (in days) and the number of positive COVID-19 cases identified on the day respectively. The initial values for the parameters were set at $[M,\alpha,t_0] = [250, 0.35, 15]$ and The nonlinear least square solver computed the final solution for the 3 parameters $M_1 = 335, \alpha_1 = 0.236$ and $t_1 = 15.3$. The squared 2-norm of the residual evaluated to $L_1 = 4085.6$ at the solution. The subscript 1 is used to differentiate between the parameters obtained in the first wave to those of the second wave.

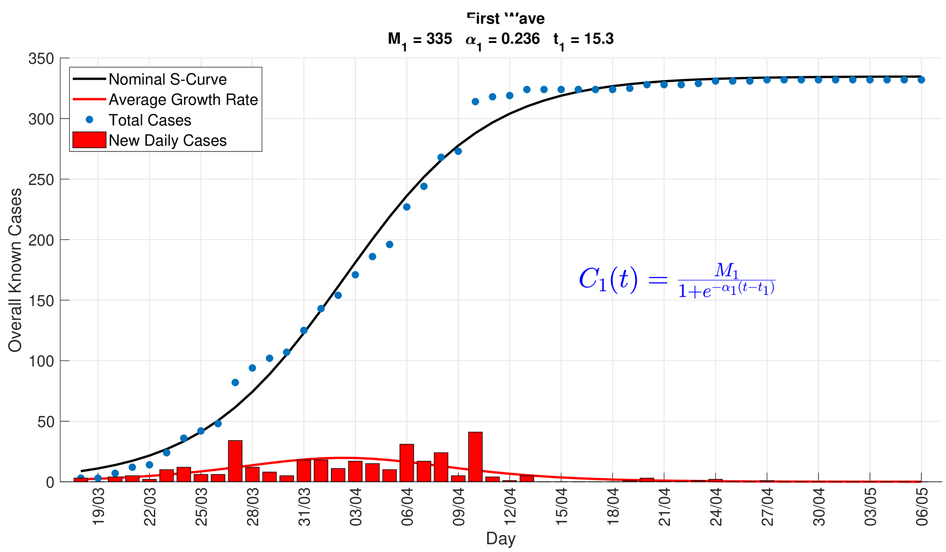

Figure 1: Nominal S-curve fit to data from first wave of COVID-19

Figure 1 shows a plot of the total number of reported cases of COVID-19 as from the 18th March 2020 up until the 6th May 2020, by which time no new cases were officially recorded for the last 12 days. The nominal S-curve as defined by the determined parameters is clearly a very good fit to the observed data. The daily number of new cases is shown by the bar chart. The slope of the curve is determined by differentiating the nominal S-curve:

$$\frac{dC}{dt} = M\alpha \frac{e^{-\alpha(t-t_0)}}{\left[1 + e^{-\alpha(t-t_0)}\right]^2} \tag{3}$$

It is referred to as the average growth rate in Figure 1 and is representative of the average daily number of new cases.

3 Relationship between ceiling value and growth rate

At the end of the first COVID-19 wave, data were gathered in various parts of the world allowing us to analyse the effect of government's actions during the pandemic. The aim of any government would be to act swiftly enough to limit the spread of the virus, thus reducing the number of positive cases (detected) and in turn keep the number of cases that require hospitalization/intensive care to a manageable level. In terms of the parameters in (1), the desired effects of government actions are three folds:

1. To delay the time to reach the peak number of new cases detected each day thus allowing for proper planning and health care readiness when it happens (make t_0 large). This would include training health care providers for suitable first care treatment when new cases are presenting and ensuring that hospitals or dedicated centers are in place and to allow time for procurement of necessary equipment (respiratory ventilators) to cater for hospitalised cases,

2. To reduce the peak number of cases (reduce α) so as not to completely burden health care services when it happens,

3. To reduce the ceiling value M representative of the total number of cases.

We first illustrate the effect of varying α and t_0 in (3) in Figure 2 for some arbitrary value set of M, α and t_0. Doubling of the growth rate α results in a doubling of the peak value of the number of cases. It should be noted that varying α alone does not affect the time at which the new peak value occurs. Furthermore, it can be seen that halving the time delay t_0 results in the peak value being left shifted. Thus, government action taken to control the virus would aim for a reduction in α, thus minimising the risk of not being able to meet the necessary medical care needed at any given time due to large demand. In addition, it would be desirable to increase t_0 in order to buy time in case the pandemic is spreading at a faster rate than can be effectively managed with existing resources. Very early through the pandemic, the urgency to flatten the bell-shaped curve became clear. The measures taken by authorities around the world did not however separately affect the three parameters described above and instead their action(s) distorted the bell-shaped curve into an asymmetric distribution. The level of distortion observed was a function of the extent of actions taken by authorities. For example, a strict lockdown reduced α considerably more than social distancing measures such social gathering restrictions would. The use of masks and travel restrictions also affected α and t_0. It is difficult to quantify the individual effect of each of these measures on the curve, as they were usually imposed in parallel. Furthermore, the slope α is in fact an average of different α's at different moment in time as a consequence of continuously changing enforcement by the authorities.

