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Undefinable True Target Learning

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Abstract

The situation where the learning true target cannot be precisely defined is quite common in various artificial intelligence (AI) application scenarios. Yet, this situation has not been systematically analysed. In this article, we formally refer to this situation as undefinable true target learning (UTTL). From the perspectives of problem definition, alternative solution, specific method, and particular application, we present the first fundamental basis for systematically analysing the UTTL situation in AI application scenarios.

1. Introduction

There is a quite common situation in various artificial intelligence (AI) application scenarios, which is that the true target for a learning task cannot be precisely defined. For example, in the scenario of applying AI technologies to implement a tool for automatically segmenting tumour/lesion areas in whole slide histopathology images, the true target tumour/lesion areas are sometimes even impossible for pathological experts to precisely label (Yang et al. 2020; Yang, Li, et al. 2024; Yang, Yang, et al. 2024). In this article, we refer to this situation in AI application scenarios as a problem of undefinable true target learning (UTTL), which belongs to the realm of machine learning (ML) (Carbonell et al. 1983; Ditterrich 1997; Jordan and Mitchell 2015).

Regarding the current literature on ML, UTTL is similar to learning with noisy labels (LWNLs) (Natarajan et al. 2013; Song et al. 2023), which is a typical type of weakly supervised learning (Z.-H. Zhou 2018). For the LWNLs problem, inaccurately labelled data are provided, mostly due to the purpose of alleviating the labour-intensive labelling (S. Zhang et al. 2024a). For the UTTL problem, only inaccurately labelled data can be provided, since the true target cannot be precisely defined. As the data prepared for addressing the two problems can be identically inaccurate, UTTL naturally shares similarity with LWNLs, which indicates that some existing approaches for addressing LWNLs can be alternatively selected to address UTTL.

However, the fundamental assumption about the true target for UTTL presented in this article is completely different from the fundamental assumption about the true target in the literature of LWNLs or even in the literature of the entire ML realm, which can be summarized in Table 1. In this article, the fundamental assumption about the true target for UTTL is that the true target does not exist in the real world, since it cannot be precisely defined in this situation. In the current literature of LWNLs or even in the literature of the entire ML realm, the acquiescent fundamental assumption about the true target for a learning task is that the true target exists in the real world, as shown in a recent work that existing strategies for

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evaluations of predictive ML models are based on the assumption that the true target exists or probably exists in the validation labels (Yang 2024). This complete difference in the fundamental assumptions for UTTL and LWNLs indicates that existing approaches for addressing LWNLs are not the best options for addressing UTTL, which has also been shown in two recent works (Yang, Li, et al. 2024; Yang, Yang, et al. 2024) in medical histopathology whole slide image analysis.

Table 1. Fundamental assumptions for UTTL and LWNLs

Fundamental assumption	Learning concept
the true target **does not exist** in the real world	UTTL
the true target **exists** in the real world	LWNLs / ML

As far as we know, this article is the first that explicitly proposes the fundamental assumption that the true target for a learning task does not exist in the real world to formally present UTTL for systematically analysing the common situation, where the learning true target cannot be precisely defined, in various AI application scenarios. From the perspectives of problem definition, alternative solution, specific method, and particular application, we present the first fundamental basis for systematically analysing the UTTL situation in AI application scenarios.

Specifically, the definition for the UTTL problem is formally presented based on the fundamental assumption that the true target for the UTTL problem does not exist in the real world. On the basis of the presented definition, the UTTL problem is transformed into mainly a combination of the ML problem and the logical reasoning problem, and an alternative solution to the transformed UTTL problem is presented. Referring to the presented alternative solution, specific methods like one-step abductive multi-target learning (OSAMTL) and its extensions, which have been proposed in recent works (Yang 2021; Yang et al. 2020; Yang, Li, et al. 2024; Yang, Yang, et al. 2024), are summarized for addressing the UTTL problem in different scenarios. Referring to the summarized specific methods OSAMTL and its extensions, implementation rules and techniques of these methods are discussed regarding particular applications in real-world scenarios.

The rest of the contents of this article are structured as follows: Primarily, in Section 2, we briefly introduce IS and analyse the similarity and difference between UTTL and IS. Subsequently, Sections 3, 4, 5, and 6 respectively present the definition, alternative solution, specific method, and particular application for UTTL. Finally, in Section 7, we present a discussion, conclusion, and future work for this article.

2. Related work

As the data prepared for addressing UTTL and LWNLs can be identically inaccurate, UTTL shares certain similarity with LWNLs. LWNLs consider the situation where the labels of the provided data contain certain noises which lead to the inaccuracy of the labels in annotating the true target (Natarajan et al. 2013; Song et al. 2023). In current literature on LWNLs, numerous approaches have been proposed to address this problem, including robust architectures, robust regularization, sample selection, and robust loss design (S. Zhang et al.

2024b).

Particularly, the objective of robust architectures (Bekker and Goldberger 2016; X. Chen and Gupta 2015; Goldberger and Ben-Reuven 2022; Han, Yao, Niu, et al. 2018; Srivastava et al. 2014; Sukhbaatar et al. 2015; Xiao et al. 2015; J. Yao et al. 2019) is to apply a noise adjustment layer over a deep neural network (DNN) to grasp how labels change or to construct a unique architectural design that accommodates a wider variety of label noise categories, which strive to hinder a DNN's tendency to overly adapt to incorrectly labelled examples through the implementation of training constraints. A key advantage of robust regularization (Hendrycks et al. 2019; Jenni and Favaro 2018; Menon et al. 2020; Tanno et al. 2019; Wei et al. 2021; Xia et al. 2020) lies in its capacity to readily acclimate to novel scenarios with minimal adjustments. Sample selection strategies (Berthelot et al. 2019; P. Chen et al. 2019; Han, Yao, Yu, et al. 2018; Jiang et al. 2018; Li et al. 2020; Malach and Shalev-Shwartz 2017; Wei et al. 2020; Yu et al. 2019; T. Zhou et al. 2020) endeavour to pinpoint and prioritize the samples deemed most plausible to be clean for the purpose of enhancing the optimization process. Robust loss (Chang et al. 2017; Englesson and Azizpour 2021; Ghosh et al. 2017; Kim et al. 2019, 2021; Lyu and Tsang 2019; Ma et al. 2018, 2020; Patrini et al. 2017; Reed et al. 2014; Song et al. 2019; Wang et al. 2019; Xia et al. 2019; Y. Yao et al. 2020; H. Zhang et al. 2021; Z. Zhang and Sabuncu 2018) design seeks to calibrate the loss value in accordance with the certainty of a particular loss (or label) through various tactics, or devise a novel loss function tailored to cope with imprecise guidance. Typically, resilient loss functions incorporate a provision that imposes a penalty on predictions made with low confidence, which are more prone to result from noisy data points. For more information about the LWNLs problem and its alternative solutions, readers can refer to (Natarajan et al. 2013; Song et al. 2023).

However, different from the literature on LWNLs or even the literature of the entire ML realm, in which the acquiescent fundamental assumption about the true target for a learning task is that the true target exists in the real world, in this article, the fundamental assumption for UTTL is that the true target does not exist in the real world. Fundamentally, this complete difference in the fundamental assumptions for UTTL and LWNLs has led to the issue that existing approaches for addressing the LWNLs problem are not suitable to handle the UTTL problem. To alleviate this issue, in this article, we formally present the first fundamental basis for systematically analysing UTTL from the perspectives of problem definition, alternative solution, specific method, and particular application.

3. Problem definition

Let us consider the situation where the true target of a learning task cannot be precisely defined. In practice, this situation inevitably leads to a big problem in the label preparation for the learning task, which is that the label prepared for an entity/event contains severe inaccuracy in representing the true target associated with the entity/event. Here, we refer to this situation as a problem of undefinable true target learning (UTTL). Since large inconsistencies usually appear among experts regarding an agreement on the true target for the UTTL problem, in this article, we explicitly propose the fundamental assumption about the true target for the UTTL problem, which is that the true target does not exist in the real world.

Based on this fundamental assumption, the UTTL problem can be described as: based on a collected number of data points, each of which consists of an entity/event and a prepared label that contains severe inaccuracy in representing the undefinable true target associated with the entity/event, to find a function that can map the entities/events into the undefinable true targets. Notably, as the label prepared for the entity/event contains severe inaccuracy due to the fact that the true target is undefinable, the properties of the label prepared for the entity/event inevitably cannot precisely represent the properties of the undefinable true target. Thus, the solution to the UTTL problem (i.e., the found function that can map the entities/events into the corresponding undefinable true targets) should be subject to the condition that the properties of the labels prepared for the entities/events are included in the properties of the undefinable true targets mapped from the entities/events.

Denote the collected number of data points as $H = \{d, l\}$, where d is the entities/events, l is the prepared labels associated with d that cannot precisely represent the undefinable true target, and the elements in d and l have a one-to-one correspondence. Denote the function that can map the entities/events into the corresponding undefinable true targets as $f: d \longmapsto t$, where t is the mapped examples of the undefinable true target and the element in d and t as well have a one-to-one correspondence. The mapping function f should be subject to the condition that the properties of l are included in the properties of t. Denote the properties of l as $prop(l)$, the properties of t as $prop(t)$, and the relation of being included in as $\subseteq$. Now, the UTTL problem is formally defined as

$$\tilde{f} = \underset{f \in \Theta_f}{finding}\, f: d \longmapsto t \qquad s.t. \quad prop(l) \subseteq prop(t). \tag{1}$$

4. Alternative solution

We propose an alternative solution to the UTTL problem. Specifically, we firstly transform the UTTL problem into mainly a combination of the machine learning (ML) problem and the logical reasoning (LR) problem, and then we propose an alternative solution to the transformed UTTL problem.

