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What connects entangled photons?

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Abstract

Entangled quantum systems can connect to the environment via a Bell state measurement. This applies, for example, to teleportation and entanglement swapping. Although the results are well understood, it is not entirely clear whether they involve nonlocal action or whether they are predetermined. This can best be decided from a model, provided it predicts the key measurement results. Models based on the fact that the partners of an entangled pair have the same value of a statistical parameter cannot be applied here. This is because the partner particles of the resulting entangled states after a teleportation or an entanglement swapping never had contact before. The question then is, what connects entangled photons? Therefore, this paper presents a local realistic model that reproduces the quantum mechanical predictions for expectation values with polarization measurements, but is not based on shared statistical parameters. Instead, the coupling of the entangled particles is based on initial conditions and conservation of spin angular momentum. The model refutes Bell's theorem and also explains teleportation and entanglement swapping in a local way. The manuscript is thus a step forward toward a complete theory describing quantum physical reality as thought possible by Einstein, Podolsky, and Rosen.

Keywords: Bell’s theorem, entanglement swapping, teleportation, local hidden variables, EPR

1. Introduction

Entanglement swapping can entangle particles that were not previously in contact. This entanglement can be achieved by Bell state measurements [1]. Many physicists are convinced that this process is non-local. This belief is ultimately based on the assumption of the validity of Bell's theorem [2]. It states that quantum mechanics cannot be local because it cannot be described by local realistic models with hidden variables. A detailed description of the literature and arguments regarding Bell’s theorem can be found in [3,4] and in the references therein. Bell's theorem was refuted by a local contextual model with hidden variables [5] which correctly predicts quantum mechanical expectation values with polarization-entangled particles. This model is based on the fact that both members of an entangled pair are connected by a shared hidden parameter.

However, the assumption of a common value of a hidden parameter for the members of an entangled pair, as also proposed by Bell [6], cannot explain phenomena such as entanglement swapping and teleportation [7-10]. When photons that did not interact before becoming entangled by entanglement swapping they cannot have a predefined shared parameter with a statistical distribution.

The fact that Bell's theorem was formally refuted does not mean that correlations at entangled photons can definitely be explained locally. It just means that Bell's statement that a local explanation is impossible is not valid. Although Bell's theorem was formally refuted with the locally realistic model [5] the question of whether Bell states can have a local explanation is thus not yet decided. That would only be the case if there were a model without common hidden parameters.

Therefore, we introduce a local realistic model in which the indistinguishability of the entangled photons explains the physical states, as in [5], but in which the photon pairs do not share the value of a statistical parameter. The question then arises as to how the photons on side B can have information regarding the position of the polarizer on side A without communication. This is answered by the model assumptions in the following section.

For each Bell state *Ψ+*, Φ-, *Φ+ and Φ- (*see equations (11-14) for definition) we show what a selection of photons by a polarizer on one side means for the state on the other side. The results are given in Table 1. From these, expectation values for correlation measurements on entangled photons and the relations for entanglement swapping and teleportation are derived.

2. A new model for polarization-entangled photons with local hidden variables

2.1 Model overview

In polarization measurements, photons can choose one of two perpendicular exits of the polarizer. A model with hidden variable must describe which of these two possible exits a photon will take. Four model assumptions are introduced, which are outlined and then described in italics:

MA1 introduces the statistical parameter λ which controls the polarizer exit that a photon will take. This model assumption is the same as MA1 in [5]. It corresponds to Malus' law.

MA2 describes the common polarization of a selection of photons from an entangled pair. This is a new model assumption.

MA3 MA3 derives the coupling of the entangled photons from the initial states on the basis of spin angular momentum conservation. This is a new model assumption.

MA4 states that photons carry the complete set of the hidden variable after a measurement. This model assumption is the same as MA4 in [5].

Figure 1 shows the coordinate systems and nomenclature of the experiments with polarization-entangled photons.

