Is the Observational Dark Energy Universe Completely a Coincidence?

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In this article, I propose a model that gives a 'Not really' answer to the question in the title: At any epoch of the universe, to an arbitrary local observer living well below the scale of Hubble horizon, the observational universe appears to be accelerated expanding. In other words, the anthropic principle might be unnecessary for the dark energy universe 'coincidence'. In this article, the negative pressure energy density results from the variance of relative acceleration of the patches of the observational universe, thus the not perfectly uniform flow of time. I will show how such a story is qualitatively compatible with the CMB and low-redshift observations on the expansion history of our universe, while providing intriguing implications to be tested against the recently puzzling high-redshift AGNs and galaxies observations.

I. INTRODUCTION

When asked why we happen to be living in an epoch of the universe where the negative pressure dark energy is taking the majority, $\sim 70\%$ of the cosmic fluid, the answer is usually the anthropic principle [1], i.e. a civilized observer needs to be born in an environment where the dense structure of the universe is diluted by dark energy. However, many find this explanation not satisfying enough, for its arbitrariness and lack of further implications. In this article, I will provide an alternative solution to this question, which appears to be less human-centric, and point to observational implications that can be used to test this proposal.

The article is organized as follows. In section II, I will reexplain Einstein equation as a conservation law of the 4D spacetime volume, in which case naturally incorporates a positive cosmological 'constant' term. In section III, I try to connect the obtained negative pressure term to the observational dark energy in cosmology. In section IV, I will point out some intriguing implications of the theory that can be tested against high-redshift observations that are collecting surging input data right now.

I will use $(-+++)$ signature in this article.

II. EINSTEIN EQUATION WITH POSITIVE COSMOLOGICAL CONSTANT

In the original Einstein's equation Λ was an extra term added with no good explanation within classical general relativity and thus has no prediction on its value [1]. Here I try to give an alternative explanation of the Einstein equation that naturally suggests the presence of such a negative pressure term and being always positive energy.

An overview of the story is as follows: I will show how Einstein equation can be explained as a differential version of the 4D spacetime volume conservation law. Such conservation law is applied to the flow along a family of time-like curves, namely a congruence, on a submanifold that we are concerned about, for example, a patch of the observational universe. The time-like congruence we consider here is not necessarily an affine parametrized geodesics, and it is the general case in the physical world. Relatively accelerated motions between patches of the universe naturally exist, for example, galaxies or clusters of galaxies colliding on each other, and the congruence formed by 4D locally hyperbolic time-like curves is not exotic to discuss . The rescaling of time is not always available globally on a hypersurface in the cosmology scenario, where we have an equal-time slice of the universe that accommodates many relatively acceleratedly moving patches/groups of masses. As we have learned from the standard general relativity, the non-vanishing relative acceleration between two observers leads to the non-uniform flow speed of their proper time. When combined with the 4D volume conservation condition as mentioned at the beginning, such macroscopic non-uniform flow speed of time between patches on a hypersurface results in a negative pressure term present in the Einstein's equation.

One important but somewhat implicit proposition in the proposed theory in this article is that I gave spacetime manifold physical entity, that it is a continuum medium with fluxes and conserved volume of dimension-4. I illustrate a time-like congruence of 4D spacetime with compactified dimension in figure 1.

Let the spacetime manifold be $\mathcal{M}$ and $\mathcal{U} \subset \mathcal{M}$ be open. Through each point $p \in \mathcal{U}$ there passes precisely one time-like curve [2], and such a family of curves I call a

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[1] QFT vacuum energy provides a prediction, though it is off by $\sim$ 100 orders of magnitudes.

[2] I am not considering the black hole defects on $\mathcal{U}$ here, and assume their effect is subdominant given the abundance of the black holes. PBH as DM is another story that could be discussed in another work.

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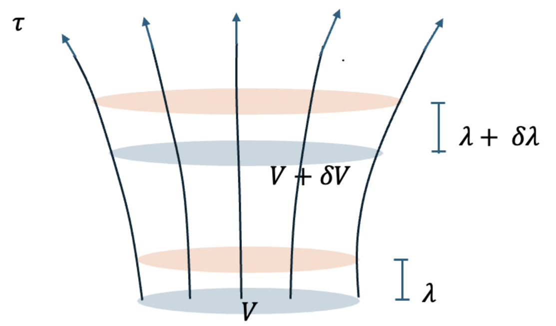

FIG. 1. An illustration of time-like congruence. A 4D cylinder confined by red and blue hyperfurfaces λ away from each other has volume $U = V\lambda$, and the discussion in section II is focused on how the evolution along a congruence vary this 4D volume $\delta U = V\delta\lambda + \lambda\delta V$.

congruence in $\mathcal{U}$. Let the normalized tangent vector field of the congruence in $\mathcal{U}$ be ξ_a.