We gathered data for the first wave at different geographical locations and fitted logistic curves to each of them. In so doing, a series of values of M, α, t_0 were generated for each location. It was noted that quick effective intervention by the authorities generally resulted in symmetric curve with a clear peak whereas delayed intervention tended to distort the curve and result in a prolonged trailing end of the bell-shaped curve. In some cases, countries developed a plateau rather than a peak which meant that they had a really hard time managing the pandemic with large number of cases over a long duration of time. The United States in particular suffered from this, primarily due to the fact that large segments of the population did not abide by instructions for social distancing until it was too late. To allow for appropriate comparison, the values of M were normalised at each of the corresponding location. These values were then plotted against α as shown in Figure 3.

A surprising correlation is found between the final number of infections and the slope of the curve. We have seen in Figure 2 that doubling α resulted in a much larger peak, showing poor control of the epidemic. One would reasonably expect therefore with a larger peak that the final total number of infections would increase, thus showing a positive correlation between M per capita and α. However, Figure 3 shows quite the opposite. A negative correlation exists between M and α, implying that a larger

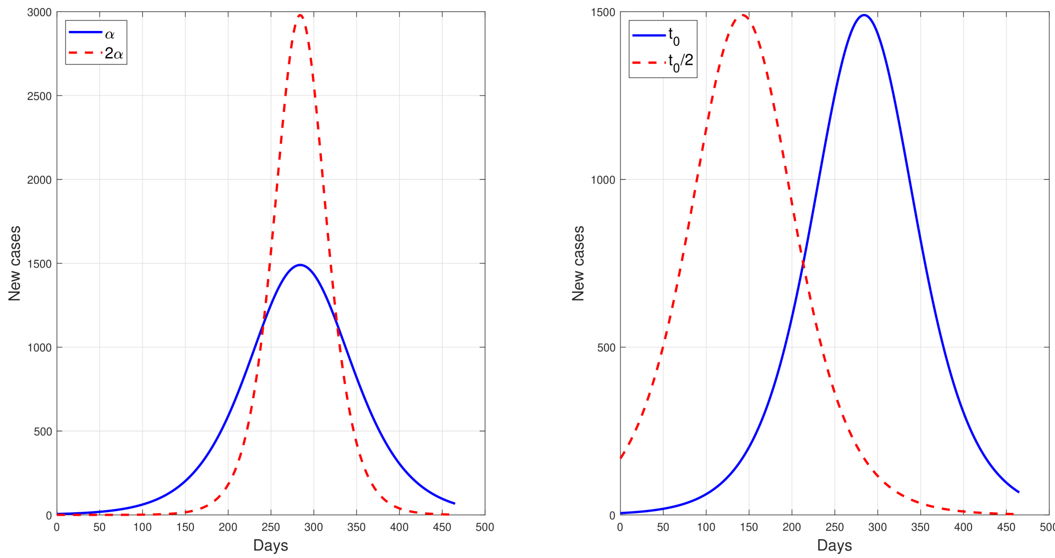

Figure 2: Effect of doubling growth rate α and halving time delay t_0

value of growth rate α results in a lesser number of final infections. This may seem counter-intuitive at first but upon further analysis, it was observed that in many countries, the actions taken resulted in a distorted flattened curve which had a plateau rather than a single peak as well as a trailing end. This trailing end in particular shows that the number of new cases stayed more or less constant for a long time at the end of the pandemic. The assumption has been all along that restrictions imposed by the government will reduce α which will in turn reduce M. The negative correlation between the two parameters in Figure 3 clearly suggests otherwise. It can be seen in Figure 3 that South-Korea had a high value of α, implying a high growth rate and a low ceiling value of M per capita. They were among the first countries to successfully control the spread of the virus at least during the first wave through intelligent and effective actions taken by the authorities which were strictly adhered to by the population. At the other end of the spectrum, we have the case of the United Kingdom where a small growth rate α resulted in a long plateau and a large number of hospitalised cases and casualties. In that respect, Mauritius (in red) did quite well and is on a par with Australia.

4 Basic Reproduction Number R_0

The basic reproduction number (R_0) is a widely used metric to determine the extent of spread of an epidemic [4]. It is calculated at the beginning of an outbreak when the social habits of a population are still unchanged and is defined as the average number of secondary cases that a primary case will generate in a given population [5]. It is a value that varies from country to country and is determined by essentially three factors: the number of people that a contagious person has been in contact with, the probability that such contacts result in the propagation of the virus and the duration over which the infected person remains contagious. The first two factors are clearly highly dependent on the social habits of a given population.