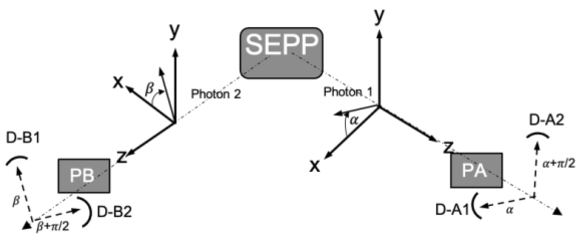

Figure 1: The SEPP (source of entangled photon pairs) emits entangled photons propagating towards the adjustable polarizers PA and PB and detectors DA-1 and DA-2 on wing A and DB-1 and DB-2 on wing B. A coincidence measuring device (not seen in the picture) encounters matching events. The polarization angles are defined in the x–y-plane, which is perpendicular to the propagation direction of the photons. The coordinate systems are left-handed with the z-axis in propagation direction for each wing, with the x-axis in horizontal and the y-axis in vertical direction.

2.2 Model assumptions

Model assumption MA1: *The statistical parameter λ, uniformly distributed between 0 and 1, controls which of the two polarizer exits the photon will take. Given the polarizer setting α and the photon polarization φ we define $\delta = \alpha - \varphi$ as the difference between the polarizer setting and the polarization of the photon. The function $A(\delta,\lambda)$ indicates which polarizer exit the photon will take.*

$A(\delta,\lambda)$ can have values +1 and -1. For $0 \le \delta < \pi/2$, we define
$A(\delta,\lambda) = +1$ for $0 \le \lambda \le \cos^2(\delta)$, (1)
meaning the photon takes polarizer exit α and

$A(\delta,\lambda) = -1$ for $\cos^2(\delta) < \lambda \le 1$, (2)
meaning the photon takes polarizer exit $\alpha + \pi/2$. MA1 is valid for single photons as well as for each wing of entangled photons.

The case $\pi/2 \le \delta < \pi$ is covered referring to the other exit of the polarizer. Then equation (2) applies with positive results and the range of values of λ is $\cos^2(\delta-\pi/2) = \sin^2(\delta) < \lambda \le 1$. The case $\delta < 0$ is covered by reversing the polarizer direction by 180°. Thus, $-\pi \le \delta < -\pi/2$ is equivalent to $0 \le \delta < \pi/2$ and $-\pi/2 \le \delta < 0$ is equivalent to $\pi/2 \le \delta < \pi$.

Thus $A(\delta,\lambda) = +1$
for $0 \le \delta < \pi/2$ and $0 \le \lambda \le \cos^2(\delta)$, (3.1)
for $-\pi/2 \le \delta < 0$ and $\sin^2(\delta) < \lambda \le 1$, (3.2)
for $\pi/2 \le \delta < \pi$ and $\sin^2(\delta) < \lambda \le 1$, (3.3)
for $-\pi \le \delta < -\pi/2$, and $0 \le \lambda \le \cos^2(\delta)$ and (3.4)
$A(\delta,\lambda) = -1$ otherwise. (3.5)

Model assumption MA2: *If the fractions of horizontally and vertically polarized photons from an entangled state that contribute to a photon stream selected by a polarizer are $\cos^2(\alpha)$ and $\sin^2(\alpha)$ respectively, then they obtain a common polarization of α or $-\alpha$, because of the indistinguishability of the photons.*

The fractions of horizontally and vertically polarized photons that would leave a polarizer exit α are $\cos^2(\alpha)$ and $\sin^2(\alpha)$ respectively. This makes up for the common polarization of the selection which comprises all photons that take the same polarizer exit. Photons with polarization α and $\alpha+\pi/2$ come in equal shares, due to symmetry reasons. MA2 accounts for the fact that the polarization of photons from the entangled state is undefined because of their indistinguishability, but is changed and re-defined by entanglement. Thus, the photons of a selection cannot be distinguished by their polarization. This argument has already been made in [5] but only for photon pairs with common hidden variables. MA2 is true for any orientation of the coordinate system. It is a contextual assumption, because the polarization of a selection coincides with the setting of a polarizer. However, this is a local realistic assumption, because it assigns a real value to the physical quantity polarization. MA2 leaves open whether the polarization of a selection is positive or negative. To distinguish this, we use the initial conditions taking into account the conservation of angular momentum. This leads to

Model assumption MA3: *Each Bell state is a mixture of indistinguishable constituent photon pairs in equal shares whose components have the same polarization 0° or 90° for Φ+ and Φ- and an offset of π/2 for Ψ+ and Ψ-. The constituent photon pairs make up the initial state.*

The coupling of a selection on wing A with polarization α and the corresponding selection of the partner photons on wing B with polarization α' is a relation between the signs of the polarizations on both sides and is given

for Ψ+ and Φ+ as $sign(\alpha)_A = sign(\alpha')_B$, *and* (4)
for Ψ- and Φ- as $sign(\alpha)_A = - sign(\alpha')_B$, (5)
where all angles are in the interval [-π/2, +π/2].
From angles outside this interval, we subtract π because α and α-π denote the same polarization.