4.1 Common ML and LR

For the common ML problem, the prepared set of labels l is usually assumed to be able to precisely represent the true targets t corresponding to the set of entities/events d in the collected number of data points $H = \{d, l\}$. Thus, in this situation, the properties of l ($prop(l)$) are equal to the properties of t ($prop(t)$) compared with formula (1). Formally, the common ML problem can be defined as

$$\tilde{f} = \underset{f \in \Theta_f}{finding}\, f: d \longmapsto t \qquad s.t. \quad prop(t) = prop(l). \tag{2}$$

Usually, the alternative solution to the common ML problem can be described as an optimized mapping function that can minimize the error between $t = f(d)$ and l, which can be formally expressed as

$$\tilde{f} = \arg \underset{f \in \Theta_f}{min}\, o(t = f(d), l). \tag{3}$$

Here, o is a predefined loss function that can estimate the error between $t = f(d)$ and l.

For the common LR problem, in addition to the prepared set of entities/events d and the corresponding set of labels l, an accumulated knowledge base (KB) containing various prior knowledge facts about the true target is both provided. The LR problem can be expressed as: to search a reasoning path (r) that can from the collected data points $H = \{d, l\}$ and KB to draw a set of conclusions (c) that are consistent with ($\cong$) some knowledge facts in KB. Formally, the common LR problem can be defined as

$$\tilde{r} = \underset{r \in \Theta_r}{searching}\, r: \{d, l\}, KB \to c \qquad s.t. \quad c \cong KB. \tag{4}$$

Usually, the alternative solution to the common LR problem can be described as a validated logical path (a series of valid logical processes) that can maintain the consistency between $c = r < \{d, L\}, KB >$ and KB, which can be formally expressed as

$$\tilde{r} = \arg \underset{r \in \Theta_r}{maint}\, cons(c = r < \{d, l\}, KB >, KB). \tag{5}$$

Here, $cons$ is a predefined procedure that can reflect the consistency between $c = r <$ $\{d, l\}, KB >$ and KB.

4.2 Transformed UTTL

Comparing the UTTL problem definition (formula (1)) with the common ML problem definition (formula (2)), we can note that the learning true target for the common ML problem can be precisely known, while the learning true target for the UTTL problem cannot be precisely known. This fact reflects that if we directly take the alternative solution to the common ML problem (formula (3)) as a solution to the UTTL problem, the finally found mapping function $\tilde{f}$ will suffer from severe inaccuracy in predicting the true target for the UTTL problem.

Referring to the common LR problem definition (formula (4)), we can observe that if we regard the conclusions c drawn from the provided data points $H = \{d, l\}$ and the accumulated knowledge base KB as some statements about the undefinable true target for the UTTL problem, then it is plausible that we can probably search a reasoning path that can draw some statements which are consistent with KB to be able to better describe the undefinable true target than the labels l in T for the UTTL problem. Thus, the alternative solution to the common LR problem (formula (5)) can probably be leveraged to propose a better alternative solution to the UTTL problem than naively employing formula (3).

We propose to transform the UTTL problem into a type of problem which is mainly a combination of the ML problem and the LR problem. Particularly, the transformed problem for UTTL can be divided into the following three sub-problems.

1) Based on a number of provided data points $H = \{d, l\}$ in which l cannot precisely describe the undefinable true target and an extra accumulated knowledge base KB which contains various prior knowledge facts about the undefinable true target, the primary sub-problem is to search a reasoning path r that can draw some statements c about the undefinable true target. The drawn c should be consistent with KB to be able to better describe the undefinable true target for UTTL than the labels l provided in the H. Formally, referring to formulas (1) and (4), this sub-problem can be defined as

$$\tilde{r} = \underset{r \in \Theta_r}{searching}\, r: \{d, l\}, KB \to c \qquad s.t. \quad prop(l) \subseteq c \cong KB. \tag{6}$$

2) Based on $H = \{d, l\}$ and the c from (6), the subsequent sub-problem is to build a programme (p) that can generate a new set of learning targets t^* corresponding to d. The properties of the generated t^* should be equal to c in describing the undefinable true target for UTTL. Formally, this sub-problem can be defined as

$$\tilde{p} = \underset{p \in \Theta_p}{building}\, p: \{d, l\}, c \rightharpoonup t^* \qquad s.t. \quad prop(t^*) = c. \tag{7}$$

3) Based on d and the t^* from 2), the final sub-problem is to find a mapping function that can map d into the corresponding final predicted true targets t for UTTL. The properties of the final predicted t should be equal to the properties of t^*. Formally, referring to formula (2), this sub-problem can be defined as

$$\tilde{f} = \underset{f \in \Theta_f}{finding}\, f: d \longmapsto t \qquad s.t. \quad prop(t) = prop(t^*). \tag{8}$$

Referring to the formulas (6), (7), and (8), the UTTL problem definition expressed in the formula (1) can be transformed as follows

$$\begin{cases} 1)\ \tilde{r} = \underset{r \in \Theta_r}{searching}\, r: \{d, l\}, KB \to c \\ 2)\ \tilde{p} = \underset{p \in \Theta_p}{building}\, p: \{d, l\}, c \rightharpoonup t^* \\ 3)\ \tilde{f} = \underset{f \in \Theta_f}{finding}\, f: d \longmapsto t \end{cases} \qquad s.t. \quad prop(l) \subseteq prop(t) \cong KB. \tag{9}$$

We can note from formula (9) that the subject condition for the transformed UTTL problem definition now is $prop(l) \subseteq prop(t) \cong KB$, which is different from the subject condition $prop(l) \subseteq prop(t)$ in the original UTTL problem definition expressed in formula (1). More details on how we get the subject condition in formula (9) from the formulas (6), (7), and (8) are provided in Proof 1 of the Appendix.

4.3 Analyses of the transformed UTTL

From the subject condition of the transformed UTTL problem definition expressed in the formula (9) ($prop(l) \subseteq prop(t) \cong KB$), we can observe that the properties of the labels L in the provided data points T ($prop(l)$) are included in ($\subseteq$) the properties of the final predicted true targets ($prop(t)$), and $prop(t)$ is also consistent with ($\cong$) the extra accumulated knowledge base KB which contains various prior knowledge facts about the undefinable true target. This subject condition reflects not only that the final predicted true targets t are able to better represent the undefinable true target for UTTL than the labels in the provided data points, but also that the properties of the final predicted true targets t are consistent with various prior knowledge facts about the undefinable true target for UTTL. This reflection indicates that the transformed UTTL problem definition is better at finding the appropriate mapping function for predicting the undefinable true target than the original UTTL problem definition.

Though the final predicted true targets t possess better properties, which are consistent with KB, compared with the labels l, we are still not sure about whether t can be precise enough to represent the undefinable true target for UTTL. Regarding the subject condition $prop(l) \subseteq prop(t) \cong KB$ in formula (9), we can deduce that how precise t can be to represent the undefinable true target for UTTL will depend on how precise the prior knowledge facts contained in KB can be to represent the undefinable true target. However, theoretically, with more knowledge facts iteratively accumulated in KB to represent the undefinable true target, the final predicted t can be iteratively more precise to represent the undefinable true target for UTTL. As a result, the transformed UTTL problem definition provides a promising foundation to approach the undefinable true target for UTTL.

4.4 Alternative solution to the transformed UTTL

Referring to the transformed UTTL problem definition expressed in the formulas (6), (7), (8), the alternative solution to the transformed UTTL problem can also be divided into three sub-solutions.

1) The first sub-solution is the solution to formula (6), which can be expressed as formula (5).
2) The second sub-solution is the solution to formula (7), which is to build a programme (p) to generate the learning targets t^* corresponding to d from $H = \{d, L\}$ and the c produced by the first sub-solution. Formally, the second sub-solution can be expressed as

$$\tilde{p} = \arg \underset{p \in \{\Theta_r \cup \Theta_f\}}{build} \ t^* = p(\{d, l\}, c). \tag{10}$$

Here, $p \in \{\Theta_r \cup \Theta_f\}$ indicates that the built programme p can be in the space of the LR-based methods (Θ_r), in the space of the ML-based methods (Θ_f), or in the space of the combined LR and ML methods ($\Theta_r \cup \Theta_f$).

3) The third sub-solution is the solution to formula (8), which can be expressed as formula (3) with the replacement of l with t^*.

In summary, the alternative solution to the transformed UTTL problem can be formally expressed as follows.

$$\begin{cases} 1)\ \tilde{r} = \arg \underset{r \in \Theta_r}{maint}\ cons(c = r < \{d, l\}, KB >, KB) \\ 2)\ \tilde{p} = \arg \underset{p \in \{\Theta_r \cup \Theta_f\}}{build}\ t^* = p(\{d, l\}, c) \\ 3)\ \tilde{f} = \arg \underset{f \in \Theta_f}{min}\ o(t = f(d), t^*) \end{cases}. \tag{11}$$

4.5 Additional notes

Notably, the optimal solution to the UTTL problem should not be limited to the alternative solution presented in this section, since the alternative solution here is proposed based on the transformed UTTL problem, which is mainly a combination of the ML problem and the LR problem. It is possible that a better problem transformation and corresponding solution for the UTTL problem defined in formula (1) can still be proposed based on other original thoughts and perspectives.