It is a number that is of critical importance to the authorities as it allows them to determine the proportion of the population that needs to be vaccinated or to have developed immunity to halter the spread of the disease. If R_0 is greater than 1, it implies an exponential growth in the number of cases while an R_0 below 1 implies an exponential decline. During a pandemic, the proportion of the population

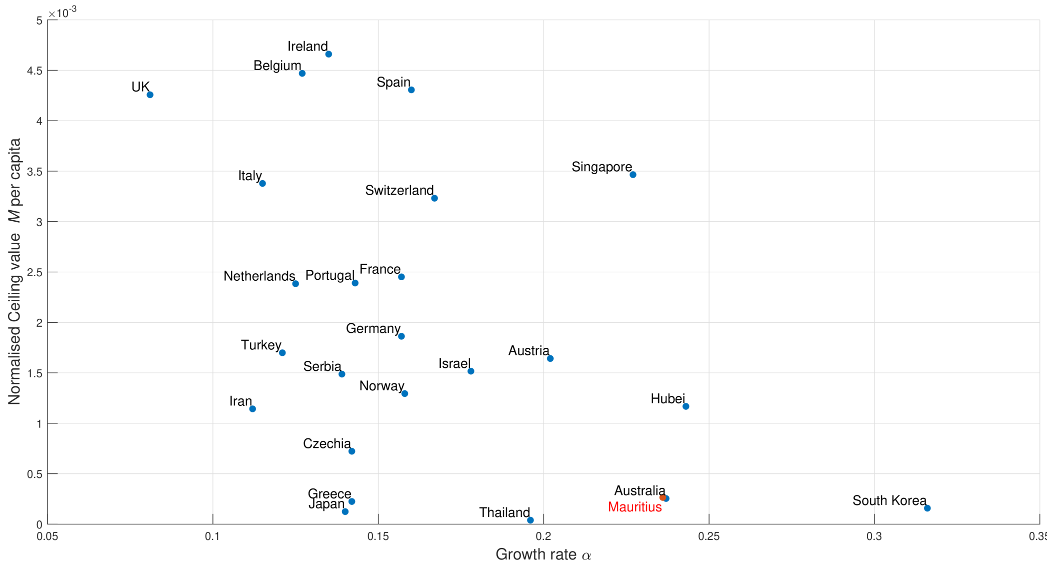

Figure 3: Scatter plot of normalised ceiling M versus growth rate α for the first wave at different locations

infected by the pandemic keeps increasing over time and after they have recovered, they become immune to the disease or to be precise, immune to a particular variant of the virus. There is also a proportion of the population who get vaccinated before they catch the virus and as such also develop immunity to the disease. If we define the proportion of the population that is immune to the virus as P and the reproduction number at any given moment in time as R, we can derive expression for the proportion of population needed to stop the spread of the virus. It is clear that $R = R_0 \cdot 1$ at the beginning, and with time $R = R_0(1 - P)$. If we want R to be 1 or less, $P \geq 1 - 1/R_0$. As an example, if $R_0 = 5$ meaning that one contagious person will on average infect 5 new subjects, then the percentage of the population that needs to become immune to stop the exponential growth of the virus is 80%.

4.1 Initial Growth Rate

To determine the basic reproduction number, we need to first calculate the initial growth rate of the disease. The reason why we choose a period that is at the beginning of the outbreak is because the social habits of the population are still unchanged by the pandemic at that time. We choose a duration of 7 days over which we plot the natural logarithm of the cumulative number of cases. If the pandemic is exponentially growing, we would expect a straight line which was the case in Mauritius as shown in Figure 4. It must be emphasised however that this analysis is statistically error prone for countries with too few cases (with daily incidence values less than 20) [7]. Mauritius unfortunately falls within that bracket but nevertheless, the exponential relationship is clearly valid.

At the start of an outbreak, the exponential growth rate ρ_i is a measure of the daily increase in the cumulative number of cases. $\rho = \log(C_{j+1}) - \log(C_j)$, where C_j is the cumulative number of cases on day j. Thus, $e^\rho = C_{j+1}/C_j$. For the case of Mauritius where the initial value of the growth rate $\rho_i = 0.44$ (from Figure 5), this implies that by multiplying the cumulative number of cases on a given day by $e^{\rho_i} = 1.55$, we obtain the cumulative number of cases for the next day. We can of course calculate the growth rate at various periods of the epidemic. However as the value of ρ is very much dependent on data on one given day, especially at the beginning of an epidemic, we smooth the data over a selected period of time to obtain an estimate of the growth rate $\hat{\rho}$. In our case, we have chosen a period of 7

