A Gaussian Kernel Similarity Approach to Multisource Information Fusion Considering the Weight of Focal Element Beliefs

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Abstract

Similarity has been extensively utilized to measure the degree of conflicts between evidences in multisource information fusion. The existent works, however, assumed that the contribution of each focal element's belief to the similarity measure is the same, and the influence of the weight of focal element belief is not considered, which is unreasonable. This article proposes a new Gaussian kernel similarity approach to measure the similarity between evidences. The proposed Gaussian kernel similarity coefficient can effectively take account of the weights of focal element beliefs. In addition, it possesses some preferable properties, such as, bounded, consistent, and symmetrical. A multisource information fusion method based on the Gaussian kernel similarity coefficient is, therefore, investigated. The developed method mainly contains three steps: (1) The Gaussian kernel similarity coefficient, as a connection, is leveraged to calculate the weight of evidences based on the weight of focal element beliefs; (2) The initial evidences are, thereby, modified based on the weight of evidence via the weight-average method; and (3) The final multisource information fusion can be achieved by the Dempster's combination rule using the modified evidences. Two illustrative examples with singletons and multi-element subsets are presented, and it is verified that the proposed method is effective in dealing with conflicting evidences.

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1. Introduction

The evidence theory, also called the Dempster-Shafer evidence theory, is an uncertain reasoning approach initiated by Dempster [1] and improved by Shafer [2]. It has been extensively implemented in the field of information fusion because of its preferable advantages in dealing with uncertain information [3]. In the evidence theory, Dempster's combination rule satisfies excellent mathematical properties, such as the exchange law and association law, which makes it widely leveraged in multisource information fusion. However, Dempster's combination rule also suffers from some shortcomings. For example, when the conflict between evidences is relatively high, using this combination rule for information fusion will result in the following defects: the result is counter-intuitive [4], "one-vote rejection", and lack of robustness. Dempster's combination rule should be, therefore, cautiously used for information fusion with conflicting evidences[5].

Many researchers have dedicated themselves to putting forth effective methods and tools to fuse the conflicting evidences under the evidence theory framework, which mainly falls into two categories: the first is to directly modify Dempster's combination rules. Representative studies have been achieved in literature[6][7][8][9][10]. Among them, Yang et al. [10] modified Dempster's combination rule with a newly defined absolute and relative difference factor based on the idea of local conflict local assignment. The second category is to modify the initial evidences first and then fuse them using Dempster's combination rule. A representative method is the average combination rule proposed by Murphy [11], which can effectively deal with the problem of conflicting evidence. However, the method does not consider the dependency on evidences. Deng et al. [12] introduced the distance function [13] based on Murphy's method to measure the similarity between evidences and modified the initial evidences with the idea of a weighted average, which has a fast convergence speed and better results. In the method of modifying initial evidences, the main work focuses on how to define the similarity of evidence to reflect the degree of conflict between evidences. As a result, many methods, such as evidence similarity, evidence distance, and divergence have subsequently emerged. For example, for the evidence similarity-based method, Jiang [14] puts forward an evidence correlation coefficient to measure the similarity between evidences. Deng and Wang [15] designed an evidence similarity method based on the Tanimoto measure. There are also methods from the distance of evidence: Lin et al. [16] used Euclidean distance to represent the differences between evidences. Ye et al. [17] used the Lance distance function and the spectral angle cosine function to describe the conflict of evidence. Zhu and Xiao [18] put forward a belief Hellinger distance to quantify the differences between evidences. Apart from the above, there are methods from the divergence: Gao and Xiao [19] put forward an RBχ2 divergence to calculate the conflict degree between evidences. Zhang and Xiao [20] proposed a SEBχ2 divergence to calculate the discrepancy between evidences. Chen and Cai [21] proposed a method for conflict management based on Renyi divergence. As the similarity of evidence is opposite to the distance of evidence and the conflict of evidence, all the above three classes of methods can be summed up as the method of evidence similarity.

It can be seen that the similarity of evidence has been intensively used to measure the degree of conflicts between evidences. However, in the existing methods, when calculating the similarity of evidence, the contribution of each focal element belief to the similarity measure is the same, which is unreasonable. All focal element beliefs should have their weights, so that when calculating the similarity of evidence, the greater the weight of a focal element belief, the greater its contribution to the similarity measure, and the smaller the opposite, which is more reasonable. To address this problem, this article proposes a new Gaussian kernel similarity coefficient to measure the similarity between evidences. The Gaussian kernel similarity coefficient considers the weight of focal element beliefs, which can quantify the contributions of focal element beliefs to the similarity of evidence and makes the measurement of the similarity more reasonable. Thereby, a multisource information fusion method based on the Gaussian kernel similarity coefficient is designed to handle the conflicting evidences. The method uses the Gaussian kernel similarity coefficient to compute the weight of evidence based on the weight of focal element beliefs. Then, according to the weighted average idea, the final modified evidence is fused using Dempster's combination rule. Finally, through two examples under single-element subset and multi-element subset, it is verified that the proposed method is more effective in dealing with conflicting evidences. In summary, the main contributions of this article are summarized as follows:

1. A new problem is defined: how to measure the similarity between evidences considering the weight of focal element beliefs.
2. To solve the above problem, a new Gaussian kernel similarity coefficient is proposed to measure the similarity between evidences. The proposed Gaussian kernel similarity coefficient, as a connection, is designed to calculate the weight of evidences based on the weight of focal element beliefs. In addition, it is bound, consistent, and symmetrical.
3. A multisource information fusion method based on the Gaussian kernel similarity coefficient is designed to handle conflicting evidences. The method takes the Gaussian kernel similarity coefficient as a connection to calculate the weight of evidence by using the weight of focal element beliefs. The final fusion results based on the proposed method are more convincible than the existing methods.

The remainder of this article is rolled out as follows. Section 2 briefly introduces some preliminaries of the evidence theory, belief entropy, and kernel functions. Section 3 presents the novel Gaussian kernel similarity coefficient to measure the similarity between evidences. The evidence information fusion method based on the Gaussian kernel similarity coefficient is articulated in Section 4. In Section 5, two illustrative examples are presented to verify the effectiveness of the proposed method. Section 6 is a summary of conclusions and future avenues.

2. Preliminaries

In this section, some basic concepts about evidence theory, belief entropy, and kernel function are presented.

2.1. The evidence theory

Evidence theory has outstanding advantages in information fusion. In this section, the evidence theory is briefly introduced as follows.