5. Specific method

Referring to the alternative solution presented for the transformed UTTL problem, which is summarized in formula (11), one-step abductive multi-target learning (OSAMTL) and its extensions have been proposed in recent works (Yang 2021; Yang et al. 2020; Yang, Li, et al. 2024; Yang, Yang, et al. 2024) to provide some specific methods for addressing the UTTL problem.

5.1 OSAMTL

OSAMTL requires as input materials a number of collected data points containing labels that cannot precisely represent the undefinable true target and an extra accumulated knowledge base that contains various prior knowledge facts about the undefinable true target. In addition to the required input materials, the key components of OSAMTL are respectively corresponding to the three sub-solutions presented in formula (11), which include the component of one-step abductive logical reasoning corresponding, the component of generation of multiple types of learning targets, and the component of multi-target learning.

5.1.1 Input materials

The input materials for the OSAMTL method include a number of collected data points $H = \{d, l\}$ where d is the entities/events, l is the prepared labels associated with d that cannot precisely represent the undefinable true target, and an extra accumulated knowledge base (KB) which contains various prior knowledge facts about the undefinable true target.

More specifically, H can be expressed as

$$H = \{d, l\} = \{\{d_1, l_1\}, \dots, \{d_n, l_n\}\}. \tag{12}$$

And KB can be more specifically expressed as

$$KB = \{k_1, \dots, k_m\}. \tag{13}$$

In formula (12), n denotes the number of data points collected in H, and each element $\{d_n, l_n\}$ represents a collected data point that consists of an entity/event d_n and its corresponding label l_n. In formula (13), m denotes the number of the prior knowledge facts, and each element k_m represents an accumulated knowledge fact about the undefinable true target.

5.1.2 One-step abductive logical reasoning

Based on the input materials H and KB, the one-step abductive logical reasoning (OSALR) component of OSAMTL draws some statements/conclusions (c) that can more accurately describe the undefinable true target than the labels provided in H. Formally, referring to the sub-solution 1) of formula (11), this component can be expressed as

$$c = \tilde{r}(H, KB) = \{c_1, \cdots, c_w\}. \tag{14}$$

More specifically, the OSALR component consists of three sub-steps as follows.

From H, the sub-step one extracts a list of groundings that can describe the logical facts contained in the given diverse noisy samples. Formally, this grounding extract (GE) step can be expressed as

$$g = GE(H) = \{g_1, \cdots, g_s\}. \tag{15}$$

Via logical reasoning, the sub-step two estimates the inconsistencies between the extracted groundings g and the prior knowledge facts accumulated in KB. Formally, this logical reasoning (R) step can be expressed as

$$ic = R(g, KB) = \{ic_1, \cdots, ic_u\}. \tag{16}$$

The sub-step three revises the groundings in g by logical abduction, which aims to reduce the estimated inconsistencies in ic. Formally, this logical abduction (LA) step can be expressed as

$$c = LA(ic) = \{c_1, \cdots, c_w\}. \tag{17}$$

With these three specific sub-steps (GE, R, LA) for implementing $\tilde{r}$ in the formula (14), the finally drawn statements/conclusions are revised groundings that are consistent with KB to be able to better describe the undefinable true target than simply the groundings of the labels l provided in H.

5.1.3 Generation of multiple types of learning targets

The generation of multiple types of learning targets (GMTLT) component aims to leverage H and c drawn by the OSALR component to abduce multiple types of learning targets. Formally, referring to the sub-solution 2) of the formula (11), this component can be expressed as

$$t^* = \tilde{p}(H, c) = \{t_1^*, \cdots, t_v^*\}. \tag{18}$$

The formula (18) indicates that the built program $\tilde{p}$ can generate multiple types of learning targets ($\{t_1^*, \cdots, t_v^*\}$) from H and c, that are associated with each data point of d in H. Usually, the program $\tilde{p}$ can be specifically implemented by logical reasoning and machine learning methods.

As the multiple types of learning targets ($\{t_1^*, \cdots, t_v^*\}$) can be generated from H with the help of the revised groundings (c) that are consistent with KB to be able to better describe the undefinable true target, the generated multiple types of learning targets in the formula (18) can also possess certain consistencies with our prior knowledge to better represent the undefinable true target.

5.1.4 Multi-target learning

The multi-target learning (MTL) component of OSAMTL is carried out on the basis of a specifically constructed machine learning (Carbonell et al. 1983; Ditterrich 1997; Jordan and Mitchell 2015) architecture (f) that can map entities/events (d) into corresponding predicted targets (t), which can be expressed as $t = f(d)$. Here, the MTL component of OSAMTL aims to optimize the parameters of f, regarding minimizing the error between the targets (t) predicted by f and the multiple types of targets (t^*) generated by the GMTLT component.

In order to estimate the error between t and t^*, a loss function (o) is commonly required. As t^* contains multiple types of targets, the error between t and the multiple types of targets in t^*can be estimated by the weighted sum of the errors between t and respective t_v^* in t^*, which can be expressed as

$$o(t, t^*) = \sum_{i=1}^{v} \alpha_i o(t, t_i^*) \quad s.t. \ \sum_{i=1}^{v} \alpha_i = 1. \tag{19}$$

Commonly, o in the formula (19) can be implemented by cross-entropy for classification and least squares for regression. Further, to produce the optimized machine learning model $\tilde{f}$, $o(t, t^*)$ should be minimized. Particularly, if f is constructed by state-of-the-art deep learning methods (LeCun et al. 2015) based on neural networks, the minimization of $o(t, t^*)$ can be implemented by stochastic gradient descent variants.

As the multiple types of learning targets (t^*) generated by the GMTLT component possess certain consistencies with our prior knowledge to better represent the undefinable true target, the produced machine learning model $\tilde{f}$ can have reasonable predictions (t)

about the undefinable true target by minimizing the error between t and t^*.

5.2 Extensions of OSAMTL

In section 5.1, we presented the formulas (12)-(19) to denote the original OSAMTL method. However, the original OSAMTL method will inevitably have limitations in handling some situations in real-world scenarios for UTTL, as the presented formulas only denote the basic components to concisely present the OSAMTL method. In this subsection, based on the original OSAMTL method presented in section 5.1, we discuss some extensions of OSAMTL to expand the usage range of OSAMTL in real-world scenarios for UTTL.

One extension of OSAMTL is that the data points provided for UTTL can be extended to diverse types instead of only a single type of data points. In contrast with the original OSAMTL, we denote this kind of extension as OSAMTL with diverse types of data points (DiTDP) (OSAMTL-DiTDP). Another extension of OSAMTL is that the label l_n corresponding to the entity/event d_n in the formula (12) can be extended to diverse types instead of only a single type of label. In contrast with the original OSAMTL, we denote this kind of extension as OSAMTL with diverse types of labels (DiTL) (OSAMTL-DiTL).

5.2.1 OSAMTL-DiTDP

For the situation of OSAMTL-DiTDP, referring to the formula (12), the provided DiTDP can be expressed as

$$H = \{H_1, \ldots, H_k\} = \{\{d_1, l_1\}, \ldots, \{d_k, l_k\}\}$$
$$= \left\{\left\{\{d_{1,1}, l_{1,1}\}, \ldots, \{d_{1,n_1}, l_{1,n_1}\}\right\}, \ldots, \left\{\{d_{k,1}, l_{k,1}\}, \ldots, \{d_{k,n_k}, l_{k,n_k}\}\right\}\right\}. \quad (20)$$

Here, k denotes the number of DiTDP and n_k denotes the number of data points for each type.

In fact, DiTDP can increase the diversity of the provided data points, which eventually leads to the labels in the provided data points representing diverse aspects of the undefinable true target. Comparing the formula (20) with formula (12), we can deduce that if the sum of the numbers for the multiple types of data points in formula (20) is equal to the number of data points in formula (12) (i.e., $\sum_{i=1}^{k} n_i = n$), the complexity of preparing DiTDP can maintain averagely unchanged as preparing a single type of data points. As a result, this extension of preparing DiTDP has the potential to significantly increase the diversity of the labels of the prepared data to represent the undefinable true target while maintaining the complexity averagely unchanged as preparing a single type of data points for OSAMTL.

In the meantime, this extension of OSAMTL is more complex to implement than the original OSAMTL, as the extension of preparing DiTDP increases the complexity in implementing the OSALR and GLTMT components of OSAMTL-DiTDP for particular applications. Specifically, for the OSALR component, the formulas (15), (16), and (17) need to be carried out multiple times regarding the prepared DiTDP to produce the final revised grounds to better describe the undefinable true target. For the GLTMT component, the formula (18) needs to be carried out by considering the possible associations among the prepared DiTDP and their corresponding revised groundings, which can make the implementation of the GLTMT component more complicated.

5.2.2 OSAMTL-DiTL

For the situation of OSAMTL-DiTL, DiTL can be expressed as $l_n = \{l_{n,1}, \ldots, l_{n,j}\}$, where j

denotes the number of the multiple types of labels included in l_n. Referring to the formula (12), the provided data points with DiTDP can be expressed as

$$H = \{d, l\} = \{\{d_1, l_1\}, \dots, \{d_n, l_n\}\}$$
$$= \left\{\left\{d_1, \{l_{1,1}, \dots, l_{1,j}\}\right\}, \dots, \left\{d_n, \{l_{n,1}, \dots, l_{n,j}\}\right\}\right\} \tag{21}$$

In fact, DiTL can significantly reduce the complexity of the original OSAMTL method, as multiple types of targets can be reasonably extracted from DiTL provided in the data points to represent the undefinable true target. As a result, this extension of OSAMTL can be less complex to implement than the original OSAMTL. In the meantime, although OSAMTL-DiTL requires diverse labels for the data points, it is practical in real-world scenarios. This is because the required diverse labels can be inaccurate, which can make the label preparation procedure much easier.