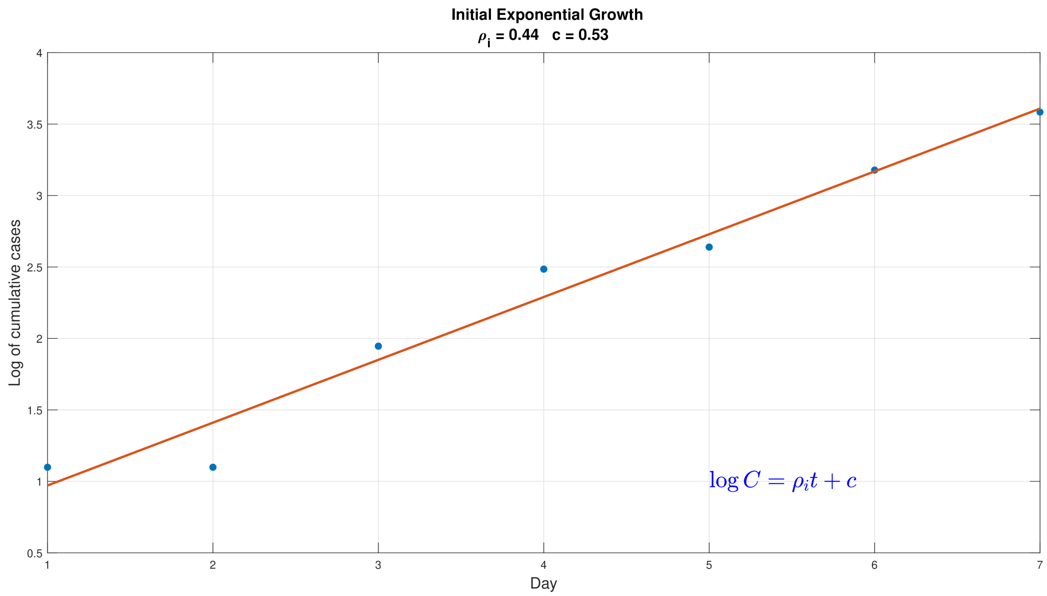

Figure 4: Initial Growth Rate

days. Figure 5 shows the variation of the growth rate during the first wave. It starts at around 0.35, corresponding to a $C_{j+1}/C_j = 1.41$, gradually converging to a value of 0 as $C_{j+1}/C_j \to 1$ by the end of the pandemic wave.

4.2 Estimating the reproduction number R_0

The exponential growth rate parameter ρ and the basic reproduction number R_0 are related via the following formula [8] :

$$R_0 = \exp\left[\frac{\mu^2}{\sigma^2}\log\left(1 + \frac{\sigma^2}{\mu}\rho\right)\right] \tag{4}$$

where μ and σ are the respective mean and the variance of a generation interval defined as the time needed for an infected person to infect another person with COVID-19. Due to a lack of data for Mauritius, we shall use the European estimates for the mean and the variance to calculate the basic reproduction number R_0; $\mu = 3.96$ and $\sigma^2 = 3.46^2$. From Figure 4, we find the peak value of $\rho = 0.44$ resulting in an estimate of $R_0 = 3.03$ (c.f. 2.21 for Western Europe). This implies that at the onset of the outbreak, we had an average of 1 contagious person infecting 3.03 individuals. Furthermore, it also gives a first indication as to the percentage of the population that needed to develop immunity to stop the spread of the virus at 67% $(1 - 1/R_0)$.

5 Modeling of the second COVID-19 wave

For the second wave, the S-curve is mathematically modeled (slightly differently from the first wave) as follows:

$$C_2 = \Phi + \frac{M_2}{1 + e^{-\alpha_2(t-t_2)}} \tag{5}$$

where the parameters M_2, α_2 and t_2 are to be determined. The constant parameter value Φ is used to simplify the model used for the second wave and is assumed to be a constant value representing the total