Definition 1. (Frame of discernment). Let Θ be a finite complete set and consist of N number of mutually exclusive elements:

$$\Theta = \left\{r_1, r_2, \cdots, r_i, \cdots, r_N\right\}, \qquad (1)$$

thereby, Θ is called the frame of discernment. Its power set 2Θ is defined as:

$$2^\Theta = \left\{\varnothing, \left\{r_1\right\}, \cdots, \left\{r_N\right\}, \left\{r_1, r_2\right\}, \cdots, \left\{r_1, r_2, r_3\right\}, \cdots, \Theta\right\}. \qquad (2)$$

Definition 2. (Mass function). The mass function maps the power set 2Θ to an interval[1] so that it is defined as:

$$m: 2^\Theta \to [0, 1], \qquad (3)$$

and satisfies the following conditions:

$$\sum_{A \in 2^\Theta} m(A) = 1, m(\varnothing) = 0, \qquad (4)$$

where $m(\varnothing) = 0$ represents the mass of empty set is always zero, the mass function is also called basic probability assignment (BPA). If m(A) > 0, then A is called the focal element, and m(A) represents the belief in focal element A [22].

Definition 3. (Dempster's combination rule). Let m_1 and m_2 be the two sets of evidence, and A_1, A_2, ⋯, A_k and B_1, B_2, ⋯, B_s be the corresponding focal elements. The new evidence after fusion is indicated by m. Then Dempster's combination rule is as follows:

$$\begin{cases} m(A) = \frac{1}{1-k} \sum_{A_i \cap B_j = A} m_1(A_i) m_2(B_j), \\ m(\varnothing) = 0 \end{cases} \qquad (5)$$

and

$$k = \sum_{A_i \cap B_j = \varnothing} m_1(A_i) m_2(B_j), \qquad (6)$$

where k is the conflict factor, and the rule fails when k=1.

Definition 4. (Average weight combination rule). If there are h pieces of evidence, $w(m_i)$ is assumed to be the weight of evidence m_i, (i=1, 2, ⋯, h), and the weighted average of the evidence MAE is given as[12]:

$$MAE(m) = \sum_{i=1}^{h} w(m_i) \times m_i, \qquad (7)$$

then Dempster's combination rule is used to combine the weighted average of the masses h-1 times[11]. Therefore, this is called the average weight combination rule.

2.2. Belief entropy

Entropy is an uncertainty measure, and to quantitatively describe the uncertainty of BPAs in evidence theory, Deng [23] proposes the belief entropy, which is defined as follows.

Definition 5. (Belief entropy). Given a BPA m defined on the frame of discernment Θ. The belief entropy is defined as:

$$E_d = -\sum_i m\left(A_i\right) log^{\frac{m\left(A_i\right)}{2^{\left|A_i\right|}-1}}, \qquad (8)$$

where A_i is a focal element, $\left|A_i\right|$ is the cardinality of set A_i, i.e., the number of elements contained in A_i. The belief entropy satisfies the probabilistic consistency, set consistency, range, subadditivity, additivity, and monotonicity [24].

2.3. Kernel function

The kernel function approach has been widely implemented in the field of machine learning, especially in support vector machines. Here its brief introduction is presented.

Definition 6. (Kernel function). Let x, z ∈ X, X belongs to space of R (n), and the input space X is mapped to feature space F by nonlinear function Φ. where F is defined in the space of R (m), and n is much smaller than m. Based on the kernel function technique, there exists [25]:

$$k(x, z) =< \Phi(x), \Phi(z) > , \qquad (9)$$

where, <, > is the inner product, k (x, z) is the kernel function. The common-used kernel functions are as follows.

1. Linear kernel

$$k(x, y) = xTy + c. \qquad (10)$$

2. Polynomial kernel:

$$k(x, y) = (axTy + c)d. \qquad (11)$$

3. Gaussian kernel

$$k(x, y) = exp - x - y22\sigma2. \qquad (12)$$

3. The proposed Gaussian kernel similarity coefficient

In evidence theory, it is an important task to measure the similarity between evidences. However, the existing methods have a flaw, i.e., each focal element's beliefs contribute equally to the similarity measure, which is unreasonable. In fact, the greater the weight of a focal element belief, the greater its contributions to the similarity measure. Therefore, before measuring the similarity between evidences, it is necessary to consider the weight of each focal element's beliefs, as shown in Figure 1. To this end, a new Gaussian kernel similarity coefficient is defined in this section, and the similarity coefficient is used to measure the similarity between evidences.

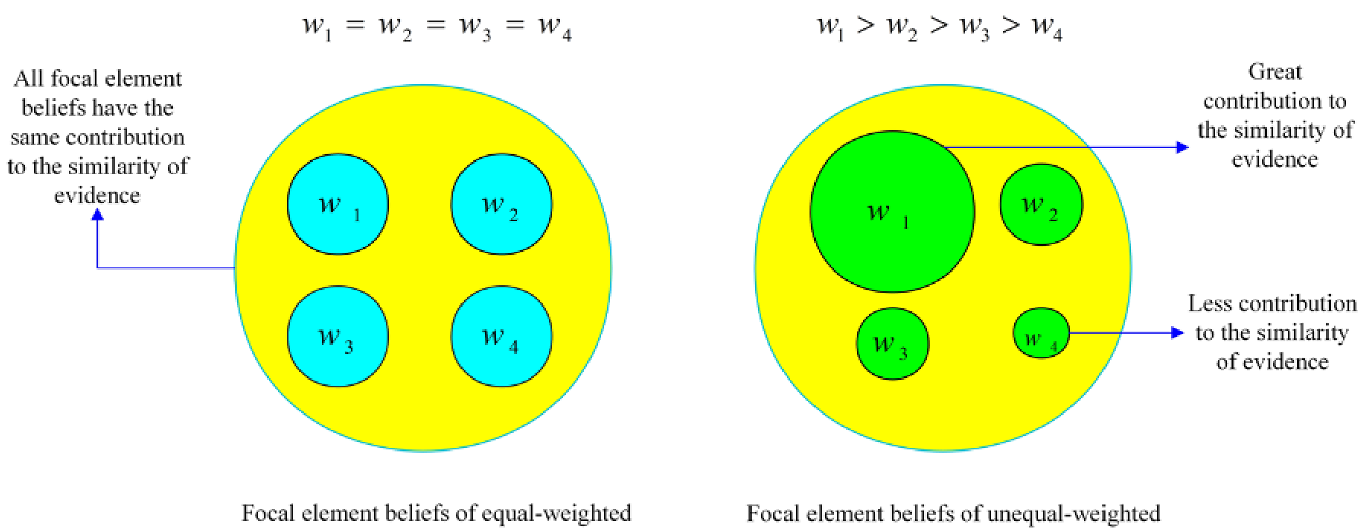

Figure 1. Focal element beliefs of equal-weighted and unequal-weighted.