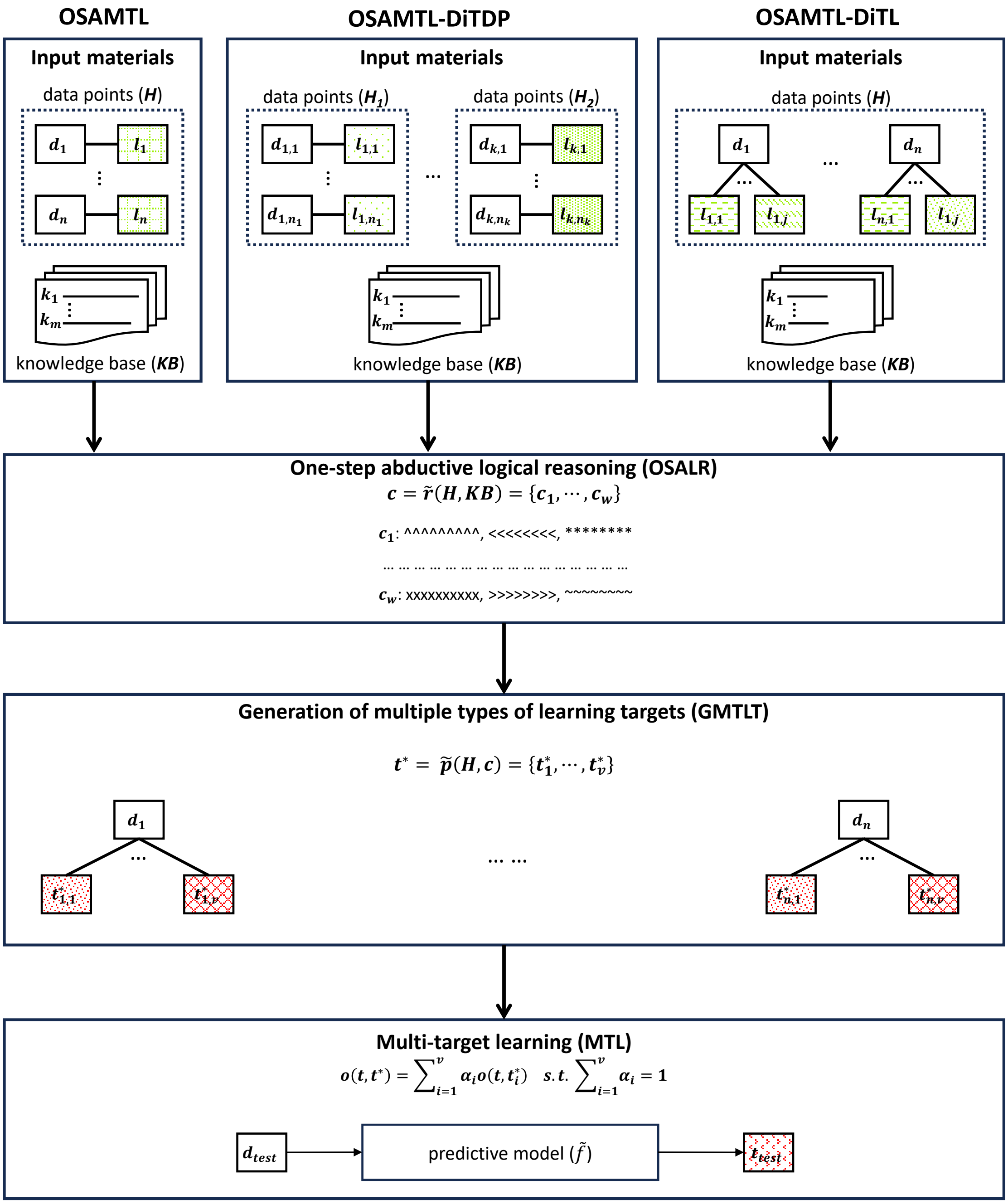


Figure. 1. Summarization of OSAMTL and its two extensions OSAMTL-DiTDP and OSAMTL-DiTL.

5.3 Summarization of OSAMTL, OSAMTL-DiTDP, and OSAMTL-DiTL

The summarization of OSAMTL and its two extensions, OSAMTL-DiTDP and OSAMTL-DiTL, can be shown as Fig. 1. The three methods of OSAMTL, OSAMTL-DiTDP, and OSAMTL-DiTL primarily differ in the preparations for the data points in the respective input materials. Because of the differences in the data points for the three methods, the complexities of implementing these three methods for UTTL tasks in real-world scenarios will also vary. Among the three methods, OSAMTL-DiTL theoretically is the easiest one to implement for real application, as the prepared data points already have similar structures to the results of the component GMTLT.

5.4 Essence of OSAMTL

The fundamental assumption for the proposal of OSAMTL is that the undefinable target can be realized as a set of multiple types of targets that possess certain consistencies with our prior knowledge about the undefinable target. Based on this fundamental assumption, the three key components of OSAMTL respectively make their contributions to realize this assumption.

Primarily, from the input materials of data points H and the knowledge base KB, the OSALR component of OSAMTL draws some revised groundings (c) that are consistent with KB to be able to better describe the undefinable true target than simply the groundings of the labels l in H. Subsequently, leveraging the provided data points H and the revised groundings c drawn by the OSALR component, the GMTLT component of OSAMTL abduces multiple types of learning targets containing information consistent with our prior knowledge KB about the undefinable true target. Finally, based on a specifically constructed machine learning architecture (f), the MTL component of OSAMTL produces the optimized machine learning model $\tilde{f}$ that can have reasonable predictions about the undefinable true target, via minimizing the error between the targets (t) predicted by f and the multiple types of targets (t^*) generated by the GMTLT component.

With these three key components of OSAMTL to realize the assumption that the undefinable target can be realized as a set of multiple types of targets that possess certain consistencies with our prior knowledge about the undefinable target, the essence of OSAMTL is that it forces the machine learning architecture to learn from the weighted summarization of multiple types of targets that possess certain consistencies with our prior knowledge about the undefinable true target. More specifically, this essence of OSAMTL reflects a result as follows.

Theorem 1. *For a classification or a regression task, the loss constructed by* $o(t, t^*) = \sum_{i=1}^{v} \alpha_i o(t, t_i^*)$, *can be theoretically expressed as* $o(t, t^*) = o(t, \sum_{i=1}^{v} \alpha_i t_i^*) + c$, *where* c *is a constant term.*

Detailed proofs for Theorem 1 are provided in Proof 2 and 3 of the Appendix. Through Theorem 1, we can declare that OSAMTL is able to reasonably force the learning model to achieve logically rational predictions about the undefinable target via learning from the weighted summarization of multiple types of targets. In fact, learning from the weighted summarization of multiple types of targets, which possess certain consistencies with our prior knowledge about the undefinable true target, can lead to a trade-off among the multiple types of targets and thus a reasonable approximation of the undefinable true target.

6. Particular application

The proposed specific method OSAMTL and its extensions for UTTL have been successfully applied to address some tasks in medical histopathology whole slide image analysis (MHWSIA). In this section, we discuss the implementation rules and techniques of these specific methods in some tasks in MHWSIA.

6.1 Application of OSAMTL

OSAMTL has been applied to the helicobacter pylori segmentation task. Precisely segmenting the helicobacter pylori areas in whole slide images digitalized from IHC slides is an unsolved task, as presenting high-quality labels to precisely annotate the helicobacter pylori areas in the whole slide images is very difficult even for pathology experts (Yang et al. 2020; Yang, Yang, et al. 2024). Taking the underlying true target of helicobacter pylori as the undefinable true target, the helicobacter pylori segmentation task can be transformed into a UTTL problem, and the OSAMTL method can just be applied to provide an alternative solution. In the following contents of this subsection, we briefly introduce the key information about the input materials required by OSAMTL and the results of the three components of OSAMTL, to illustrate the application of OSAMTL to the helicobacter pylori segmentation task.

6.1.1 Input materials

Referring to the formulas (12) and (13), the input materials for the application of OSAMTL to the helicobacter pylori segmentation task include a number of collected data points that consist of entities and their corresponding labels, and an accumulated knowledge base that contains factual descriptions about the undefinable true target for the task. For the helicobacter pylori segmentation task, the entities of the collected data points are a number of image patches cropped from whole slide images digitalized from IHC slides, and the corresponding labels are same-sized frames that contain polygons annotating the helicobacter pylori areas in the image patches. A mimic example of the collected data points and correspondingly related contents for illustration is provided in Fig. 2, and the accumulated knowledge base is shown as Table. 2.

From Fig. 2, we can note that the label l associated with the image patch d is quite inaccurate to represent the underlying true target t, as the image 'l shown on d' shows that the provided label l for the image patch d includes many background areas as the target, though it probably enclosed the entire underlying true target t. As the pieces of knowledge listed in Table 1 are provided by related experts for identifying the underlying true target of helicobacter pylori, the provided pieces of knowledge can to some extent describe the key features of the underlying true target, though they are semantic and unquantifiable.