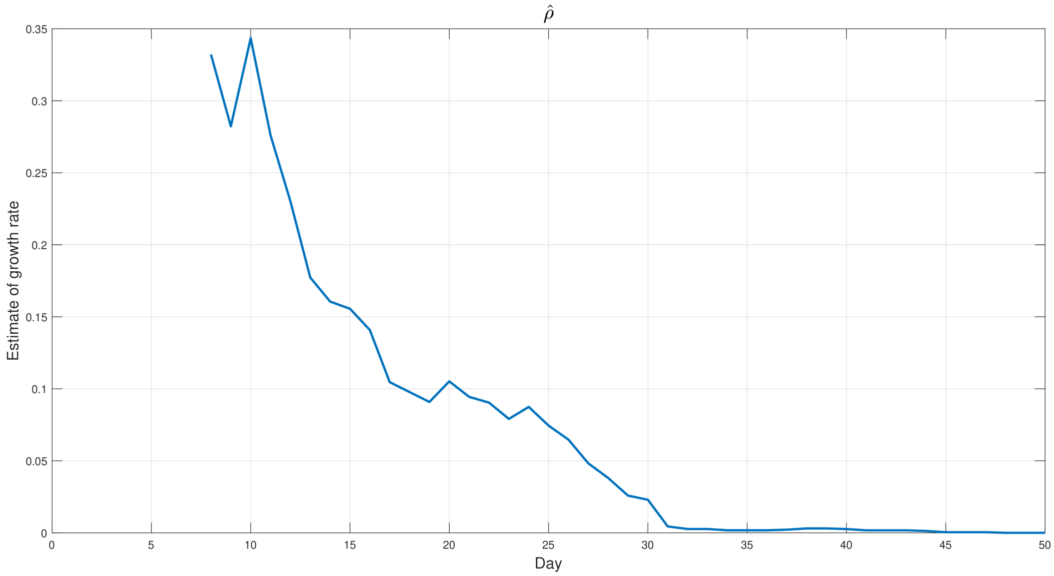

Figure 5: Estimate of growth rate $\hat{\rho}$

number of cases just before the start of the second wave. In this case, it was 1204. Ideally, the model to represent the second wave should have been the sum of two logistic curves, accounting for the first wave and the subsequent wave. However, the number of new cases of the virus immediately after the first wave in Mauritius was primarily due to the re-opening of the air space. The spread of the virus was very well controlled through strict quarantine measures on new arrivals. As such these data would not fit on a logistic curve which requires a natural spread of the virus through interaction between infected cases. The quarantine measures were however relaxed over time and the surfacing of a much more contagious variant of the virus (believed to have mutated in India) gave rise to the second wave in Mauritius. It began on the 5th March 2021 when four new positive local cases were detected and the country was put under lockdown again on the 9th March 2021, initially for a period of 14 days. Varying levels of confinement were then imposed until the 1st July 2022 when authorities relaxed sanitary measures and allowed incoming travelers who were unvaccinated to clear immigration without undergoing a test on arrival or needing to self-isolate.

We collected data up to the end of June 2022 from [1] and assumed that the data collected represented 80% of the S-curve. We fitted the S-curve as previously done for the first wave and predicted the number of cases for the remaining 20%. Data collected had a high level of uncertainty as can be seen in Figure 6. Accurate data and the larger the percentage of the S-curve that they cover allows for a better fit and a more accurate determination of the three parameters. In particular, for the case of Mauritius, it can be noted that on numerous days, the number of new cases were officially recorded as zero (which is not correct) and the cumulative sum of all those cases were added together on just a single day later on. It is worthwhile noting that the data was not smoothed before being used to determine the parameters of the second wave. The initial values assumed for the parameters were $[M, \alpha, t_0] = [250\,000, 0.05, 200]$. The least square solver computer the final solution for the 3 parameters $M_2 = 241\,419, \alpha_2 = 0.025$ and $t_2 = 284.2$ with the squared 2-norm of the residual evaluating to $L_2 = 2.089 \times 10^{10}$ at the solution. This figure is clearly quite high and can be explained by the fact that data were not reported on a daily basis.

We also quantified the uncertainties arising from the data set and in this way allow for some variation in the parameters. We used a χ^2 minimization technique to numerically determine the uncertainties in the parameters assuming a given confidence interval. From a large set of fits obtained using simulated

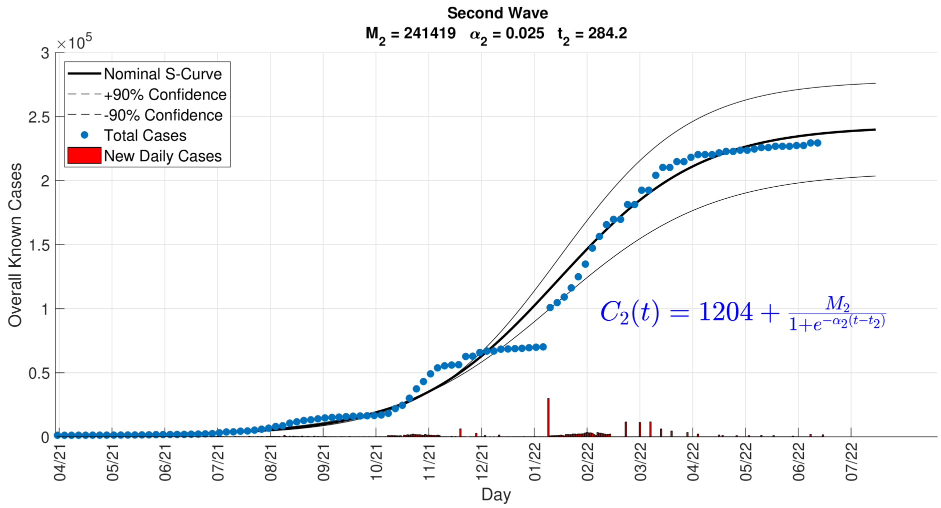

Figure 6: S-curve fit to data from second wave of COVID-19

data that is statistically deviated from the true values, a database of tables was built to establish uncertainties in the parameters for different scenarios depending on the percentage of data collected and the corresponding confidence levels [6]. In our case, we consider the scenario where we have 80% of the data and determine bounds for the three parameters M_2, α_2 and t_2, using a 90% confidence level. A chosen confidence interval will indicate a range above and below the nominal value and provide a bound in which the number of cases will lie with 90% degree of certainty. The corresponding values used in the determination of the bounds are shown in Table 1.