3.1. Gaussian kernel similarity coefficient

For a power set 2Θ, it consists of 2N elements, thereby, when N is large enough, the number of focal elements are larger, making the dimension of the evidence vector is higher. The kernel function is an effective tool to deal with high-dimensional problems, among them, the Gaussian kernel function is a distance function, which can calculate the similarity between two vectors [26][27]. However, the Gaussian kernel function has a disadvantage, i.e., it cannot reflect the local similarity of data, because its hyperparameter σ is a fixed value. Inspired by reference[28], to reflect the local similarity of data, the term σ^2 of Gaussian kernel function is changed to the term of σiσj. According to this idea, it can be achieved to measure the similarity between evidences, considering the weight of focal element beliefs. Therefore, a Gaussian kernel similarity coefficient is defined as follows:

Definition 7. (Single-element Gaussian kernel similarity coefficient, S-GKSC). Let Θ be the frame of discernment, n be the number of focal elements, and h be the number of evidence to be fused. S-GKSC is defined as:

$$k_{ij} = exp\left(-\sum_{p=1}^{n} \frac{\left(m_i(A_p) - m_j(A_p)\right)^2}{2w_{ip}w_{jp}(^{\frac{h}{2}})} \right), \qquad (13)$$

where m_i (A_p) and m_j (A_p) are the belief of the p-th focal element in the evidence i and j, respectively, (i, j=1, 2, ···, h; p=1, 2, ···, n); w_{ip} and w_{jp} are the weights of m_i (A_p) and m_j (A_p), respectively, which are local scale parameters to measure the similarity of evidence, the larger the value, the greater the contribution to the similarity; $h/2$ is an adaptive parameter, which can adaptively adjust the size of $w_{ip}w_{jp}$ according to the number of evidences to be fused. Thus, the situation that $w_{ip}w_{jp}$ being too small when h is large can be avoided.

Definition 8: (Multi-element Gaussian kernel similarity coefficient, M-GKSC). Let Θ be the frame of discernment, n be the number of focal elements, and h be the number of evidence to be fused. M-GKSC is defined as:

$$k_{ij} = exp\left(- \sum_{p=1}^{n} \frac{\left(m_i(A_p) - m_j(A_p)\right)^2}{2w_{ip}w_{jp}^{\left(\frac{h}{2}\right)}(2^{\left|A_p\right|} - 1)} \right), \qquad (14)$$

where the value $2^{\left|A_p\right|} - 1$ comes from the idea of belief entropy and can be used in a situation where the focal element is a multi-element subset. It should be noted that M-GKSC degenerates to S-GKSC when $\left|A_p\right| = 1$, i.e., all elements are singletons. Therefore, M-GKSC is a general expression.

The similarity of any two evidences can be obtained from Eq. (14), and it is mapped to the range of[1]. The more the similarity tends to one, the smaller the conflict of evidence and the more the similarity tends to zero, the larger the conflict of evidence. It can be seen that the weight of focal element beliefs needs to be calculated before the calculation of M-GKSC. The method of calculating the weight of focal element beliefs is given in Section 4.

3.2. Properties of Gaussian kernel similarity coefficient

Let m1 and m2 be two BPAs defined on Θ, three properties of the Gaussian kernel similarity coefficient can be proofed as follows:

1. Boundedness: 0< k (m1, m2) ≤1.
2. Consistency: k (m1, m2) =1, if and only if m1=m2.
3. Symmetry: k (m1, m2) = k (m2, m1).

Proof 1. Given two BPAs m1 and m2 defined on Θ. It can be seen that,

$$\sum_{p=1}^{n} \frac{\left(m_1(A_p) - m_2(A_p)\right)^2}{2w_{1p}w_{2p}^{\left(\frac{h}{2}\right)}(2^{\left|A_p\right|} - 1)} \geq 0,$$

and when x is non-negative, one has 0 < e-x ≤1, thereby, the following inequation can be achieved:

$$0 < k(m_1, m_2) \leq 1$$

which completes the proof.

Proof 2. Given two BPAs m1=m2 defined on Θ. It can be seen that,

$$\sum_{p=1}^{n} \frac{\left(m_1(A_p) - m_2(A_p)\right)^2}{2w_{1p}w_{2p}(^{\frac{h}{2}})(2\left|A_p\right| - 1)} = 0,$$

so,

$$k(m_1, m_2) = 1.$$

Therefore, the consistency of the Gaussian kernel similarity coefficient has been proofed.

Proof 3. Given two BPAs m1 and m2 defined on Θ, thereby, the Gaussian kernel similarity coefficients of these two BPAs can be calculated as:

$$k(m_1, m_2) = exp\left(- \sum_{p=1}^{n} \frac{\left(m_1(A_p) - m_2(A_p)\right)^2}{2w_{1p}w_{2p}(^{\frac{h}{2}})(2\left|A_p\right| - 1)} \right),$$

$$k(m_2, m_1) = exp\left(- \sum_{p=1}^{n} \frac{\left(m_2(A_p) - m_1(A_p)\right)^2}{2w_{2p}w_{1p}(^{\frac{h}{2}})(2\left|A_p\right| - 1)} \right),$$

therefore,

$$k(m_1, m_2) = k(m_2, m_1).$$

which completes the proof of the symmetry of the Gaussian kernel similarity coefficient.

4. Gaussian kernel similarity coefficient-based Conflict Evidence Fusion

In this section, a fusion method of conflict evidence based on the Gaussian kernel similarity coefficient is proposed. In this method, two kinds of weights are considered, one is the weight of focal element beliefs, the other is the weight of evidence, and the Gaussian kernel similarity coefficient is used to calculate the weight of evidence based on the weight of focal element beliefs. Then, the initial evidences can be modified according to the weight of evidence, and the fusion of conflicting evidence can be realized based on the modified evidences via Dempster's combination rule. As can be seen, the core of this method is that the two weights are connected by the Gaussian kernel similarity coefficient, and the detailed process is shown in Figure 2.