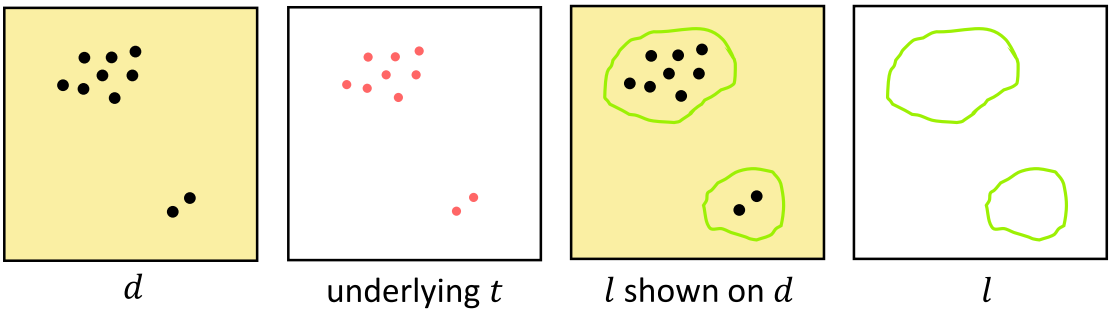


Figure. 2. A mimic example of the collected data points and correspondingly related contents for illustration. The first and the last images (d and l) constitute the example of the collected data points, which are an image patch cropped from a whole slide image digitalized from an IHC slide and its corresponding label that annotates the helicobacter pylori areas in the image patch. The second image (underlying t) is assumed to illustrate the underlying true target corresponding to the image patch d. The third image (l shown on d) illustrates the helicobacter pylori areas annotated in the image patch.

Tabel 2. Details of the accumulated knowledge base (Yang, Yang, et al. 2024)

Accumulated Knowledge Base
k_1: Helicobacter pylori distributes in luminal areas
k_2: Helicobacter pylori are black dot-like regions
k_3: An obvious gradient exists between the location of helicobacter pylori and its neighbourhood

6.1.2 Results of OSALR

Based on the input materials, the OSALR component of OSAMTL was particularly implemented for the helicobacter pylori segmentation task via a series of logical reasoning processes (Yang, Yang, et al. 2024). The particularly implemented OSALR component of OSAMTL finally resulted in a number of revised groundings that more accurately describe the undefinable true target than the labels provided in the collected data points of the input materials. Details of the revised groundings are shown in Table 3.

Table 3. Details of the revised groundings (Yang, Yang, et al. 2024)

Revised Groundings
c_1: Pixels of images outside the polygons of labels are helicobacter pylori negatives
c_2: Pixels of images inside the polygons of labels are helicobacter pylori positives
c_3: Black dot-like pixels of images inside the polygons of labels which distribute in luminal areas and have an obvious gradient with their neighbourhood are true helicobacter pylori positives with high probability

6.1.3 Results of GMTLT

Based on the revised groundings produced by the OSALR component of OSAMT, the

GMTLT component of OSAMTL was particularly implemented for the helicobacter pylori segmentation task via a series of image processing algorithms and procedures (Yang, Yang, et al. 2024). The particularly implemented GMTLT component of OSAMTL finally resulted in two types of inaccurate targets to represent the underlying true target associated with the helicobacter pylori segmentation task. Mimic examples of the produced two types of inaccurate targets and their masks shown on the corresponding image patch are shown as Fig. 3.

From Fig. 3, we can observe that the target type t_1^* can probably enclose the entire underlying true target while including many backgrounds as the target, just exactly like the labels provided in the input materials for the task. In addition, the target type t_2^* can probably be accurate in representing the underlying true target while excluding some parts of the underlying true target as the background. In summary, the two types of targets are both inaccurate but complementary to each other. Thus, the union of the two types of inaccurate targets is reasonable to represent the underlying true target.

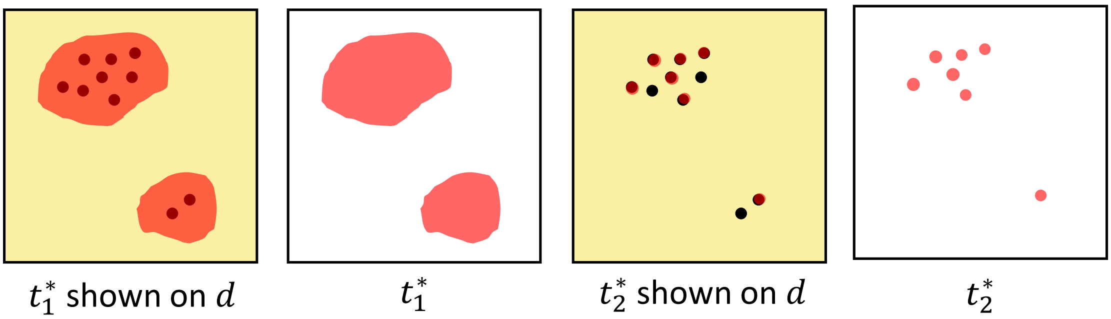


Figure 3. Mimic examples of the produced two types of inaccurate targets and their masks shown on the corresponding image patch. The second and fourth images are examples of the produced two types of inaccurate targets t_1^* and t_2^*. The first and the third images are the masks of t_1^* and t_2^* shown on the corresponding image patch.

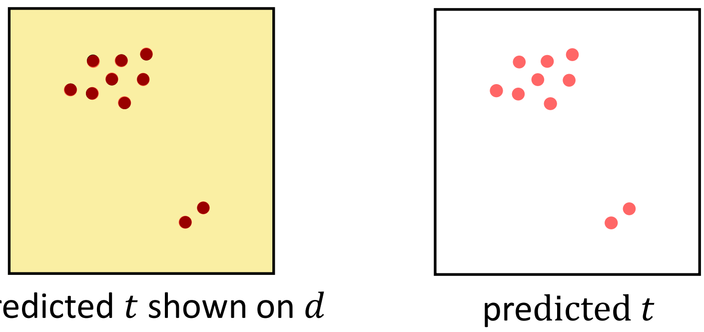


Figure. 4. Mimic example of the predicted target and its mask shown on the corresponding image patch.

6.1.4 Results of MTL

Based on the two types of inaccurate targets generated by the component GLTMT of OSAMTL and their corresponding image patches, the MTL component of OSAMTL was particularly implemented for the helicobacter pylori segmentation task via minimizing the summary error between the two types of inaccurate targets and the predictions of the image patches corresponding to the two types of inaccurate targets from a small deep convolutional neural network (Yang, Yang, et al. 2024). The particularly implemented MTL component of OSAMTL finally produced a predictive model that can map an image patch into the predicted target, which can more reasonably represent the underlying true target than the two types of inaccurate targets. A mimic example of the predicted target and its mask shown on the corresponding image patch is shown as Fig. 4.

6.1.5 Summarization

Regarding the helicobacter pylori segmentation task as a UTTL problem, the application of OSAMTL to this task can be summarized as follows.

1) One type of data point is prepared, in which one type of labels for annotating the underlying true target of helicobacter pylori areas is associated with corresponding image patches. Pieces of knowledge from related experts for identifying the underlying true target of helicobacter pylori are collected. The one type of labels in the prepared data points is quite inaccurate to represent the true helicobacter pylori areas in the corresponding image patches. The collected pieces of knowledge can to some extent precisely describe the key features of the underlying true target of helicobacter pylori, though they are semantic and unquantifiable. Particularly, the labels in the prepared one type of data points include many background areas as the helicobacter pylori areas in the corresponding image patches.
2) Based on the input materials, the OSALR component of OSAMTL, particularly implemented via a series of logical reasoning processes, finally resulted in a number of revised groundings that more accurately describe the undefinable true target than the labels provided in the collected data points of the input materials.
3) Based on the revised groundings, the GMTLT component of OSAMTL, particularly implemented via a series of image processing algorithms and procedures, finally resulted in two types of inaccurate targets to represent the underlying true target associated with the image patches in the collected data points for the helicobacter pylori segmentation task. The two types of targets are both inaccurate but complementary to each other.
4) Based on the two types of inaccurate targets and their corresponding image patches, the MTL component of OSAMTL, particularly implemented via minimizing the summary error between the two types of inaccurate targets and the predictions of the image patches corresponding to the two types of inaccurate targets from a small deep convolutional neural network, finally produced a predictive model that can map an image patch into the predicted target. The predicted target can more reasonably represent the underlying true target than the two types of inaccurate targets for the helicobacter pylori segmentation task.

More details of the application to the helicobacter pylori segmentation task in MHWSIA can be found in (Yang et al. 2020; Yang, Yang, et al. 2024).

6.2 Application of OSAMTL-DiTDP

OSAMTL-DiTDP has been applied to the tumour segmentation task for breast cancer. Precisely segmenting the tumour areas for breast cancer in whole slide images digitalized from IHC slides is also an unsolved task, since presenting high-quality labels to precisely annotate the tumour areas for breast cancer in the whole slide images is very difficult even for pathology experts (Yang, Li, et al. 2024). Identically, taking the underlying true target of tumour for breast cancer as the undefinable true target, the tumour segmentation task for breast cancer can also be transformed into a UTTL problem, and the OSAMTL-DiTDP method can just be applied to provide an alternative solution. In the following contents of this subsection, we briefly introduce the key information about the input materials required by OSAMTL-DiTDP and the results of the three components of OSAMTL-DiTDP, to illustrate the application of OSAMTL-DiTDP to the tumour segmentation task for breast cancer. Particularly, for simplicity, the illustration is based on the task of tumour segmentation in HE-stained pre-treatment biopsy images (Yang, Li, et al. 2024).