These values provide a grid of uncertainty which is then interpolated to determine $\Delta M, \Delta\alpha$ and Δt_0. Two additional logistic curves are thus included in Figure 6 using the parameters $(M + \Delta M, \alpha + \Delta\alpha, t_0 + \Delta t_0)$ and $(M - \Delta M, \alpha - \Delta\alpha, t_0 - \Delta t_0)$ respectively with $[\Delta M, \Delta\alpha, \Delta t_0] = [35617, 0.00211, 0.862]$. It can be seen that the forecasted data fit quite nicely within those bounds. A higher confidence level could have been used but the lack of data on certain days would not do justice to the quality of the fit.

6 Case Fatality Rate and Crude Mortality Rate

6.1 Case Fatality Rate

The Case Fatality Rate (CFR) is the most widely used ratio to determine the extent of damage that a virus has done. At the beginning of a pandemic, the authorities want to have an estimate of the proportion of the population that will not survive the pandemic. There are various metrics that are used to quantify this value and they vary in subtle ways and should not be confused with each other. Moreover, they invariably change as the pandemic progresses, i.e., these metrics are strong functions of time. It is usually found that the probability of not surviving a pandemic is much higher at the beginning of the pandemic and there are various reasons for this observation. The most common reason is that the population doesn't usually change its social habits at the beginning of a pandemic and can tend to trivialise its consequences. This has been noted in the United States where beaches and night clubs were kept open and people were allowed to mingle freely during the early stage of the virus hitting the US. The second reason is that the virus is usually poorly understood at the beginning of a pandemic and as a result effective first treatment may still be under trial. Furthermore, during the early stage of a pandemic, the population tends to become irrational and develop emotional and behavioural responses, creating a psychosis around the

Parameter M							
11.00	5.90	3.60	2.60	1.70	1.50	1.10	0.60
48.00	22.00	13.00	8.80	6.60	4.30	3.30	2.50
120.00	48.00	28.00	18.00	12.00	8.50	6.30	4.70
300.00	110.00	50.00	29.00	19.00	14.00	9.10	7.00
440.00	210.00	77.00	42.00	29.00	16.00	13.00	10.00
720.00	470.00	140.00	48.00	36.00	22.00	16.00	11.00
Parameter α							
1.10	1.10	0.70	0.70	0.70	0.70	0.60	0.50
4.50	3.40	2.90	2.60	2.40	2.30	2.00	1.90
8.10	6.70	5.80	5.40	4.80	4.40	3.90	3.50
11.00	9.40	8.40	8.40	6.70	6.70	5.90	5.60
14.00	12.00	11.00	9.90	9.60	8.70	8.30	7.00
19.00	15.00	15.00	12.00	12.00	11.00	8.70	8.40
Parameter t_0							
0.15	0.08	0.06	0.04	0.03	0.03	0.02	0.01
0.53	0.32	0.20	0.15	0.13	0.09	0.07	0.05
1.10	0.64	0.43	0.30	0.24	0.17	0.13	0.09
1.70	0.99	0.66	0.48	0.34	0.38	0.19	0.14
2.00	1.50	0.91	0.65	0.51	0.35	0.28	0.20
2.40	2.00	1.30	0.73	0.67	0.44	0.32	0.23

Table 1: Look-up tables obtained through χ^2 minimization technique for determining uncertainties of the three parameters

virus. This does not help the infected people at the time and they may be deliberately ignored by health care workers out of fear of being infected themselves.

CFR is defined as follows:

$$\text{Case Fatality Rate} = \frac{\text{Number of deaths from the disease}}{\text{Number of infected cases}} \tag{6}$$

We used (6) to calculate the variation of the CFR for both COVID-19 waves in Mauritius, and the result is presented in Figure 7. Due to a large variation in the number of cases in the first wave and the second wave, the y-axis on the left hand side is set to be logarithmic. The bar chart at the bottom represents the official number of COVID-19 related deaths per day and the corresponding values are represented on the linear y-axis on the right hand side. It can be noted that at the start of the pandemic, the CFR peaked at about 5% before settling down to a value of 3% by the end of the first wave. The country officially did not have any COVID-19 related casualty from May 2020 to March 2021, although we kept having new cases (coming from overseas which were isolated) explaining the gradual decline seen in the CFR during that time. From March to September 2021, the CFR kept decreasing and reached a remarkably low value of just under 0.1% at that time before the country officially restarted registering COVID-related deaths. In some instances, data is not available on each day and the total cumulative over a few days are added together and represented as the data for the day. This explains the sudden jump observed in the calculated CFR value in the figure. Based on the information provided, the CFR for Mauritius at the end of the second wave of the pandemic was around 0.45%.