Consider a bounded $\mathcal{U}$ by two hypersurfaces λ away from each other along the time-like curves and 2D boundaries on hypersurfaces, thus I have a finite 4D volume U cylinder. I can decompose the 4-D volume into the 3-D volume of the hypersurface orthogonal to ξ_a, V, times the range of $\mathcal{U}$ along the congruence, λ, i.e. $U = V\lambda$.

Such orthogonal choice of hypersurface is not a condition with physical significance, only for the simplicity of calculating 4D volume variance. Frobenius' theorem gives the sufficient and necessary condition allowing this choice, which I will mention in the following derivation. The metric on $\mathcal{U}$ is still an undetermined variable, and Einstein's equation is to be derived from here.

Denote the variation from the motion along the congruence as δ. The 4-D volume variation of $\mathcal{U}$ is thus

$$\delta U = \lambda\delta V + V\delta\lambda. \tag{1}$$

Proposition: Let us enforce the conservation of 4-D spacetime volume along time-like congruence by:

$$\delta U + \delta Q = 0, \tag{2}$$

where δQ is the in/out-flow of the 4-D volume current along the congruence:

$$\delta Q = T^{ab}\xi_a\xi_b V\lambda\delta\tau \tag{3}$$

I have not made any statement of T_{ab} at this point, just noting that such tensor expression of flows in a continuum fluid is in general legal. The anatomy of the first term of equation (2) lies in the core of the derivation of Einstein's equation with cosmological constant.

I can denote the normalized basis vector fields on the hypersurface Σ orthogonal to ξ_a as η_a. As they can be chosen to form a set of coordinate vectors of $\mathcal{U}$, they commute in derivatives:

$$\eta^a\nabla_a\xi_b = \xi^a\nabla_a\eta_b \tag{4}$$

$\xi^b\nabla_b$ is exactly the variation along the time-like congruence $\frac{\delta}{\delta\tau}$ that I concern about in the problem.

Thus the tensor $B_{ab} = \nabla_a\xi_b$ describes the failure of η_b and ξ_a being parallelly transported along the congruence. In a addition, the failure of ξ_a's trivial transportation should not be ignored like one conventionally does, for the following reason.

In the case that ξ^a does get trivially transported along the congruence, we have $\xi^a\nabla_a\xi_b = 0$, namely the time-like congruence is generated by geodesics. We learned in the textbook that such geodesics congruence is the only type that we need to study because $\xi^a\nabla_a\xi_b = \alpha\xi_b$ can always be regulated to $\xi^a\nabla_a\xi_b = 0$ by reparametrizing τ. It is a mathematically correct but often disobeyed statement in the physical world. Astrophysical objects' motion often deviates from background geodesics [3], because of the accelerations from non-gravitational forces and/or initial conditions. The recalibration of proper time is not always possible when we consider two relatively acceleratedly moving groups in a problem, where we can only choose one time-flow to be the benchmark. For all of those reasons, it is reasonable, if not, by no means mistaken to study the more general case of $\xi^a\nabla_a\xi_b = \alpha\xi_b$, where $\alpha = 0$ is always a special case.

I decompose B_{ab} into time-like, spacial-trace, spacial-traceless symmetric and antisymmetric parts:

$$B_{ab} = -\alpha\xi_a\xi_b + \frac{1}{3}\theta h_{ab} + \sigma_{ab} + \omega_{ab} \tag{5}$$

where $h_{ab} = g_{ab} + \xi_a\xi_b$ is the spacial metric, characterized by $h^{ab}\xi_a = 0$ for time-like normalized ξ_a.

According to Frobenius's theorem[2], the antisymmetric term $\omega_{ab} = 0$ is the necessary and sufficient condition of ξ^a being hypersurface orthogonal, so let this term vanish.