6.2.1 Input materials

Referring to the formulas (20) and (13), the input materials for the application of OSAMTL-DiTDP to the tumour segmentation task for breast cancer include a number of collected two types of data points that respectively consist of entities and their corresponding labels, and an accumulated knowledge base that contains factual descriptions about the undefinable true target for the task. For the tumour segmentation task for breast cancer, the entities for each type of the collected data points are a number of image patches cropped from whole slide images digitalized from IHC slides, and the corresponding labels are same-sized frames that contain polygons annotating the tumour areas for breast cancer in the image patches. Two mimic examples respectively for the collected two types of data points and correspondingly related contents for illustration are provided in Fig 5, and the accumulated knowledge base is shown as Table 4.

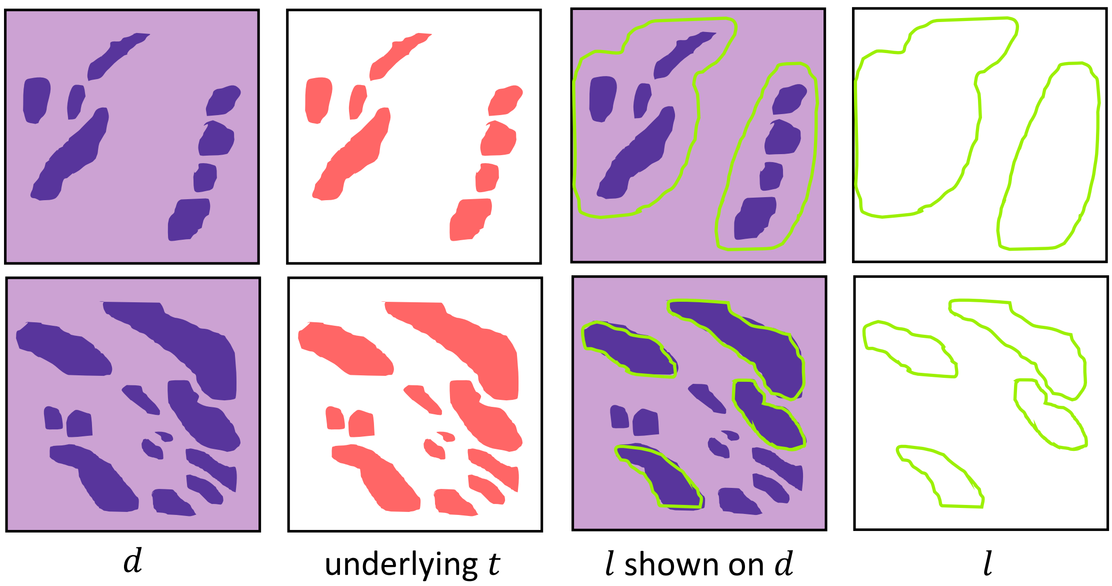


Figure. 5. Two mimic examples respectively for the two types of collected data points and their correspondingly related contents for illustration. The top row is for the type one of the collected data points, and the bottom row is for the type two of the collected data points.

From Fig. 4, we can note that, for each type of the collected data points, the label l associated with the image patch d is quite inaccurate to represent the underlying true target

t. The image 'l shown on d' for type one of the collected data points shows that the provided label l for the image patch d included many background areas as the target, though it probably enclosed the entire underlying true target t. On the contrary, the image 'l shown on d' for type two of the collected data points shows that the provided label l for the image patch d excluded some target areas as the background, though it probably eliminated the entire background. The labels respectively prepared for the two types of collected data points are complementary to each other in representing the underlying true target. Identically, as the pieces of knowledge listed in Table 3 are also provided by related experts for identifying the underlying true target of tumour for breast cancer, the provided pieces of knowledge can to some extent describe the key features of the underlying true target, though they are semantic and unquantifiable.

Tabel 4. Details of the accumulated knowledge base (Yang, Li, et al. 2024)

Accumulated Knowledge Base
k_1: Tumour is composed of tumour cells.
k_2: Tumour cells may be arranged in cords, clusters, and trabeculae.
k_3: Some tumours are characterized by a predominantly solid or syncytial infiltrative pattern with little associated stroma.
k_4: The cytoplasm of a tumour cell is eosinophilic and vacuolated.
k_5: The nuclei of tumour cells are enlarged, and the chromatin of tumour cells is vacuolated.
k_6: The nuclei of tumour cells are degenerated.

6.2.2 Results of OSALR

Based on the input materials, the OSALR component of OSAMTL-DiTDP was particularly implemented for the tumour segmentation task for breast cancer via a series of logical reasoning processes (Yang, Li, et al. 2024). The particularly implemented OSALR component of OSAMTL-DiTDP finally resulted in a number of revised groundings that more accurately describe the undefinable true target than the labels provided in the two types of collected data points of the input materials. Details of the revised groundings are shown in Table 5.

Table 5. Details of the revised groundings (Yang, Li, et al. 2024)

Revised Groundings
c_1: pixels of type-one images outside the polygons of type-one labels are tumour negatives
c_2: pixels of type-one images inside the polygons of type-one labels are tumour positives
c_3: pixels of type-one images outside the polygons of type-one labels are not exactly true tumour negatives
c_4: pixels of type-one images inside the polygons of type-one labels are not exactly true tumour positives
c_5: pixels of type-two images inside the polygons of type-two labels are tumour positives
c_6: pixels of type-two images outside the polygons of type-two labels are tumour negatives
RG_7: pixels of type-two images inside the polygons of type-two labels are not exactly true tumour positives
RG_8: pixels of type-two images outside the polygons of type-two labels are not exactly true tumour negatives

6.2.3 Results of GMTLT

Based on the revised groundings produced by the OSALR component of OSAMT-DiTDP, the GMTLT component of OSAMTL-DiTDP was particularly implemented for the tumour segmentation task for breast cancer via a series of logical reasoning and machine learning procedures (Yang, Li, et al. 2024). The particularly implemented GMTLT component of OSAMTL-DiTDP finally resulted in two types of inaccurate targets to represent the underlying true target associated with the tumour segmentation task for breast cancer. Mimic examples of the produced two types of inaccurate targets and their masks shown on the corresponding image patches are shown in Fig. 6.

From Fig. 6, we can observe that the target type t_1^* can probably enclose the entire underlying true target while including many background pixels as the target, just like the type-one labels provided in the input materials for the task. In addition, the target type t_2^* can probably be accurate to represent the underlying true target while excluding some parts of the underlying true target as the background, just like the type-two labels provided in the input materials for the task. In summary, the two types of targets are both inaccurate but complementary to each other. Thus, the union of the two types of inaccurate targets is reasonable to represent the underlying true target.

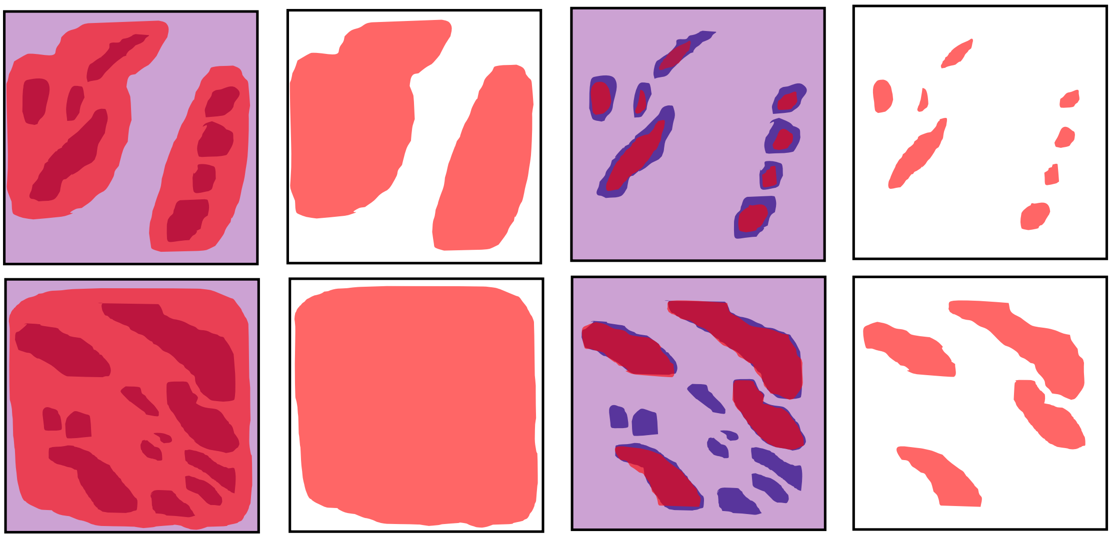


Figure. 6. Mimic examples of the produced two types of inaccurate targets and their masks shown on the corresponding image patches. The second and fourth column images are examples of the produced two types of inaccurate targets t_1^* and t_2^*. The first and the third column images are the masks of t_1^* and t_2^* shown on the corresponding image patches.

6.2.4 Results of MTL

Based on the two types of inaccurate targets generated by the component GLTMT of OSAMTL-DiTDP and their corresponding image patches, the MTL component of OSAMTL-DiTDP was particularly implemented for the tumour segmentation task for breast cancer via minimizing the summary error between the two types of inaccurate targets and the predictions of the image patches corresponding to the two types of inaccurate targets from a small deep convolutional neural network (Yang, Li, et al. 2024). The particularly implemented MTL component of OSAMTL-DiTDP finally produced a predictive model that can map an image patch into the predicted target, which can more reasonably represent the underlying true target than the two types of inaccurate targets. Mimic examples of the predicted targets and their masks shown on the corresponding image patches are shown as Fig. 7.