To compare the performance of Mauritius with the rest of the world, we also plotted the CFR for four other countries during the same period as shown in Figure 8. We chose the United States and the United Kingdom as references, assuming that their reported data present an accurate reflection of the true state

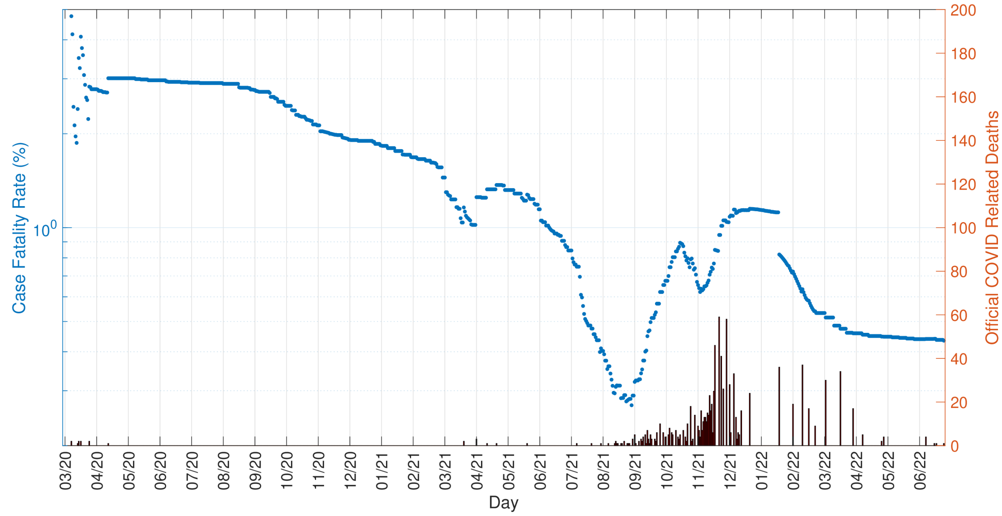

Figure 7: Case Fatality Rate for Mauritius

with regards to COVID-19 at the time. We included India as we expect it to be one of the countries with the highest CFR, given the international media coverage it warranted during the second wave with the very poor management of the pandemic. And finally, we chose Fiji due to its demographic, geographic location as well as ethnic similarity with Mauritius.