Let us look into the evolution of B_{ab} along the congruence [2].

$$\xi^c\nabla_c B_{ab} = \xi^c\nabla_c\nabla_a\xi_b \tag{6}$$

$$= \xi^c\nabla_a\nabla_c\xi_b + R_{cab}{}^d\xi^c\xi_d \tag{7}$$

$$= \nabla_a(\xi^c\nabla_c\xi_b) - (\nabla_a\xi^c)(\nabla_c\xi_b) + R_{cab}{}^d\xi^c\xi_d \tag{8}$$

$$= \xi_b\nabla_a\alpha + \alpha B_{ab} - B_a{}^c B_{cb} + R_{cab}{}^d\xi^c\xi_d \tag{9}$$

[3] In some literature geodesics is referred to curves with tangent vectors obeying $\xi^a\nabla_a\xi_b = \alpha\xi_b$. Here to make them distinguished we refer geodesics, affine-parametrized curves, only to those $\xi^a\nabla_a\xi_b = 0$.

Contracting equation (6, 9) with h^{ab}, I can get the famous Raychaudhuri's equation:

$$\xi^a \nabla_a \theta = \alpha\theta - \frac{1}{3}\theta^2 - \sigma^{ab}\sigma_{ab} - R_{ab}\xi^a\xi^b + R_{cabd}\xi^c\xi^d\xi^a\xi^b \tag{10}$$

Because R_{cabd} has antisymmetry, the last term goes to zero.

Contracting equation (6) with $\xi^a\xi^b$,

$$\xi^a\xi^b\xi^c\nabla_c\nabla_a\xi_b = -\xi^a\xi^c(\nabla_c\xi^b)(\nabla_a\xi_b) = -\xi^a\xi^c B_c{}^b B_{ab} \tag{11}$$

which cancels the B^2 term in equation (9), thus

$$\xi^a\xi^b\xi_b\nabla_a\alpha = -\alpha B_{ab}\xi^a\xi^b \tag{12}$$

$$\xi^a\nabla_a\alpha = -\alpha^2 \tag{13}$$

Equation (10) and (13) are purely geometrical identities, and they give us information on $\delta V = \delta\theta V$ and $\delta\lambda = \delta\alpha\lambda$ that I can substitute back to equation (2).

$\delta\alpha = -\alpha^2\delta\tau$ is easy to see from equation (13]) For $\delta\theta$, the first term on the right-hand side of equation (10) gives an exponentially diverging or decaying mode. Dropping the second-order terms, I get:

$$\delta\theta = -R_{ab}\xi^a\xi^b\delta\tau \tag{14}$$

Substituting those back to equation (2), I get:

$$-\lambda V R_{ab}\xi^a\xi^b\delta\tau - \lambda V\alpha^2\delta\tau + T^{ab}\xi_a\xi_b V\lambda\delta\tau = 0 \tag{15}$$

Noting $-1 = g^{ab}\xi_a\xi_b$, for arbitrary ξ_a that is time-like but not necessarily geodesic, I have

$$T^{ab} = R^{ab} - \alpha^2 g^{ab} \tag{16}$$

It looks like the Einstein equation that we are familiar with, but not exactly. I have made no statement about the spacetime medium current tensor T^{ab} by far, and it needs a little bit of dressing to be connected to the energy-momentum tensor. According to the Bianchi identity, T^{ab} is not conserved on $\mathcal{U}$:

$$\nabla_a T^a{}_b = \nabla_a R^a{}_b - 2\alpha\nabla_b\alpha \tag{17}$$

$$= \nabla_b R - 2\alpha\nabla_b\alpha \tag{18}$$

define $\tilde{T}^{ab} = T^{ab} - \frac{1}{2}Tg^{ab}$, I get

$$R^{ab} - \frac{1}{2}Rg^{ab} + \alpha^2 g^{ab} = \tilde{T}^{ab} \tag{19}$$

Now it seems that $\tilde{T}^{ab}$ is the conserved energy-momentum tensor that we are familiar with to describe our cosmic fluid excluding dark energy. Its dual $T^{ab} = \tilde{T}^{ab} - \frac{1}{2}\tilde{T}g^{ab}$ is the spacetime volume current tensor that I introduced before.

The derivation here is much motivated by the thermodynamics explanation of the Einstein's equation by Ted Jacobson [3]. Instead of looking into the black hole case where one of the space dimensions is highly compressed, here the subjects are the less-special, well-behaved 4D spacetime submanifolds, and the conservation of energy $dQ = TdS$ in [3] is substituted by the conservation of 4D volume proposition $dU + dQ = 0$. The fundamental arguments are quite the same, that the Einstein's equation is describing how the spacetime distortion is driven by the flow of thermal energy/4D volume current tensor, under the constraint of energy/volume being a conservation law.