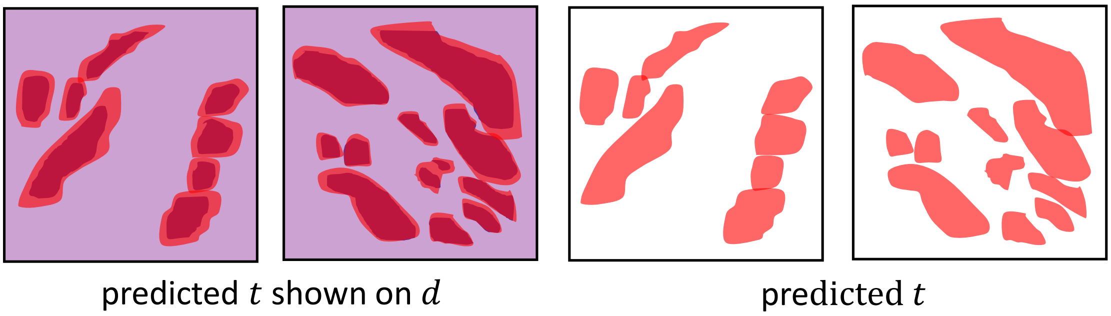


Figure. 7. Mimic examples of the predicted targets and their masks shown on the corresponding image patches.

6.2.5 Summarization

Regarding the tumour segmentation task for breast cancer as a UTTL problem, the application of OSAMTL-DiTDP to this task can be summarized as follows.

1) Two types of data points are prepared, respectively, in which one type of labels for annotating the underlying true target of tumour areas for breast cancer is associated with corresponding image patches. And pieces of knowledge from related experts for identifying the underlying true target of tumours for breast cancer are collected. Each one type of labels in the prepared two types of data points is quite inaccurate to represent the true tumour areas for breast cancer in the image patches. And the collected pieces of knowledge can to some extent precisely describe the key features of the underlying true target of tumours for breast cancer, though they are semantic and unquantifiable. Particularly, the labels in the prepared type-one data points include many background areas as the tumour areas for breast cancer, and the labels in the prepared type-two data points exclude some tumour areas as background areas in corresponding image patches.
2) Based on the input materials, the OSALR component of OSAMTL-DiTDP particularly implemented via a series of logical reasoning processes finally resulted in a number of revised groundings that more accurately describe the undefinable true target than the labels provided in the two types of collected data points of the input materials.
3) Based on the revised groundings, the GMTLT component of OSAMTL-DiTDP particularly implemented via a series of logical reasoning and machine learning procedures finally resulted in two types of inaccurate targets to represent the underlying true target associated with the image patches in the collected data points for the tumour segmentation task for breast cancer. The two types of targets are both inaccurate but complementary to each other.
4) Based on the two types of inaccurate targets and their corresponding image patches, the MTL component of OSAMTL-DiTDP particularly implemented via minimizing the

summary error between the two types of inaccurate targets and the predictions of the image patches corresponding to the two types of inaccurate targets from a small deep convolutional neural network finally produced a predictive model that can map an image patch into the predicted target. The predicted target can more reasonably represent the underlying true target than the two types of inaccurate targets for the tumour segmentation task for breast cancer.

More details of the application to the tumour segmentation task for breast cancer in MHWSIA can be found in (Yang, Li, et al. 2024).

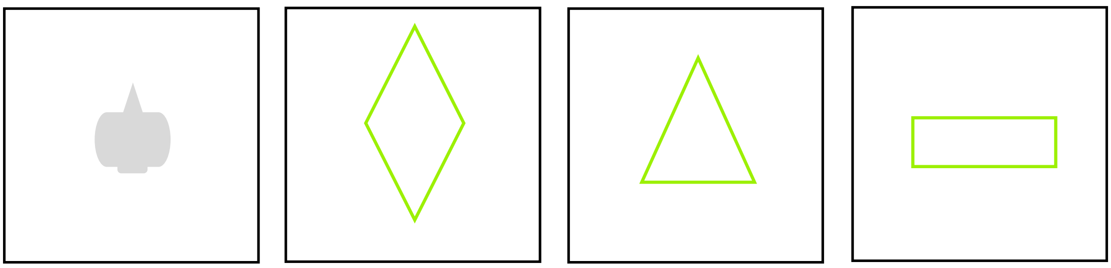
6.3 Application of OSAMTL-DiTL

To the best of our knowledge, there is no specific work that explored OSAMTL-DiTL in a real-world application for the UTLL problem. In this subsection, we focus more on discussing the similarities and differences between the application of OSAMTL-DiTL and the applications of OSAMTL and OSAMTL-DiTDP to reveal the potential of the application of OSAMTL-DiTL (Yang 2021).

6.3.1 Input materials

Identical to the former two applications of OSAMTL and OSAMTL-DiTDP to the two image segmentation tasks in MHWSIA, the input materials (referring to the formulas (20) and (13)) the input materials of the application of OSAMTL-DiTL to a real-world task also include a number of collected data points that respectively consist of entities and their corresponding labels, and an accumulated knowledge base that contains factual descriptions about the undefinable true target for the task.

The accumulated knowledge base is similar to the knowledge bases for the former two applications of OSAMTL and OSAMTL-DiTDP, which are shown as Table. 1 and Table. 3. But, different from the former two applications of OSAMTL and OSAMTL-DiTDP, in which each entity in the collected data points only has one inaccurate label, each entity in the collected data points for OSAMTL-DiTL has multiple (more than one) inaccurate labels that can describe partial properties of the underlying true target. A mimic example of three inaccurate labels assigned to the same entity for the collected data points is shown as Fig. 8.



d l_1 l_2 l_3

Figure. 8. A mimic example of three inaccurate labels assigned to the same entity for the collected data points. The first column d is the entity, and the rest three columns l_1, l_2 and l_3 are the inaccurate labels assigned to d.

6.3.2 Results of OSALR

Identical to the former two applications of OSAMTL and OSAMTL-DiTDP to the two image segmentation tasks in MHWSIA, the OSALR component of OSAMTL-DiTL can be particularly implemented for a specific task via a series of logical reasoning processes and some other possible procedures on the basis of the input materials. The particularly implemented OSALR component of OSAMTL-DiTL finally resulted in a number of revised groundings that can more accurately describe the undefinable true target than the multiple types of labels provided in the collected data points of the input materials.

Via some basic logical reasoning processes based on the current mimic input materials, the revised groundings can possibly contain contents like Table 6, in addition to the groundings contained in the three inaccurate labels.

Table 6. Possible revised groundings

Revised Groundings
$c_1, \ldots, c_3$ c_4: the union of the three inaccurate labels (l_1, l_2 and l_3) can probably contain the entire underlying true target while including some background areas as the target c_5: the intersection of the three inaccurate labels (l_1, l_2 and l_3) can probably be accurate to represent the underlying true target while excluding some parts of the target as the back-ground

The contents in Table 6 can reflect that the implementation of the OSALR component of OSAMTL-DiTL can be much easier than the implementations of the OSALR components of OSAMTL and OSAMTL-DiTDP.

6.3.3 Results of GMTLT

Based on the revised groundings produced by the OSALR component of OSAMTL-DiTL, the GMTLT component of OSAMTL-DiTL can be particularly implemented for a specific task via some specifically designed procedures. The particularly implemented GMTLT component of OSAMTL-DiTL will finally result in multiple types of inaccurate targets to represent the underlying true target associated with a specific task. Two series of possible mimic examples for the produced multiple inaccurate targets can be shown as Fig. 9.

Three types of inaccurate targets are presented in the top row series, and two types of inaccurate targets are presented in the bottom row series. In summary, the two possible series of inaccurate target types are inaccurate but complementary to each other. Thus, the union of the multiple types of inaccurate targets in the respective series can also be reasonable to represent the underlying true target.

From the top row series of multiple inaccurate targets, we can note that they are just exactly like the inaccurate labels provided in the collected data points for the input materials. And, from the bottom row series of multiple inaccurate targets, we can note that they are some results of logical processes based on the top row series of multiple inaccurate targets. These facts can reflect that the implementation of the GMTLT component of OSAMTL-DiTL can be much easier than the implementations of the GMTLT components of OSAMTL and OSAMTL-DiTDP.

6.3.4 Results of MTL

Based on one series of the multiple types of inaccurate targets generated by the component GLTMT of OSAMTL-DiTDP and their corresponding entities, similar to the former two applications, the MTL component of OSAMTL-DiTL can be particularly implemented for a specific task via minimizing the summary error between the multiple types of inaccurate targets and the predictions of the entities corresponding to the multiple types of inaccurate targets from a machine learning model. The particularly implemented MTL component of OSAMTL-DiTL can finally produce a predictive model that can map an image patch into the predicted target, which can more reasonably represent the underlying true target than the multiple types of inaccurate targets. A mimic example of the predicted target can be shown as Fig. 10.