Step 1: Calculate the weight of each focal element belief under each set of evidence.

In general, when a focal element's belief in one evidence is significantly different from that in other evidence, its credibility should be low, and a small weight should be assigned so that the contribution to the similarity of evidence is small. To this end, the weight of each focal element belief under each set of evidence is calculated. Before this, it is necessary to define the distance of focal element beliefs.

Definition 9. (Distance of focal element beliefs) Let Θ be the frame of discernment, n be the number of focal elements, h be the number of evidence to be fused, and the beliefs of evidences i and j on focal element A_p are $m_i(A_p)$ and $m_j(A_p)$, respectively, (i, j=1, 2, ⋯, h; p=1, 2, ⋯, n). Then the distance of focal element beliefs between $m_i(A_p)$ and $m_j(A_p)$ is defined as:

$$d_{ij}(A_p) = \left| m_i(A_p) - m_j(A_p) \right|. \qquad (15)$$

The smaller the distance of focal element beliefs, the higher the degree of mutual support. Therefore, the mutual support between mi (Ap) and mj (Ap) is defined as:

$$s_{ij}(A_p) = 0.998\left[1 - d_{ij}(A_p)\right] + 0.002 \qquad (16)$$

where 0.998 and 0.002 can avoid the situation that the denominator of Eqs. (13) and (14) being zero. Then, the support matrix $S(A_p)$ of focal element A_p is

$$S(A_p) = \begin{bmatrix} s_{11}(A_p) & s_{12}(A_p) & \cdots & s_{1j}(A_p) & \cdots & s_{1h}(A_p) \\ \vdots & \vdots & & \vdots & & \vdots \\ s_{i1}(A_p) & s_{i2}(A_p) & & s_{ij}(A_p) & \cdots & s_{ih}(A_p) \\ \vdots & \vdots & & \vdots & & \vdots \\ s_{h1}(A_p) & s_{h2}(A_p) & \cdots & s_{hj}(A_p) & \cdots & s_{hh}(A_p) \end{bmatrix}, \qquad (17)$$

then, the total support of mi (Ap) by other evidence is:

$$\text{Sup}\left\{m_i\left(A_p\right)\right\} = \sum_{\substack{j=1 \\ j \neq i}}^{h} s_{ij}\left(A_p\right) \qquad (18)$$

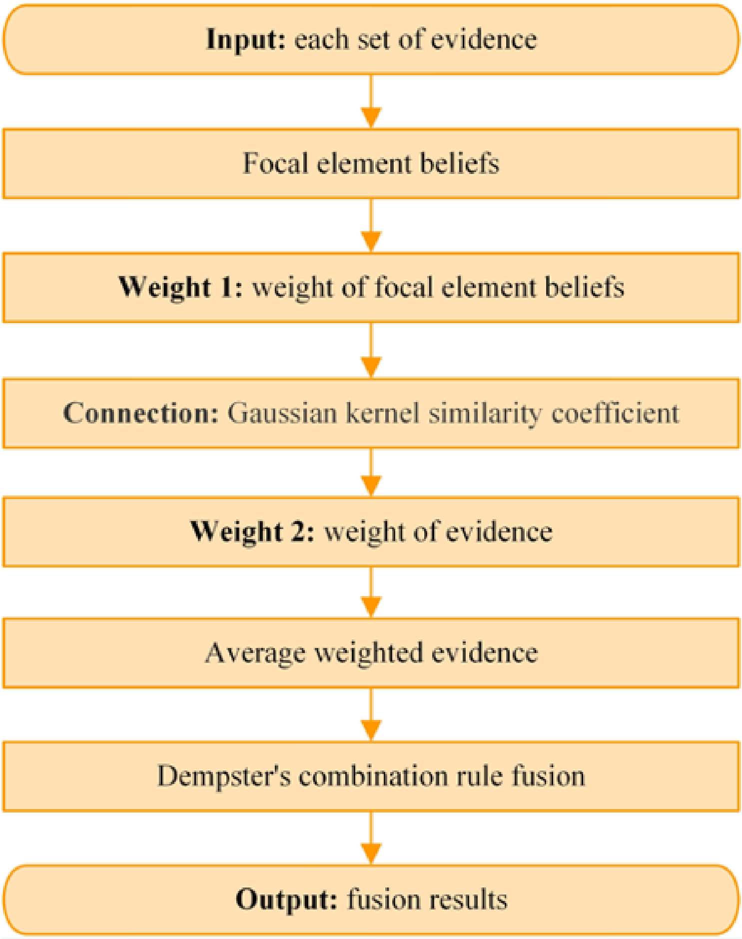

Figure 2. Flowchart of the proposed method.

If the total support of mi (A_p) is large, the importance of mi (A_p) is accordingly high [12]. Therefore, the weight wip of mi (A_p) can be defined as:

$$w_{ip} = w_i(A_p) = \frac{Sup\left\{m_i(A_p)\right\}}{\sum_{i=1}^{h} Sup\left\{m_i(A_p)\right\}}. \qquad (19)$$

Based on Eq. (19), it can be seen that,

$$\sum_{i=1}^{h} w_{ip} = 1. \qquad (20)$$

Similarly, the weights of other focal element beliefs can be obtained, and the weight matrix W is:

$$W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{h1} & w_{h2} & \cdots & w_{hn} \end{bmatrix}. \qquad (21)$$

The generation process of the weight of focal element beliefs is depicted in Figure 3.