Back to equation (19), the first desirable feature we can see is that the cosmological 'constant' $\Lambda = \alpha(\tau)^2$ is always positive. In the theory here, negative pressure dark energy comes from the patch-wise non-uniform time flow.

The integral of equation (13) tells us $\alpha \sim \frac{1}{\tau}$, namely after long-enough time the congruence saturates to geodesics one in any case. On the other hand, if we regard the integrated τ as the lifetime of a patch of observational universe, the above relationship suggests $\sqrt{\Lambda} = |\alpha| \sim \frac{1}{\tau}$ always holds, just like what we have found out about our own observational universe.

III. COSMOLOGICAL EFFECT

Now that I have Einstein's equation with a positive cosmological constant, I want to see if I can connect it with the observational 'dark energy'.

In the observational cosmology field, dark energy has been a placeholder for the unexplained fact that we see the universe accelerated expanding around us. The astrophysical objects at distances far enough to be in the 'Hubble flow' run away from us with increasing velocity. Such accelerated expansion reality is fairly homogeneous, and the negative pressure portion of the energy density of the cosmic fluid has an equation of state very close to $w \equiv \frac{p}{\rho} \sim -1$ [4–6]. Those are about the uncontroversial part of what we know of the observational dark energy so far.

Dark energy has no observed perturbative effects so far, most of the time it is only discussed on the background level, in Friedmann equations. I will focus on the background cosmology in this article.

Let us denote the average over position in the celestial as $\bar{x}$ and the expectation value over the full phase space as $\langle x\rangle$. The two Friedmann's equations are the time and space components of

$$R^{ab} - \frac{1}{2}Rg^{ab} + \langle\bar{\alpha}^2\rangle g^{ab} = \langle\bar{\tilde{T}}\rangle^{ab} \tag{20}$$

$\langle\bar{\alpha}\rangle$'s calibration to zero is hidden in the intuitive pressumption that our whole observational universe is 'free-falling', namely evolving in shortest path along the time direction that we choose as an observer. Hence the cosmological constant that we observe in the astrophysical surveys is the variance of time-flow in the whole observational volume, $\sigma^2(\bar{\alpha})$.

Now the mystery is the incredibly stable scaling of $\sigma^2(\bar{\alpha})$ with the scale factor a and its homogeneity. Here I do not serve an ultimate solution to the homogeneous principle of the universe, but qualitatively provide an argument attaching those feature of the dark energy density to the matter density.

The argument is that since the time-flow non-uniformity is caused by relative acceleration motion, and the forces are mainly caused by matter in the recently matter-dominated universe, I deduce $\sigma^2(\alpha) \sim \rho_m$, namely, the variance of time-flow non-uniformity is the same order of magnitude as matter density in a matter dominated universe.

This assumption gives the right units and respects the linear order approximation I adopted earlier in Raychaudhuri's equation. Denoting the order one proportional parameter as d_F, we have

$$\Lambda = \sigma^2(\alpha) \approx d_F \rho_m \tag{21}$$

In our local universe with $\Omega_\Lambda = 0.7$, $d_F \approx 2.4$. The dimension analysis in natural unit suggests such a linear relationship and that d_F is dimensionless, and the d_F notation is taken from the fractal dimension of the Poission-like distribution of the matter in our universe [7], which has a measured value of 2.4. However, failing to come up with a detailed argument justifying the fractal dimension being THE dimensionless parameter here, I will not conclude it to be just a number game or an implication of a deeper connection [4].

In either case, given equation (21), we have the relationship between matter density and the variance of the timeflow on a causally connected patch. Matter distribution beyond Hubble horizon does not act any forces on a group, so when considering the homogeneous isotropic universe beyond Hubble horizon, local matter density alone does not provide the whole story.

As shown in figure 2, on the shell of $\chi(a)$ away from us, there are number of $N = V_{\text{shell}}/V_{\text{Hubble}}$ patches that follow the same distribution of α in their local Hubble volume, given the homogeneous distribution of the matter field. Hence the variance $\sigma^2(\bar{\alpha})$ is suppressed by $1/N$. In the regime of $\chi(a) >> 1/H(a)$ where the above approximations applies, we have:

$$\Lambda(a) \equiv \sigma^2(\bar{\alpha}) \approx \frac{\sigma^2(\alpha)}{N} \tag{22}$$

$$\approx \frac{d_F \rho_m(a)}{V_{\text{shell}}(\chi(a))/V_{\text{Hubble}}(a)} \tag{23}$$

$$= \frac{d_F \rho_m^0 a^{-3}}{4\pi\chi^2\lambda_H/(4\pi/3\lambda_H^3)} \tag{24}$$

$$= \frac{d_F \rho_m^0 a^{-3}}{3\chi^2/\lambda_H^2} \tag{25}$$

[4] An even more intriguing number game could be that the fraction $\Omega_b : \Omega_m : \Omega_\Lambda$ is well captured by $1 : d_F^2 : d_F^3$.

