Review of: "The Generalized ETA Transformation Formulas as the Hecke Modular Relation"

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Potential competing interests: No potential competing interests to declare.

The main aim of the article is to elucidate the Hecke modular structure involved in earlier works by Rademacher, Apostol, Dieter, etc., and make further developments. The authors confine themselves to the Lambert series case only.

§2 deals with elucidation of the works of Rademacher's integral transform method by showing the functional equation for the zeta function and the general eta transform formula as developed by Rademacher for the Dedekind eta function and also by Apostol in the relevant Lambert series case. The main result here is

Theorem 0.1. The Rademacher-Apostol zeta function Zp(s, h) satisfies the Hecke functional equation relating s and 1 − p − s, where p ≥ 1 is an odd integer.

Another result, which is a nice transformation formula for the Lambert series gp, has also been proved.

§3 deals with the second main result involving the Kratzel case. Kratzel introduced the zeta function Za,b(s) with co-prime natural numbers a, b. The functional equation satisfied by Z is reproduced here, which involves s and −s. The authors pointed out that

Kratzel's method is actually that of Rademacher, not Rademacher's one, only, and it has been elucidated in Theorem 2, which also proves a transformation formula for the Dedekind eta function.

In §4, the authors unify the method of Rademacher and that of Dieter. The authors here prove the modular relation structure of the zeta function and the general eta transformation formulas contained in the Rademacher-Apostol and that of Dieter.

In §5, the case of Schoenberg has been considered, and the functional equation relation relating Z(s, α, β) and that of ˜Z(Sα, β) is shown.

Conclusion: The manuscript is well written, barring a few typos here and there. The zeta functions considered and the methods elaborated could be useful to the researchers working in this area.

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