With this definition we obtain
$sign(\alpha) = - sign(\alpha+ \pi/2) = sign(-\alpha- \pi/2)$. (6)

As shown below, MA3 expresses the conservation of spin angular momentum and is thus a local assumption.

Model assumption MA4: *Photons having left a polarizer exit α have polarization α with λ evenly distributed in the range 0 ≤ λ ≤ 1.*

MA4 emphasizes that photons carry the full set of hidden variables after leaving the polarizer.

2.3 Predicting measurement results for single photons

Using equations (3.1 or 3.4), a photon with polarization φ is found behind the exit α of a polarizer with probability

$$P_\delta = \int_0^{\cos^2(\delta)} d\lambda = \cos^2(\delta), \quad (7)$$

where $\delta = \alpha - \varphi$ with $0 \le \delta < \pi/2$ or $-\pi \le \delta < -\pi/2$.
Using equations (3.2 or 3.3) for $-\pi/2 \le \delta < 0$ or $\pi/2 \le \delta < \pi$ we refer to the other exit of the polarizer and have, with $\vartheta^* = \delta - \pi/2$

$$P_\delta = \int_{\cos^2(\delta)}^1 d\lambda = 1-\cos^2(\vartheta^*) = \cos^2(\vartheta), \text{ as well.} \quad (8)$$

With $\delta = \alpha - \varphi$ we obtain the same P_δ for a photon in state cos(φ)*|H> + sin(φ)*|V> by projection onto cos(α)*<H| + sin(α)*<V| according to QM (i.e., Born's rule).

2.4 Conclusions from the model assumptions

MA2 has the consequence that the selection by a polarizer in position α on one side corresponds to a selection with polarization α+π/2 or -α-π/2 on the other side. (for Ψ+ or Ψ-) This can be seen from the following consideration: According to equations (7,8) a polarizer PA set to α selects a fraction of $\cos^2(\alpha)$ of horizontally polarized photons 1 and a fraction of $\sin^2(\alpha)$ of vertically polarized photons 1. This means that partner photons 2 are also selected, but with perpendicular polarization, resulting in a selected fraction of $\cos^2(\alpha) = \sin^2(\alpha+ \pi/2)$ of vertically polarized photons 2 and a selected fraction of $\sin^2(\alpha) = \cos^2(\alpha+ \pi/2)$ of horizontally polarized photons 2. Due to MA2, the polarization of the selected photons 2 is α+ π/2 or -α- π/2.

From equations (4) and (6) we obtain for Ψ+ the polarization -α- π/2 of the partner photon 2 with the same sign as that of the polarization α. For Ψ- we obtain the polarization α+ π/2 of partner photon 2 with an opposite sign of the polarization α in accordance with equations (5) and (6).

For Φ+ and Φ- we find that the selection by a polarizer in position α on one side corresponds to a selection with polarization α or -α on the other side. Again, a polarizer PA set to α selects a fraction of $\cos^2(\alpha)$ of horizontally polarized photons 1 and a fraction of $\sin^2(\alpha)$ of vertically polarized photons 1. This means that partner photons 2 are also selected, resulting in a selected fraction of $\cos^2(\alpha)$ of horizontally polarized photons 2 and a selected fraction of $\sin^2(\alpha)$ of vertically polarized photons 2. Due to MA2 the polarization of the selected photons 2 is α or -α.

According to equation (4) we obtain the polarization of the partner photons 2 of α for Φ+ as $sign(\alpha)_A = sign(\alpha)_B$ and for Φ- the polarization of partner photon 2 is -α as $sign(\alpha)_A = - sign(-\alpha)_B$ in accordance with equation (5). The results for all four Bell states are presented in Table 1.

Bell state	A	B
Ψ^-	α	α + π/2
Φ^+	α	α
Ψ^+	α	-α -π/2
Φ^-	α	-α

Table 1: polarization of partner photons 2 at wing B for different Bell states for a selection of photons 1 with a polarizer set to α at wing A.