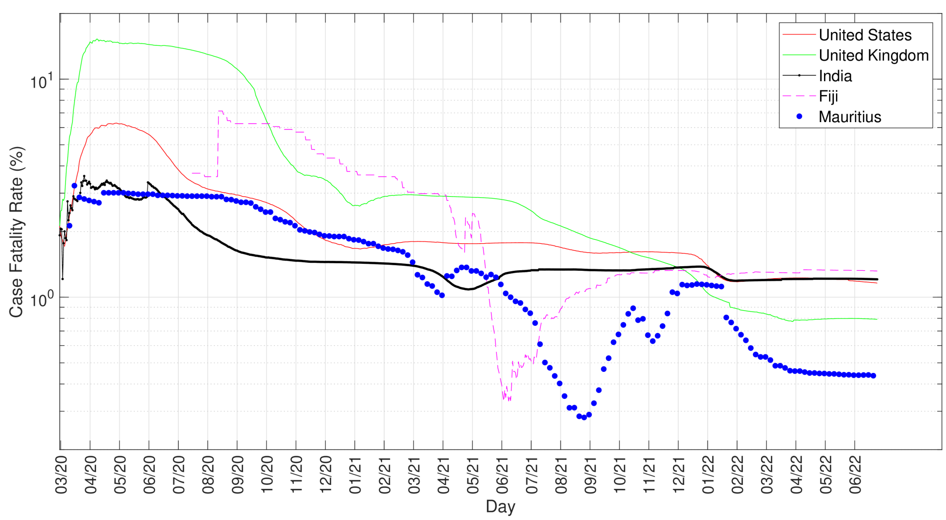

Figure 8: A comparison of Case Fatality Rate

We note a very high fatality rate for the UK at the beginning of the pandemic, reaching a value of just over 10% before subsiding by September 2020 and beautifully managing the pandemic from that point onwards to the extent of being almost unaffected by the second wave. The CFR kept decreasing all along before finally settling down at a very low value of 0.8%. A similar behaviour is seen for the US except that they peaked at a value of around 6%. India surprisingly has a peak CFR of only 2.5% which then went down and stayed at a value close to 1% throughout the second wave. This clearly reflects the problem with using CFR blindly as a measure of performance by authorities in the face of a pandemic. CFR is calculated based on the number of "confirmed deaths" and "confirmed infections". In the case of India, it was officially recorded that there were hundreds of thousands of people infected per day during the months of March-April-May 2021 (peak of 314835 new cases in 24 hours on 21st April 2021 [9]). However, due to scarcity of health facilities, sick and infected people were not able to receive appropriate treatment and hundreds passed away in their own premises, with their deaths not being recorded as COVID-19 related. This resulted in a large number of confirmed infections but the number of official deaths did not align with the number of infections. So much so that firewood for funeral pyres were being rationed leaving the dead partly cremated and the authorities were asked to cut down trees in parks for kindling. While it can be argued that there were also many more infections which were not officially recorded, especially in rural areas, so were the corresponding fatalities.

Fiji officially recorded its first COVID-19 case quite late (mid July 2020), probably due to the fact that it is an insular island, just like Mauritius and managed to control its borders quite well at the beginning of the worldwide pandemic. Its CFR peaked at 7% before dropping considerably before the second wave kicked in and finally settled at a value of just above 1%. We note that numerous countries (other than the ones mentioned here) converged to this value of about 1% at the end of the second wave. Mauritius however has a much lower CFR of just under 0.5% which is remarkably better than that of other countries, representing half the number of confirmed fatalities for the same number of confirmed infections. This figure begs for answers. While Mauritius did not do too badly for a *developing* country in terms of its sanitary capacity, it would seem that some under reporting of confirmed deaths may have been used to lower the figure. Mauritius has one of the highest prevalence of diabetes in the world which represents a serious comorbidity in the case of an infection with COVID-19. There were numerous reports in the local media that the cause of deaths were being wrongly reported as renal failure among other things, as a result of complications in diabetic patients arising from infection with COVID-19. It is also asserted that the government introduced new policies not to test asymptotic cases, in turn reducing the number of officially recorded infections and that only patients admitted to specially designated treatment centers were recorded as being COVID-19 positive [10]. It is difficult to validate these claims but it is interesting to note that after their publication in the local media, the CFR started shooting up again.

6.2 Crude Mortality Rate

Another figure of merit that is used to determine the efficacy of authorities and health care services to deal with the pandemic is the "Crude Mortality Rate" (CMR). CMR measures the proportion of the population that have died from a disease. It is the ratio of the number of deaths (from the disease) and that of the entire population. In the case of Mauritius, as at end of June 2022, the CMR for COVID-19 stood at 0.079%. Table 2 presents the crude mortality rate for some select countries. The United States has the highest CMR and surprisingly India has one of the lowest value, about 10 times less than that of the United States. Again, one can only conclude massive under reporting of the fatalities by the Indian authorities, as reported by some selected Indian and international media [12]. Surprisingly, even though Japan is the country with the largest population share (standing at 26%) in the world that is 65 years old or more [18], it has the lowest CMR. Mauritius has a lower CMR than the United States, United Kingdom as well as Fiji.

7 Conclusion

We have successfully fitted mathematical models to the two COVID-19 waves that hit Mauritius during the period February 2020 to July 2022. In so doing, we also derived similar models for numerous countries and found a surprising relationship in the data for the first COVID-19 wave. In particular, the ceiling value of the number of cases was inversely correlated to the slope of mathematical models fitted to the

Country	Number of Deaths	Population	Crude Mortality Rate (%)
India	526,033	1,407,563,842	0.037
United States	1,026,937	336,997,624	**0.305**
Japan	31,277	124,612,530	**0.025**
United Kingdom	182,912	67,281,040	0.271
Italy	168,353	59,240,330	0.284
Australia	9,930	25,921,089	0.038
Singapore	1,413	5,453,600	0.026
Mauritius	1,003	1,273,428	0.079
Fiji	807	924,610	0.087

Table 2: Crude Mortality Rate

curve, implying that a fast growth rate of the virus results in an lower and earlier plateau, hence in fewer infected cases at the end. Mauritius was found to have a fast growth rate resulting in a reduced ceiling value during the first COVID-19 wave Figure 3, thus showing good management of the early pandemic by authorities. We then estimated the initial growth rate of the pandemic to calculate the reproduction number $R_0 = 3.03$ at the beginning of the pandemic, implying that 1 contagious person would infect 3.03 subjects on average. This figure also allowed for the determination of the immunity percentage of the population that was needed to stop the spread of the various at 67%. Mauritius again did exceptionally well when we compare its face value performance using the Case Fatality Rate (CFR), although there remains some doubt as to the validity of the official numbers reported. And finally we summarise our result with the Crude Mortality Rate (CMR) where Japan stood out with a remarkably low figure, even though it is the country which has the highest population proportion above 65 years of age. Future work would involve designing a systematic control scheme using information available from the developed model to optimally manage the timing and level of confinement imposed on the population.

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