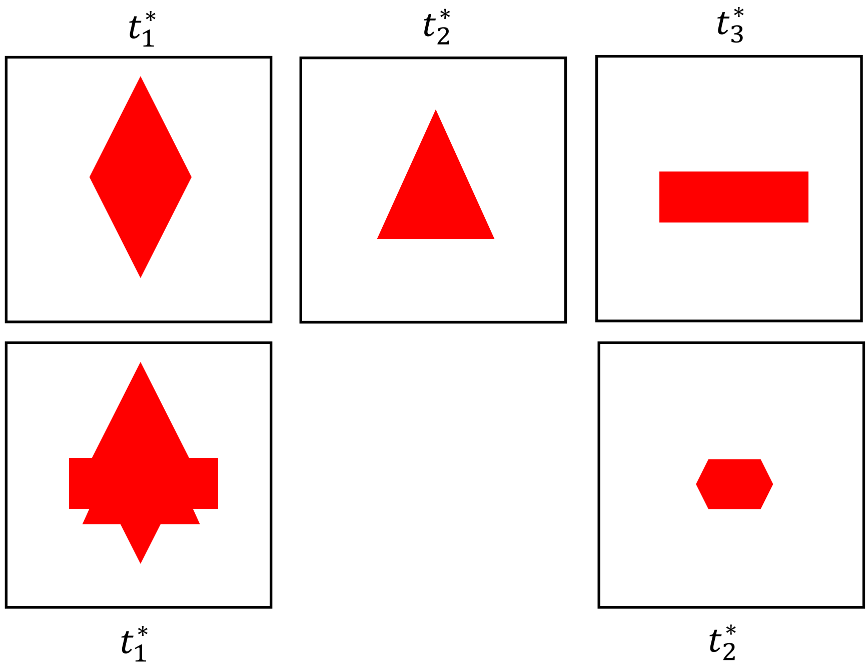


Figure. 9. Two series of possible mimic examples for the produced multiple types of inaccurate targets. The top row series contain three types of inaccurate targets and the bottom row series contain two types of inaccurate targets.

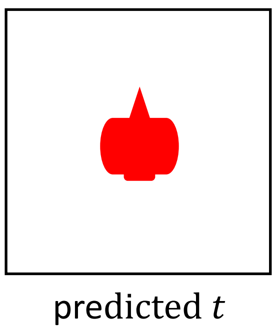


Figure 10. A mimic example of the predicted target.

6.3.5 Summarization

The possible application of OSAMTL-DiTL for a UTTL problem can be summarized as follows.

1) One type of data point is prepared, in which multiple types of labels for annotating the

underlying true target are associated with the corresponding entities. Pieces of knowledge from related experts for identifying the underlying true target are collected. Each one type of the multiple types of labels in the prepared data points can be inaccurate in representing the true target associated with the corresponding entities. The collected pieces of knowledge can to some extent precisely describe the key features of the underlying true target, though they can be semantic and unquantifiable.

2) Based on the input materials, the OSALR component of OSAMTL-DiTL, particularly implemented via a series of logical reasoning processes, can finally result in a number of revised groundings that more accurately describe the undefinable true target.
3) Based on the revised groundings, the GMTLT component of OSAMTL-DiTL, particularly implemented via some specifically designed procedures, can finally result in two or more types of inaccurate targets to represent the underlying true target associated with the entities in the collected data points for a UTTL problem. The two or more types of targets are inaccurate but can be complementary to each other.
4) Based on the two or more types of inaccurate targets and their corresponding entities, the MTL component of OSAMTL-DiTL can be particularly implemented via minimizing the summary error between the two or more types of inaccurate targets and the predictions of the entities corresponding to the two or more types of inaccurate targets from a learning algorithm, which finally produces a predictive model that can map an entity into the predicted target. The predicted target can more reasonably represent the underlying true target than the two or more types of inaccurate targets for a UTTL problem.

7. Discussion, conclusion, and future work

In this article, we explicitly propose the fundamental assumption that the true target for a learning task does not exist in the real world to formally present the concept of UTTL for the common situation where the learning true target cannot be precisely defined in various AI application scenarios.

Primarily, based on the fundamental assumption that the true target for the UTTL problem does not exist in the real world, the definition for the UTTL problem is formally presented. Subsequently, on the basis of the presented definition, the UTTL problem is transformed into mainly a combination of the ML problem and the logical reasoning problem, and an alternative solution to the transformed UTTL problem is presented. In addition, referring to the presented alternative solution, specific methods like one-step abductive multi-target learning (OSAMTL) and its extensions (OSAMTL-DiTDP and OSAMTL-DiTL) are summarized for addressing the UTTL problem in different scenarios. Finally, referring to the summarized OSAMTL and its extensions (OSAMTL-DiTDP and OSAMTL-DiTL), implementation rules and techniques of these methods are discussed regarding particular real-world application scenarios. The discussions include applying OSMTL to precisely segmenting the helicobacter pylori areas in whole slide images (Yang et al. 2020; Yang, Yang, et al. 2024) and applying OSAMTL-DiTDP to tumour segmentation in HE-stained pre-treatment biopsy images (Yang, Li, et al. 2024), and discussing the similarities and differences between the application of OSAMTL-DiTL and the applications of OSAMTL and OSAMTL-DiTDP to reveal the potentials of the application of OSAMTL-DiTL (Yang 2021).

In conclusion, from the perspectives of problem definition, alternative solution, specific method, and particular application, this article has established the first fundamental basis for systematically analysing the UTTL situation, where the learning true target cannot be precisely defined, in various AI application scenarios.

As we have analysed in Section 4.5, the optimal solution to the UTTL problem should not be limited to the alternative solution presented in the article, since it is based on the transformed UTTL problem, which is mainly a combination of the ML problem and the LR problem. It is probable that better problem transformations and corresponding solutions for the UTTL problem defined in formula (1) can still be proposed, regarding other original thoughts and perspectives. In addition, with the fundamental assumption that the true target for a learning task does not exist in the real world, the concept of UTTL can also be applied in various other AI application scenarios to establish different perspectives for addressing related tasks. These works need to / can be done in the future. We hope that accomplishing these future works can attract more researchers to establish a community for studying UTTL.

Declaration

The author, Yongquan Yang, established the conceptualization and did the initial work for this article while he was taking a vacation (from 2023.09 to 2024.01) in Chengdu, Sichuan, China. The rest of the work for this article was done in his spare time when he was a contract employee (starting from 2024.02) at Zhongjiu Flash Medical Technology Co., Ltd. in Mianyang, Sichuan, China.

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Appendix

Proof 1. From the formulas (6), (7), and (8), we have the following subject conditions:

$$prop(l) \subseteq c \cong KB, \tag{1}$$

$$prop(t^{*}) = c, \tag{2}$$

$$prop(t) = prop(t^{*}). \tag{3}$$

Referring to the subject conditions (2) and (3), we have

$$c = prop(t). \tag{4}$$

Substituting the subject condition (4) into (1), we have the final subject condition

$$prop(l) \subseteq prop(t) \cong KB. \tag{5}$$

Proof 2. When we use average cross entropy (ACE) to estimate the error between two elements for a two-class classification task, the basic loss function $o(\cdot,\cdot)$ can be denoted by

$$o(t, t_0^*) = -\left[t_0^{*,f} \, log(t) + \left(1 - t_0^{*,f}\right) log(1 - t)\right]$$
$$s.t. \; t_0^{*,f} \cup \left(1 - t_0^{*,f}\right) = t_0^*. \tag{1}$$

Here, $t_0^{*,f}$ is the foreground class of the target $t^*{}_0$, and $1 - t_0^{*,f}$ is the background class of the target t_0^*. Referring to formula (1), we rewrite $o(t, t^*) = \sum_{i=1}^{v} \alpha_i o(t, t_i^*)$ by

$$o(t, t^*) = \sum_{i=1}^{v} \alpha_i \left\{-\left[t_i^{*,f} \, log(t) + \left(1 - t_i^{*,f}\right) log(1 - t)\right]\right\}$$
$$= -\left[\sum_{i=1}^{v} \alpha_i \, t_i^{*,f} \, log(t) + \sum_{i=1}^{v} \alpha_i \left(1 - t_i^{*,f}\right) log(1 - t)\right]. \tag{2}$$

Plugging $t^*{}_0 = \sum_{i=1}^{v} \alpha_i t_i^*$ and substituting into formula (1), we have

$$o(t, \sum_{i=1}^{v} \alpha_i t_i^*) = -\left[\sum_{i=1}^{v} \alpha_i \, t_i^{*,f} \, log(t) + \sum_{i=1}^{v} \alpha_i \left(1 - t_i^{*,f}\right) log(1 - t)\right]. \tag{3}$$

Comparing formula (3) with formula (2), theoretically we can have

$$\mathrm{o}(t, t^*) = o(t, \sum_{i=1}^{v} \alpha_i t_i^*). \tag{4}$$

Proof 3. When we use the mean squared error (MSE) to estimate the error between two elements for a regression task, the basic loss function $o(\cdot,\cdot)$ can be denoted by

$$o(t, t_0^*) = (t - t_0^*)^2. \tag{1}$$

Referring to formula (1), we rewrite $o(t, t^*) = \sum_{i=1}^{v} \alpha_i o(t, t_i^*)$ by

$$o(t, t^*) = \sum_{i=1}^{v} \alpha_i \, (t - t_i^*)^2$$
$$= (t - \sum_{i=1}^{v} \alpha_i t_i^*)^2 + \sum_{i=1}^{v} \alpha_i {t_i^*}^2 - (\sum_{i=1}^{v} \alpha_i t_i^*)^2$$
$$= (t - \sum_{i=1}^{v} \alpha_i t_i^*)^2 + D(t^*). \tag{2}$$

Here, $D(t^*) = \sum_{i=1}^{v} \alpha_i {t_i^*}^2 - (\sum_{i=1}^{v} \alpha_i t_i^*)^2$ is the variance for the multiple targets of t^* and is a constant. Plugging $t^*{}_0 = \sum_{i=1}^{v} \alpha_i t_i^*$ and substituting into the formula (1), we have

$$o(t, \sum_{i=1}^{v} \alpha_i t_i^*) = (t - \sum_{i=1}^{v} \alpha_i t_i^*)^2. \tag{3}$$

Comparing formula (3) with formula (2), theoretically we can have

$$o(t, t^*) = o(t, \sum_{i=1}^{v} \alpha_i t_i^*) + \; D(t^*). \tag{4}$$