The Bell states Ψ- and Φ+ are known to be rotationally invariant. The same applies to the states Ψ+ and Φ- as well if the coordinate system on wing B is changed from left- to right-handed. In this case, the polarization values for $\Psi+$ and $\Phi-$ in column B in Table 1 change sign, so that the difference between A and B is constant and therefore independent of α. Model assumption MA3 reproduces the conservation of spin angular momentum. This is shown in the following section.

2.5 Conclusions from conservation of spin angular momentum

Conservation of spin angular momentum requires that the total spin of a Bell state is zero. Let $|R>$ and $|L>$ denote the state of the right and left polarized photons, respectively. These are related to the spin direction. The connection to the linear polarization is given by

$$|R> = 1/\sqrt{2} *(|H> + i|V>) \text{ and}$$

$$|L> = 1/\sqrt{2} *(|H> - i|V>) \text{ with} \quad (9)$$

$$|H> = 1/\sqrt{2} *(|R> + |L>) \text{ and}$$

$$|V> = -i/\sqrt{2} *(|R> - |L>). \quad (10)$$

This gives for the four Bell states with the suffixes A and B denoting the wings of the entangled states:

$$\Phi^+= 1/\sqrt{2} *(|H_A>|H_B> + |V_A>|V_B>)$$

$$= 1/\sqrt{2} *(|R_A>|L_B> + |L_A>|R_B>), \quad (11)$$

$$\Psi^- = 1/\sqrt{2} *(|H_A>|V_B> - |V_A>|H_B>)$$

$$= i/\sqrt{2} *(|R_A>|L_B> - |L_A>|R_B>), \quad (12)$$

$$\Phi^- = 1/\sqrt{2} *(|H_A>|H_B> - |V_A>|V_B>)$$

$$= 1/\sqrt{2} *(|R_A>|R_B> + |L_A>|L_B>), \quad (13)$$

$$\Psi^+ = 1/\sqrt{2} *(|H_A>|V_B> + |V_A>|H_B>)$$

$$= -i/\sqrt{2} *(|R_A>|R_B> - |L_A>|L_B>). \quad (14)$$

For $\Phi+$ and $\Psi-$ the total spin of the photon pairs vanishes because left and right polarization cancel. This also applies to $\Phi-$ and $\Psi+$ if the coordinate system on wing B is rotated by 180°, i.e., the photons exit the source in the opposite direction.

Φ^+ and Ψ^- are rotationally symmetrical. So, it also applies

$$\Phi^+= 1/\sqrt{2} *(|H'_A>|H'_B> + |V'_A>|V'_B>) \text{ and} \quad (15)$$

$$\Psi^- = 1/\sqrt{2} *(|H'_A>|V'_B> - |V'_A>|H'_B>) \quad (16)$$

for each angle α of a rotation of the coordinate system, with

$$|H'> = \cos(\alpha) *|H> + \sin (\alpha) *|V> \text{ and}$$
$$|V'> = -\sin(\alpha) *|H> + \cos (\alpha) *|V> . \quad (17)$$

Projection of Φ^+ onto $<H'_A|$ yields

$$<H'_A| \Phi^+> = |H'_B> = \cos(\alpha) *|H_B > + \sin (\alpha) *|V_B >. \quad (18)$$

So we see that a projection of Φ^+ by a polarizer PA in position α means the polarization of the partner photons is in direction α. The projection of Ψ^- yields

$$<H'_A| \Psi^- > = |V'_B> = -\sin (\alpha) *|H_B > + \cos(\alpha) *|V_B >. \quad (19)$$

This state is orthogonal to α. A projection of Ψ^- by a polarizer PA in position α results in the direction $\alpha + \pi/2$. for the state or polarization of the partner photons.

Φ^- and Ψ^+ are also rotationally symmetrical if the coordinate system on wing B is rotated by 180°, i.e. the photons exit the source in the opposite direction.

Reversing direction on side B is the same as replacing each angle on side B with its negative value.

So we obtain that projection of Φ^- by a polarizer PA in position α means the state or polarization of the partner photons in direction $-\alpha$ in the original coordinate system. A projection of Ψ^+ by a polarizer PA in position α results in the direction $-\alpha - \pi/2$. for the state or polarization of the partner photons on side B.