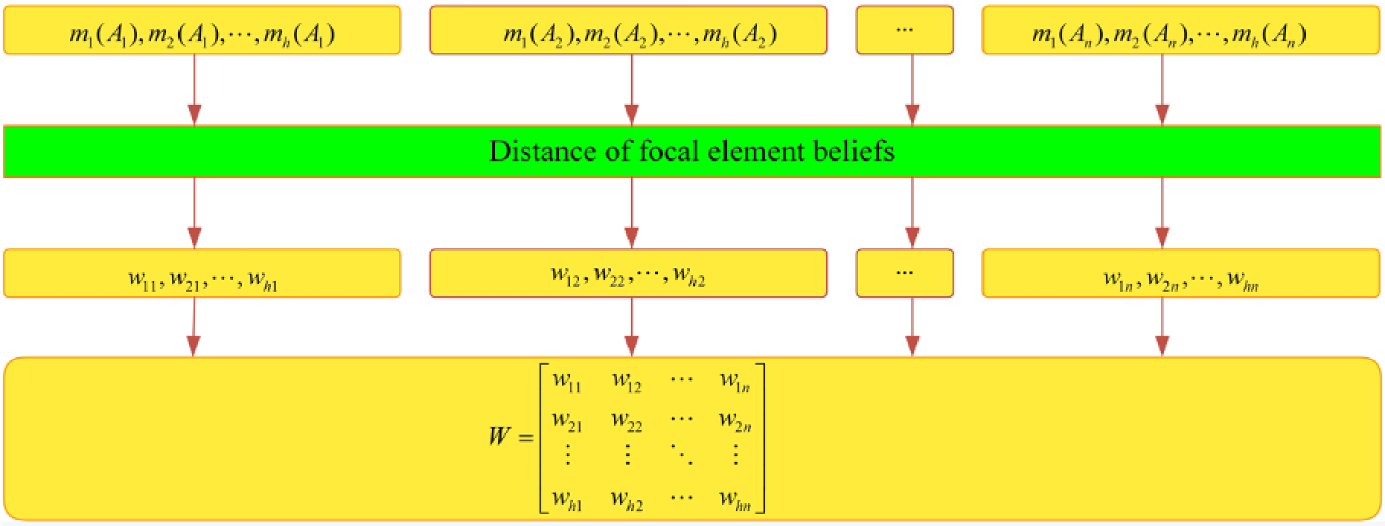

Figure 3. The generation process of the weight of the focal element beliefs.

Step 2: Calculate the similarity among the evidences according to the Gaussian kernel similarity coefficient.

According to the Eq. (14), the similarity kij between mi and mj can be obtained, and the similarity matrix K is:

$$K = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1j} & \cdots & k_{1h} \\ \vdots & \vdots & & \vdots & & \vdots \\ k_{i1} & k_{i2} & & k_{ij} & \cdots & k_{ih} \\ \vdots & \vdots & & \vdots & & \vdots \\ k_{h1} & k_{h2} & \cdots & k_{hj} & \cdots & k_{hh} \end{bmatrix}. \qquad (22)$$

Step 3: Calculate the weight of each group of evidence. (Weight 2)

The similarity of evidence can be regarded as support, and the total support of evidence mi by other evidences is:

$$\mathrm{Sup}\left(m_i\right) = \sum_{\substack{j=1 \\ j \neq i}}^{h} k_{ij} \qquad (23)$$

If the total support of mi is large, the importance of mi is accordingly high. Therefore, the weight $w(m_i)$ of mi is:

$$w(m_i) = \frac{Sup(m_i)}{\sum_{i=1}^{h} Sup(m_i)}. \qquad (24)$$

From Eq. (24), it can be seen that,

$$\sum_{i=1}^{h} w(m_i) = 1. \qquad (25)$$

The combination process of two weights is shown in Figure 4.

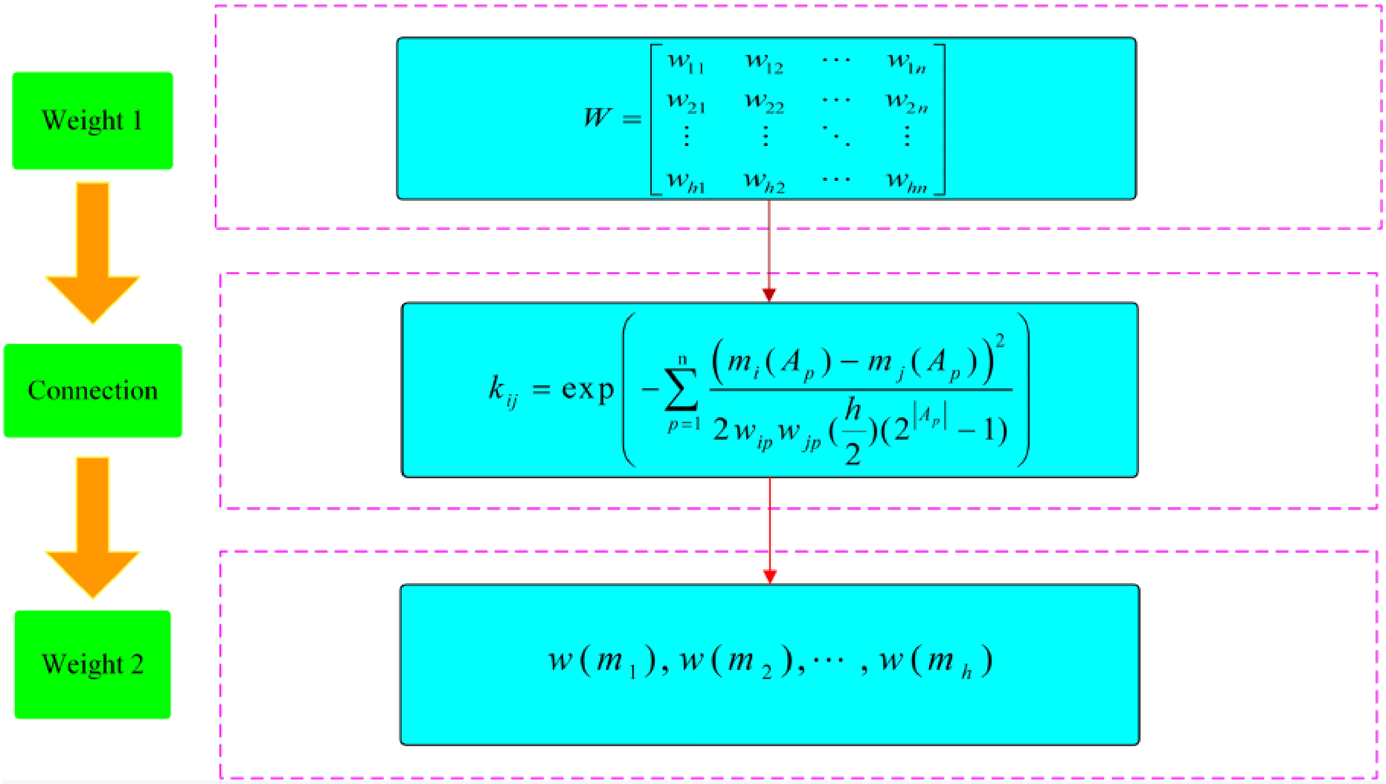

Figure 4. Combination process of two weights.