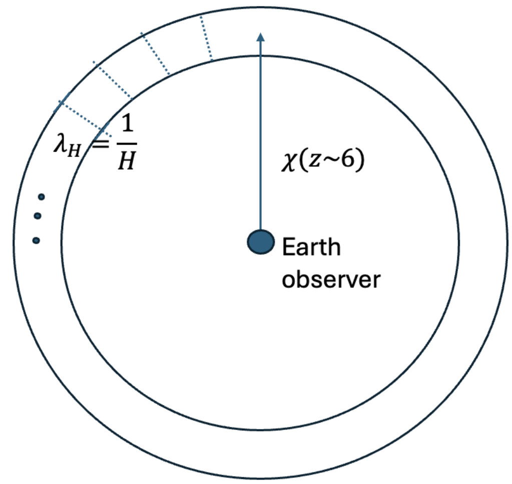

FIG. 2. All the currently observable galaxies around certain redshift, say $z \sim 6$, reside on a shell χ away from us. The Hubble horizon $\lambda_H = 1/H$ at that epoch (in our chronicle) is much smaller than us, thus such a shell accommodates many Hubble bubbles.

We can solve for the evolution of dark energy density by taking derivative of the integral equation (25) up to $N = \chi/\lambda_H = 10$, corresponding to a = 0.2 thus redshift z = 4 in a universe with $\Omega_\Lambda = 0.7$. Denoting $X(a) = \frac{\Omega_\Lambda(a)}{\Omega_\Lambda}$, from equation (25) we get

$$X'(a) = 2\sqrt{3}\frac{X^{3/2}(a)}{a^{1/2}} - 3\frac{X(a)}{a} - 2E'(a)E^{-1}(a)X(a) \tag{26}$$

where $E(a) = H(a)/H_0 = \sqrt{\Omega_\Lambda(a) + \Omega_m a^{-3}}$, and we used the relationships $\chi = \int_a^1 \frac{1}{a'^2 H(a)} da'$ and $\lambda_H = 1/H$.

On the other hand, in the regime $\chi(a) << \lambda_H$, roughly $z < 0.1$, $\Lambda(a)$ in our Hubble volume is expected to saturate as in equation (21). The intermediate regime $0.1 < z < 4.0$ needs more dedicated modeling, which we leave for future work.

Figure 3 shows the density of dark energy evolution in the far field $z > 4$ by solving the ordinary differential equation (26). Reassuringly, it is not scaling up as a^{-3} with the matter density, instead decreasing to a smaller platform, thus agreeing with the observation that the dark energy was subdominant in the early universe. The decreasing rate varies with the initial guess of $\Lambda(a_p)$, but the trend stays stable with reasonable trial values that confine $X(a_p)$ between 0 and 1. In a sentence, it seems regardless of the initial/boundary fraction of dark energy at far-field redshift a_p, a general case is that the $1/N$ suppression dominates thus diluting dark energy out when we look out toward smaller scale factor a.

Although for convenience, I have been using the terms dark energy and $\sigma^2(\bar{\alpha})$ inter-changeably, but $\alpha^2 g^{ab}$ term in equation (19) is not an energy-momentum tensor and

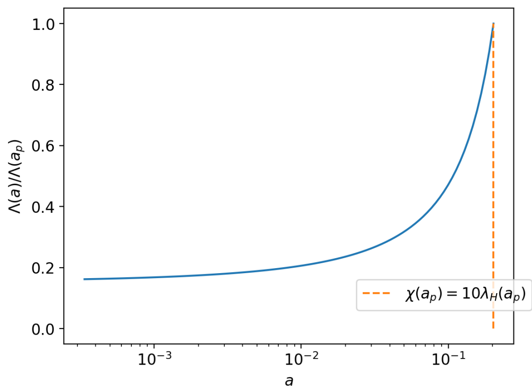

FIG. 3. Dark energy density, or the variance of time-flow $\sigma^2(\bar{\alpha})$ as a function of scale factor a. In the regime where far-field approximation holds, $\chi >> \lambda_H$, dark energy density is suppressed by $1/N$ factor towards high redshift, as required by the CMB observation.