Altogether it follows that of the two possibilities given by MA2, only the one given by MA3 is consistent with the conservation of spin angular momentum. For the relationship between the position of the selective polarizer and the polarization of the partner photons, the conservation of the spin angular momentum means the same sign of α on both sides for Φ^+ and Ψ^- and the opposite sign for Φ^- and Ψ^+ as shown in Table 1. As the conservation of spin angular momentum is a given there is no action associated with it. MA3 is thus a local assumption.

2.6 Calculating expectation values for photons in the singlet state

We have seen above that all selected photons 1 from the singlet state which take PA exit α have polarization α while their partner photons 2 have polarization $\alpha+\pi/2$. Matching events occur if those photons 2 with polarization $\alpha+\pi/2$ would hit PB exit β. Note, that λ is evenly distributed in the value range $0 \le \lambda \le 1$ for the photons 2 with polarization $\alpha+\pi/2$. This can be seen by assuming a polarizer setting PB at $\alpha+\pi/2$ and examining the initial states by applying equations (3.1) - (3.4) to horizontally polarized photons and vertically polarized photons.

For example, for $\pi/2 \le \alpha+\pi/2 < \pi$ the horizontally polarized photons 2 have $\delta = \alpha+\pi/2 -0$. From eq. 3.3 we obtain for $\sin^2(\alpha+\pi/2) = \cos^2(\alpha) < \lambda \le 1$ that those photons contribute to a selection with polarization $\alpha+\pi/2$. The vertically polarized photons 2 have $\delta = \alpha+\pi/2 -\pi/2 = \alpha$. From eq. 3.1 we obtain for $0 < \lambda \leq \cos^2(\alpha)$ that those photons contribute to a selection with polarization $\alpha+\pi/2$ as well.

Thus, the probability that photons 2 with polarization $\alpha + \pi/2$ would pass PB at β can be obtained by equations (7) and (8), using $\delta = \beta -\alpha-\pi/2$ thus yielding

$$P_\delta = \cos^2(\delta) = \cos^2(\beta -\alpha-\pi/2) = \sin^2(\alpha-\beta), \quad (20)$$

where δ is the angle between the PB polarizer setting β and the polarization $\alpha + \pi/2$ of photons 2 selected by PA.

Equation (20) can be directly obtained from MA2. As MA2 is true for any orientation of the coordinate system we choose $\alpha + \pi/2$ as the horizontal base of photons 2. As shown above, all the photons that have polarization $\alpha + \pi/2$ encompass the value range $0 \leq \lambda \leq 1$ so that MA1 is also valid in the new coordinate system. This is done by applying MA1 to a fictitious polarizer on wing B with the setting $\alpha + \pi/2$.

Then the polarizer PB setting in the new coordinate system is $\beta' = \beta -\alpha-\pi/2$. From MA2 we obtain the contribution of the horizontally polarized photons to a common polarization β' as $\cos^2(\beta') = \cos^2(\beta -\alpha-\pi/2)$ in accordance with equation (20). Those photons 2 would hit PB at β and match with partner photons 1 which hit PA at α.

The expectation value for a joint measurement with photon 1 detected behind detector PA at α and partner photon 2 detected behind detector PB at β is as obtained from ([5], equation (13))

$$E(\alpha,\beta) = -\cos(2(\alpha-\beta)), \quad (21)$$

in accordance with QM. As the expectation value $E(\alpha,\beta)$ in Equation (21) exactly matches the predictions of quantum physics, it also violates Bell's inequality.

2.7 Applying the model to entanglement swapping

Entanglement swapping uses a protocol in which two wings of different systems, each in the singlet state, are entangled by a Bell state measurement of the two remaining wings [1,2,9].

Let AB and CD be the two initial systems in the singlet state. Then we define the outer pair AD and the inner pair BC. With a Bell state measurement between B and C, we want to entangle A and D. However, this coupling is random in the case of entanglement swapping. Therefore, four resulting Bell states are possible. How are these results for the inner pair BC related to the state of the outer pair AD? This is determined by applying Table 1 to the pairs of channels. AB and CD are always in the state Ψ-. BC is obtained by the Bell state measurement.