Step 4: Calculate the average weighted evidence mD using Eq. (26),

$$m_D = \sum_{i=1}^{h} w(m_i) \times m_i. \qquad (26)$$

Step 5: The average weighted evidence m_D is fused h-1 times by Dempster's combination rule, and the final fusion result is m_E,

$$m_E = \underbrace{m_D \oplus m_D \oplus \cdots \oplus m_D}_{h-1 \text{ times}}. \qquad (27)$$

The following two equations are a form of multiple evidence fusion of Dempster's combination rule. When h groups of average weighted evidence mD are fused, Eq. (28) is used. If the focal element of mD is a single subset, Eq. (29) should be used.

$$\begin{cases} m(A) = \dfrac{\sum_{\cap A_i = A} \prod_{i=1}^{m} m_i(A_i)}{\sum_{\cap A_i \neq \varnothing} \prod_{i=1}^{m} m_i(A_i)}, A \neq \varnothing, \\ m(\varnothing) = 0 \end{cases} \qquad (28)$$

$$\begin{cases} m(A_i)^* = \dfrac{m(A_i)^m}{\sum_{i=1}^{n} m(A_i)^m}, A_i \neq \varnothing. \\ m(\varnothing) = 0 \end{cases} \qquad (29)$$

5. Illustrative Examples

The evidence with low credibility can be identified in multi-evidence fusion, which makes the final fusion result conform to common sense. To demonstrate the effectiveness of the proposed method, two examples of multi-evidence fusion are given in this section, and the data are from references[12]and [29] respectively.

5.1. Information fusion of singletons

Example 1: Let the frame of discernment $\Theta = \{F_1, F_2, F_3\}$, and the BPAs of the five sets of evidences are as follows.

Table 1. BPAs of the five sets of evidences.

BPAs	F_1	F_2	F_3
m_1	0.5	0.2	0.3
m_2	0	0.9	0.1
m_3	0.55	0.1	0.35
m_4	0.55	0.1	0.35
m_5	0.55	0.1	0.35

Step 1: Calculate the weight of each focal element belief under each set of evidence.

Using the focal element F_1 as an example, the support matrix $S(F_1)$ of $m(F_1)$ under each set of evidence is generated by Eqs. (15) and (16) as:

$$S(F_1) = \begin{bmatrix} 1 & 0.501 & 0.9501 & 0.9501 & 0.9501 \\ 0.501 & 1 & 0.4511 & 0.4511 & 0.4511 \\ 0.9501 & 0.4511 & 1 & 1 & 1 \\ 0.9501 & 0.4511 & 1 & 1 & 1 \\ 0.9511 & 0.4511 & 1 & 1 & 1 \end{bmatrix}.$$

The weight of $m(F_1)$ calculated by Eqs. (18) and (19) is

$$w_1(F_1) = 0.21748, w_2(F_1) = 0.12033, w_3(F_1) = 0.22073$$
$$w_4(F_1) = 0.22073, w_5(F_1) = 0.22073$$

Similarly, the weights of $m(F_2)$ and $m(F_3)$ under each group of evidence can be obtained as follows

$$w_1(F_1) = 0.21748, w_1(F_2) = 0.22720, w_1(F_3) = 0.20504$$
$$w_2(F_1) = 0.12033, w_2(F_2) = 0.06858, w_2(F_3) = 0.17141,$$
$$w_3(F_1) = 0.22073, w_3(F_2) = 0.23474, w_3(F_3) = 0.20785$$
$$w_4(F_1) = 0.22073, w_4(F_2) = 0.23474, w_4(F_3) = 0.20785$$
$$w_5(F_1) = 0.22073, w_5(F_2) = 0.23474, w_5(F_3) = 0.20785.$$

Step 2: Calculate the similarity between the five sets of evidence by Eq. (14), and the similarity matrix K is obtained as

$$K = \begin{bmatrix} 1 & 0.00022 & 0.94225 & 0.94225 & 0.94225 \\ 0.00022 & 1 & 0.00003 & 0.00003 & 0.00003 \\ 0.94225 & 0.00003 & 1 & 1 & 1 \\ 0.94225 & 0.00003 & 1 & 1 & 1 \\ 0.94225 & 0.00003 & 1 & 1 & 1 \end{bmatrix}.$$

Step 3: Calculate the weight of five sets of evidence by Eqs. (23) and (24) as:

$$w(m_1) = 0.242572, w(m_2) = 0.000027, w(m_3) = 0.252467,$$
$$w(m_4) = 0.252467, w(m_5) = 0.252467.$$

Step 4: Calculate the average weighted evidence mD by Eq. (26) as:

$$m_D(F_1) = 0.5379, m_D(F_2) = 0.1243, m_D(F_3) = 0.3378.$$

Step 5: Calculate the final fusion result mE by Eq. (29) as:

$$m_E\left(F_1\right) = 0.9105,\ m_E\left(F_2\right) = 0.0006,\ m_E\left(F_3\right) = 0.0889.$$

The fusion results are compared with those by other methods as shown in Table 2, Table 3, and Figure 5.

Table 2. Comparison of fusion results.

Method	m_{12}	m_{123}	m_{1234}	m_{12345}
Dempster's rule	$m(F_1) = 0$ $m(F_2) = 0.8571$ $m(F_3) = 0.1429$	$m(F_1) = 0$ $m(F_2) = 0.6316$ $m(F_3) = 0.3684$	$m(F_1) = 0$ $m(F_2) = 0.3288$ $m(F_3) = 0.6712$	$m(F_1) = 0$ $m(F_2) = 0.1228$ $m(F_3) = 0.8772$
Murphy's method[11]	$m(F_1) = 0.1543$ $m(F_2) = 0.7469$ $m(F_3) = 0.0988$	$m(F_1) = 0.3500$ $m(F_2) = 0.5224$ $m(F_3) = 0.1276$	$m(F_1) = 0.6027$ $m(F_2) = 0.2676$ $m(F_3) = 0.1346$	$m(F_1) = 0.7958$ $m(F_2) = 0.0932$ $m(F_3) = 0.1110$
Deng's method[12]	$m(F_1) = 0.1543$ $m(F_2) = 0.7469$ $m(F_3) = 0.0988$	$m(F_1) = 0.5816$ $m(F_2) = 0.2439$ $m(F_3) = 0.1745$	$m(F_1) = 0.8060$ $m(F_2) = 0.0482$ $m(F_3) = 0.1458$	$m(F_1) = 0.8909$ $m(F_2) = 0.0086$ $m(F_3) = 0.1005$
Proposed method	$m(F_1) = 0.6907$ $m(F_2) = 0.0369$ $m(F_3) = 0.2724$	$m(F_1) = 0.7936$ $m(F_2) = 0.0098$ $m(F_3) = 0.1966$	$m(F_1) = 0.8633$ $m(F_2) = 0.0024$ $m(F_3) = 0.1343$	$m(F_1) = 0.9105$ $m(F_2) = 0.0006$ $m(F_3) = 0.0889$