might not be able to be investigated by the currently available Boltzmann codes, especially on perturbative level, if it ever proves relevant in the future. There are two straightforward reasons. First, this tensor does not conserve. $\nabla_a \langle \bar{\alpha}^2 \rangle = 2\langle \bar{\alpha} \nabla_a \bar{\alpha} \rangle$ only effectively vanishes when $\langle \bar{\alpha} \rangle = 0$, calibration of time-flow is chosen for certain hypersurface. Secondly, when regarding $\langle \bar{\alpha}^2 \rangle g^{ab}$ as an energy-momentum tensor, it always has an equation of state -1, even though $\langle \bar{\alpha} \rangle$ could vary when course-grained on different scales. The theory and the corresponding derivation suggests that α only enters Einstein's equation as a coefficient in front of the metric g^{ab}, thus only affecting the spacetime in the way of scaling the whole metric instead of distorting the spacetime with Weyl or Newtonian potential.

IV. DISCUSSIONS

One counter-intuitive but very intriguing implication of $\sigma^2(\bar{\alpha})$ as dark energy is that, our observational universe that is acceleratedly expanding is not special. Anthropic principle is not needed in this picture, because at any redshift, an observer living well below the Hubble horizon scale would see an acceleratedly expanding universe around them. Inside a smaller (to us) Hubble horizon bubble at higher redshift, always resides another acceleratedly expanding universe, and when the residents in that Hubble bubble look outwards in a universe centered on themselves, they would see a similar expansion history of the universe as us. An absolute chronicle of the universe since 'The Big Bang' loses its meaning in this picture, when looking far enough every observer reaches their own 'Big Bangs', the scale factor is in the place of time for any local (below Hubble size) observer. Namely, forward-time might be a concept as trivial as downward-direction of the universe, and the whole universe could be seen as a series of indefinitely unfolding self-similar structures at hierarchical scales, pivot at the observer's scale when a chronicle story needs to be told. Our chronicle of the universe is only one among many observer-dependent ones.

An important consequence is that the lifetime of the observational universe for a civilization living in a galaxy at, say $z = 6$, could be longer than 1Gyr as calculated from our origin of the time, when calculated in an acceleratedly expanding universe around them. The difference in the universe histories viewed by different redshift observers echoes a fact that we already knew since the birth of general relativity, that the history of our universe may not exactly be what we try to construct by tracing back to higher redshift galaxies. Those high-z observable patches are light-like connected to us, not time-like, thus strictly speaking none of those high-redshift galaxies and black holes we can see at this moment will evolve into the local objects seen around us, on the (roughly, cosmology-scale) same equal-time space-hypersurface, a more formal expression of the phrase 'in our epoch'.

A longer proper universe lifetime gives longer accretion time for those high-redshift supermassive black holes, whose overabundance and overweight have been a concerning confusion in recent high redshift observations [8]. The discovery of many $> 10^9 M_\odot$ supermassive black holes (SMBH) above redshift $z > 6$ forcing astrophysicists to look for exotic mechanisms to allow super-Eddington accretion of the black holes, where Eddington limit is the accretion rate at which the radiation pressure force cancels the gravity. Even with a relatively heavy black hole seed $\sim 100 M_\odot$, the Eddington limit accretion needs at least ~ 0.8 Gyr to form a SMBH $\sim 10^9 M_\odot$, and the universe lifetime at redshift 6 based on Big Bang theory is just about enough. Many cosmological approaches to the problem rearrange the expansion history of ΛCDM; However, in the picture proposed by this article, an observer-dependent origin thus the lifetime of the universe could be an alternative cure. With the presence of effective 'dark energy' in a Hubble volume bubble, the proper universe lifetime to a high-redshift observer is always dragged longer, and the same effect applies to any localized physical processes.

In the same logic, astrophysicists might find some of the high redshift galaxies behave older than theory predictions. In recent and upcoming high-redshift astrophysical surveys like JWST [9, 10], those kinds of puzzling early-universe but highly-evolved galaxies have already been found, with arguably indecisive detection evidences. Although now still troubled by systematics and selection effects, those high-redshift galaxy properties, especially the charts on their ages, will be crucial to test the implications pointed out in this article.

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