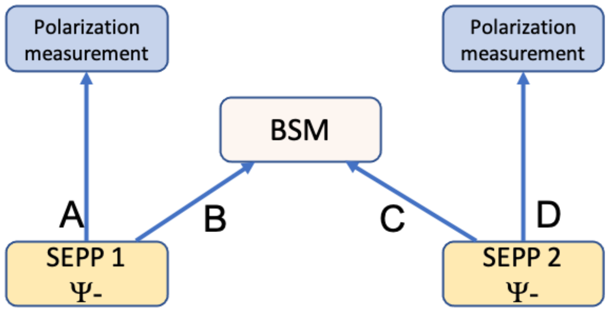

Figure 2: Entanglement swapping entangles wings A and D by a Bell state measurement between B and C.

Thus, we obtained the results of Table 2. Compared with Table 1 we see that the Bell state of the outer pair AD is equal to the measured Bell state of the inner pair BC according to QM [9]. Note that the polarizations $\alpha + \pi$ and α are equal

Bell state BC	A	B	C	D
Ψ^-	α	$\alpha + \pi/2$	α (+ π)	$\alpha + \pi/2$
Φ^+	α	$\alpha + \pi/2$	$\alpha + \pi/2$	α (+ π)
Ψ^+	α	$\alpha + \pi/2$	$-\alpha - \pi$	$-\alpha - \pi/2$
Φ^-	α	$\alpha + \pi/2$	$-\alpha - \pi/2$	$-\alpha$

Table 2: polarization of the photons of wings B,C and A,D for different Bell states obtained between B and C by applying table 1 with an assumed selection of photons by a polarizer set to α at wing A.

2.8 Applying the model to teleportation

Teleportation uses a protocol in which an unknown state β is transferred to another wing B of a singlet state by Bell state measurement between the unknown β and wing A of the singlet state [10]. Using MA3 and Table 1 we obtain the polarizations at wings A and B. AB are always in the Ψ- state. The polarization of the pair βA is obtained by measuring the Bell state.

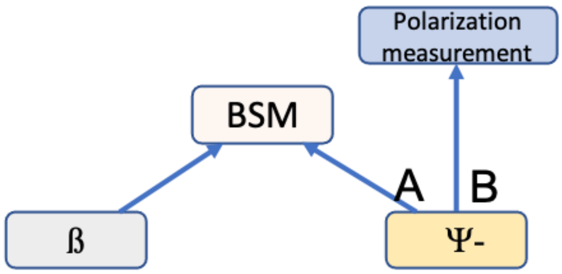

Figure 3: Teleportation of an unknown state ß to a remote wing B by a Bell state measurement between ß and Wing A

Thus, we obtained the results shown in Table 3. The results at wing B can be converted to the state β by simple rotation or mirroring. This result is in accordance with quantum mechanical calculations [10]. Note that the polarizations β + π and β are equal.

Bell state βA	A	B
Ψ^-	β + π/2	β (+ π)
Φ^+	β	β + π/2
Ψ^+	-β-π/2	-β
Φ^-	-β	-β +π/2

Table 3: polarization of the photons of wings A and B for different Bell states obtained between the unknown β and wing A.

3. Results, discussion and conclusions

The title question, what connects the photons of an entangled pair, can now be answered on the basis of the presented model: it is the conservation of spin angular momentum (MA3). Another relationship between the two sides of an entangled pair is of a statistical nature. The proportions of horizontally and vertically polarized photons are the same on both sides or vice versa depending on the initial conditions (MA2). The model presented here is based on the selection of the photons by a polarizer (on one side of a photon pair in a Bell state). The polarization of photons from an entangled state can change if the photons are indistinguishable. Owing to their indistinguishability, the selected photons have a common polarization that depends on the mixing ratio of the constituent horizontally and vertically polarized components. A selection on one wing means a corresponding selection on the other wing as well. The physical state of a selection of photons on one wing thus depends on the selection on the other wing depending on the Bell state. There is no action involved and no common parameter needed to explain entangled states

Thus, the model presented is definitely local. It is the basis to explain the phenomena of entanglement swapping and teleportation. QM expectation values are predicted accordingly. The manuscript is thus a step forward toward a complete theory describing quantum physical reality as thought possible by Einstein, Podolsky, and Rosen [11].

Conflict of Interests/ Competing Interests

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Ethical compliance
No human participants are involved
Data Access Statement
No Data were produced

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