Table 3. Comparison of the final fusion results

Method	$m\ (F_1)$	$m\ (F_2)$	$m\ (F_3)$
Dempster's rule	0	0.1228	0.8772
Murphy's method[11]	0.7958	0.0932	0.0110
Deng's method[12]	0.8909	0.0086	0.1005
Proposed method	0.9105	0.0006	0.0889

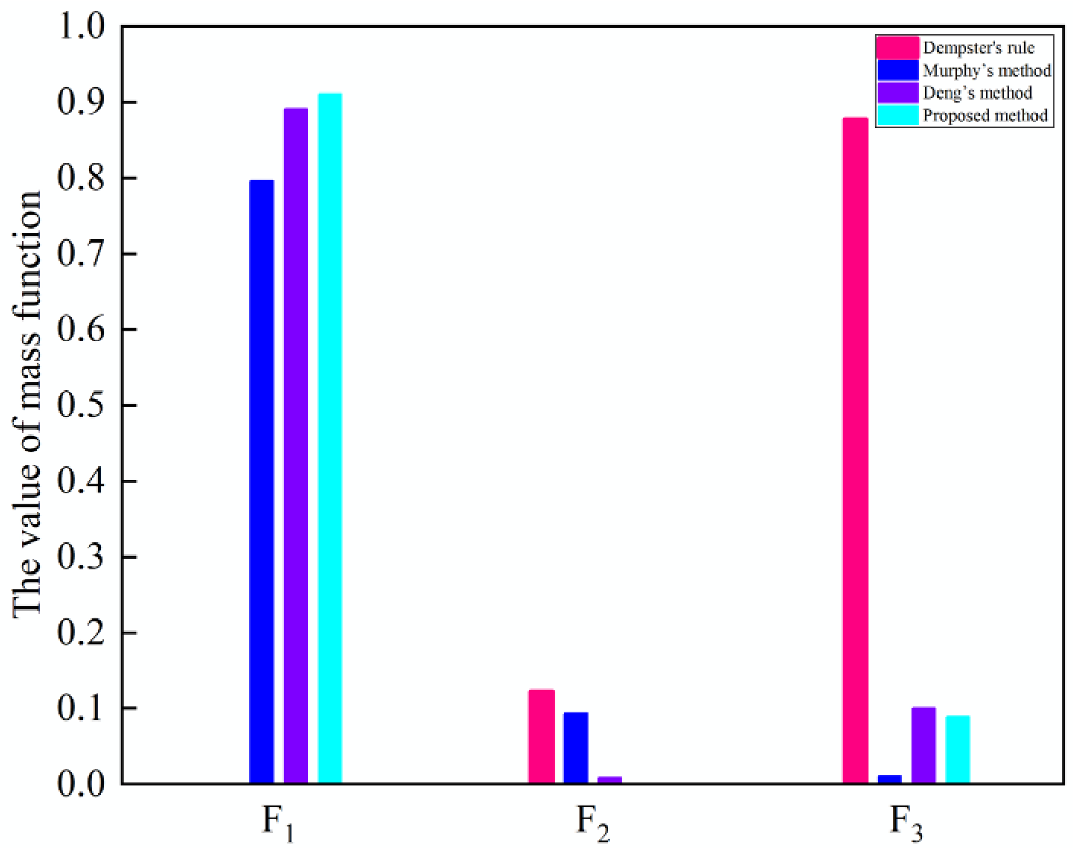

Figure 5. Combination results of different methods.

As can be seen from Table 2, Table 3, and Figure 5, when Dempster's combination rule is used, the problem of "one-vote rejection" appears, because $m_2(F_1)$ =0, and the rule cannot identify F_1, which is obviously against common sense. Murphy's method, Deng's method, and the proposed method can all make effective decisions. However, Murphy's method does not consider the dependency of evidences, and its convergence speed is relatively slow. Deng's method considers the dependency of evidences but does not consider the influence of weights of focal element beliefs, so the convergence speed is still relatively slow. The proposed method considers the dependency of evidences and the influence of weights of focal element beliefs, and weakens the weight of unreliable evidence, making it more effective in dealing with conflicting evidence. Therefore, the proposed method outperforms other combination rules for the single-element subset.

5.2. Information fusion of focal elements with multi-element subsets

Example 2: Let the framework of discernment Θ = {F1, F2, F3}, and the BPAs of the three sets of evidences are as follows. It can be seen that the second BPA is contradictory with the other two mass functions, which results in conflicting evidences.

Table 4. BPAs of the three sets of evidences.

BPAs	F_1	F_2	F_2, F_3	F_1, F_2, F_3
m_1	0.60	0.10	0.10	0.20
m_2	0.05	0.80	0.05	0.10
m_3	0.70	0.10	0.10	0.10

Step 1: Calculate the weight of m (F1), m (F2), m (F2, F3), and m (F1, F2, F3) under each set of evidence by Eqs. (15), (16), (18), and (19) as follows,

$$w_1\left(F_1\right) = 0.39683, w_1\left(F_2\right) = 0.40598, w_1\left(F_2, F_3\right) = 0.33620, w_1\left(F_1, F_2, F_3\right) = 0.32140$$
$$w_2\left(F_1\right) = 0.23564, w_2\left(F_2\right) = 0.18804, w_2\left(F_2, F_3\right) = 0.32760, w_2\left(F_1, F_2, F_3\right) = 0.33930$$
$$w_3\left(F_1\right) = 0.36753, w_3\left(F_2\right) = 0.40598, w_3\left(F_2, F_3\right) = 0.33620, w_3\left(F_1, F_2, F_3\right) = 0.33930$$

Step 2: Calculate the similarity between the three sets of evidence by Eq. (14), and the similarity matrix K is obtained as:

$$K = \begin{bmatrix} 1 & 0.03977 & 0.94497 \\ 0.03977 & 1 & 0.02309 \\ 0.94497 & 0.02309 & 1 \end{bmatrix}.$$

Step 3: Calculate the weight of three sets of evidence by Eqs. (23) and (24) as:

$$w\left(m_1\right) = 0.4885, w\left(m_2\right) = 0.0312, w\left(m_3\right) = 0.4803.$$

Step 4: Calculate the average weighted evidence mD by Eq. (26) as:

$$m_D\left(F_1\right) = 0.63087, \quad m_D\left(F_2\right) = 0.12184, \quad m_D\left(F_2, F_3\right) = 0.09844, m_D\left(F_1, F_2, F_3\right) = 0.14885$$

Step 5: Calculate the final fusion result mE by Eq. (29) as:

$$m_E\left(F_1\right) = 0.9367, \quad m_E\left(F_2\right) = 0.0405, \quad m_E\left(F_2, F_3\right) = 0.0147, m_E\left(F_1, F_2, F_3\right) = 0.0081.$$

Table 5. Comparison of the final fusion results.

Method	$m(F_1)$	$m(F_2)$	$m(F_2, F_3)$	$m(F_1, F_2, F_3)$
Dempster's rule	0.4519	0.5048	0.0336	0.0096
Fan and Zou's method[29]	0.8119	0.1096	0.0526	0.0259
Chen and Cai's method[21]	0.5928	0.3802	0.0210	0.0060
Xiao's method[30]	0.8973	0.0688	0.0254	0.0080
Proposed method	0.9367	0.0405	0.0147	0.0081

As can be seen from Table 5 and Figure 6, when Dempster's combination rule is used, the final fusion result is F_2, but the common-sense result is F_1, which shows that Dempster's combination rule is prone to a paradox when fusing conflicting evidences. Fan and Zou's method, Chen and Cai's method, Xiao's method, and the proposed method all correctly identified F_1. However, compared with their methods, the proposed method achieves the highest mass function on the focal element F_1, as it considers the weight of focal element beliefs that weakens the weight of unreliable evidences. Therefore, the proposed method also outperforms other methods for BPAs with multi-element subsets.

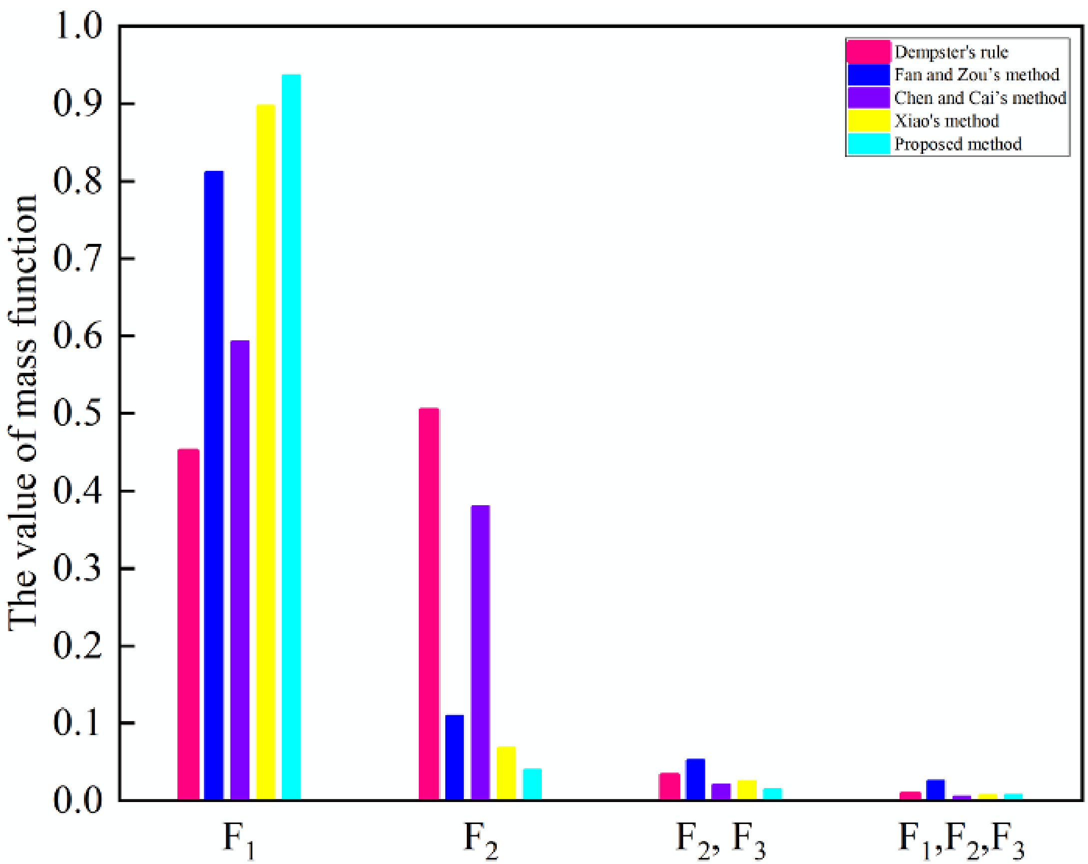

Figure 6. Combination results of different methods.

6. Conclusion

In this article, the shortcomings of the existing evidence similarity-based multisource information fusion methods are analyzed, and a Gaussian kernel similarity coefficient is proposed to measure the similarity between evidences from the focal element beliefs perspective. The Gaussian kernel similarity coefficient considered the weight of focal element beliefs, which quantified the contributions of focal element beliefs to the evidence similarity and makes the measurement of the similarity of evidence more reasonable. Based on the newly developed Gaussian kernel similarity coefficient, a fusion method of conflict evidence is designed, the core of this method is to use the Gaussian kernel similarity coefficient to combine two weights, i.e., the weight of focal element beliefs and the weight of evidences. Through two illustrative examples under single-element subset and multi-element subset, some conclusions can be drawn that the proposed method is more effective in dealing with conflicting evidence than the existing methods. In addition, it is a good choice to use high-dimensional tools in evidence theory, because the focal elements are a high-dimensional relationship. Therefore, further research can be done in the future.

CRediT authorship contribution statement

Ruishi Yang: Methodology, Writing - original draft, Writing - review & editing. Haibin Li: Validation, Writing - review & editing, Funding acquisition. Hongzhong Huang: Writing - review & editing.

Declaration of competing interest

The author states that there are no conflicts of interest.

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