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Remarks on "Perov Fixed-Point Results on F-contraction Mappings Equipped with Binary Relation"

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Abstract: Since 1964, when I.A. Perov introduced the so-called generalized metric space where $d(x,y)$ is an element of the vector space $\mathbb{R}^m$. Since then, many researchers have considered various contractive conditions on this type of spaces. In this paper, we generalize, extend and unify some of those established results. It is primarily about examining the existence of a fixed point of some mapping from X to itself, but if (x,y) belong to some relation R on the set X. Then the binary relation R and some F contraction defined on the space cone $\mathbb{R}^m$ are combined. We start our consideration on the paper [6] and give strict critical remarks on the results published in there. Also, we improve their result by weakening one condition.

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1 Introduction and Preliminaries

In **1906**, the French mathematician Maurice Fréchet, for the first time in the history of mathematics, introduced the abstract measurement of the distance between two points in an arbitrary non-empty set X. In order for that distance to correspond to the ordinary distance that a person understands intuitively, knowing it from Euclidean geometry, M. Fréchet introduced it axiomatically on the following way.

Let X be a given non-empty set and d a mapping (function) defined on the Cartesian product $X^2 = X \times X$ with values in the set $[0, +\infty)$ of non-negative real numbers that satisfies the following axioms for each x, y, z from X:

d1) $d(x,y) = 0$ if and only if $x = y$;

d2) $d(x,y) = d(y,x)$;

d3) $d(x,z) \leq d(x,y) + d(y,z)$.

Then the pair (X,d) is called a metric space and the mapping d a distance or metric on a non-empty set X.

Since S. Banach discovered his famous theorem in **1922** about the uniqueness of the fixed point of every contraction f defined on the complete metric space (X,d), numerous mathematicians tried to generalize his result. These generalizations basically went into the following two directions:

1. Some of the three above possible metric space axioms are broken.

2. The right side of the Banach condition

$$d(fx,fy) \leq k \cdot d(x,y),\ k \in [0,1)$$

was replaced by new expressions such as, for example, $ad(x,y)+bd(x,fx)+cd(y,fy)$, where a,b,c are non-negative real numbers such that $a+b+c<1$. Or for example with $\varphi(d(x,y))$,where $\varphi:[0,+\infty)\to[0,+\infty)$ and has some additional property.

In case 1, several so-called general metric spaces have been considered such as: *partial metric spaces*, *metric like spaces*, *b-metric spaces*, *partial b-metric spaces* and *b-metric like spaces* (total of 6 new different types of spaces including metric spaces). For more details, see: ([1]-[4]).

In the second direction of generalization, new contractive conditions were obtained, as is well-known in the literature: Kannan, Chatterjea, Zamfirescu, Hardy-Rogers, Ćirić, Boyd-Wong, Meir-Keeler, and others. For details see [19]. In all 6 spaces mentioned above, the mapping d is from $X^2=X\times X$ to $[0,+\infty)$, where X is different from the empty set.

In **1933**, the Serbian mathematician Đ.Kurepa instead of $[0,+\infty)$ considered the vector space V with the cone P and defined the so-called cone metric spaces. So, he considered the mapping d from $X^2=X\times X$ to the cone P of the real vector space V, i.e., $d(x,y)$ is in the new situation a vector and not a non-negative real number (for more details see ([1], [8], [11], [12]).

After Kurepa's introduction of cone metric spaces in 1933, Perov **1964** defined one of their special types (see [14]). Namely, instead of vector space V with cone P, he takes the special case $V=\mathbb{R}^n$ with cone $P=\{(x_1,x_2,,,x_n):x_i\geq 0$ for all $i=1,2,...,n\}$. Then $d(x,y)=(x_1,x_2,...,x_n)\in P$. To move on to work with matrices, Perov takes $d(x,y)=(x_1,x_2,...,x_n)^T$ i.e., the matrix of type $n\times 1$. From [10] we know that the generalized metric d, i.e., valued cone metric d can be represented as a column matrix of pseudo-metric $p_i, i=1,2,...,n$.

Therefore, $d(x,y)=(p_1(x,y),p_2(x,y),...,p_n(x,y))^T$ and at least one of the pseudo-metric p_i is a real metric.

Let us now write the contractive condition that Perov stated as a generalization of the Banach condition in his famous result from 1922. Let M be the given square matrix of order n and Γ the mapping from X to X where (X,d) is the given generalized metric proctor, i.e., valued metric space. If there is a square matrix M that converges to 0 such that $d(\Gamma x,\Gamma y)\leq M\cdot d(x,y)$ for every x,y in X then Γ has a fixed point in X. Here $d(x,y)$ and $d(\Gamma x,\Gamma y)$ are columns that is, the matrix of type $n\times 1$. If $d(x,y)$ is a row, i.e. ordered n-tuple then the previous condition has the notation $d(\Gamma x,\Gamma y)\leq d(x,y)\cdot M^T$.

Otherwise, throughout this manuscript, we represent as in [6] $\mathbb{N},\mathbb{Z},\mathbb{R},\mathbb{R}_{\geq 0},\mathbb{R}_t$, and $\mathbb{R}_{t>k}$ the set of all natural numbers, all integers, real numbers, non-negative real numbers, real matrices of the order $t\times 1$, and real matrices of the order $t\times 1$ with entries greater than k, respectively.

We can freely say that set, relation, function and operation are the four basic pillars of mathematics. We take the set as the basic concept. The operation is a special function from $X \times X$ to X while the function is a special binary relation on X. The binary relation R in the set X means any subset of $X \times X$. All this is previously well-known from the school course of study. Here we will state the basic terms from binary relations on a given non-empty set X if there is also a mapping T from X to itself.

Now we will state the basic properties of an arbitrary relation R considered on a non-empty set X. Also, if X is provided with some metric or vector metric and if Γ is a mapping from X to itself, we will state the properties that we will need to prove the result in the rest of the paper.

Let X be a non-empty set. Then, the Cartesian product on X is defined as follows:

$$X^2 = X \times X = \{(a, b) : a, b \in X\}.$$

All subsets of X^2 are known as the binary relations on X.

Let R be any subset of X^2. Then, notice that for each pair $a, b \in X$, there are two possibilities: either $(a, b) \in R$ or $(a, b) \notin R$. In the first case, we mean that a relates to b under R. For $(a, b) \notin R$, we mean that a does not relate to b under R. For all other things on the binary relations see ([6], Definitions 3., 4., 5., 6., 7., 8., 9., 10. as well as Propositions 2., and 3., Lemma 1., Example 3.). See also ([3], Definitions 2.7., 2.8., 2.9., 2.10, 2.13., 2.14, 2.16., 2.17., 2.18., Propositions 2.11., 2.12., Example 2.15.).

The following two lemmas are quite useful, and they can be used in proofs that the introduced Picard sequence $x_n = Tx_{n-1}$ is a Cauchy one where T is the mapping of the metric space (X, d) into itself where x_0 is a given point.

Lemma 1 *Let (X, d) be a metric space and $\{x_n\}$ be a Picard sequence in it. If*

$$d(x_{n+1}, x_n) < d(x_n, x_{n-1})$$

for all $n \in \mathbb{N}$ then $x_n \neq x_m$ whenever $n \neq m$.

Lemma 2 *Let x_n be a sequence in metric space (X, d) such that $\lim_{n\to+\infty} d(x_n, x_{n+1}) = 0$. If x_n is not a Cauchy sequence in (X, d), as a result, there exist two sequences $\{n_k\}$ and $\{m_k\}$ of natural numbers such that $n_k > m_k > k$ and the sequences*

$$d(x_{n_k}, x_{m_k}), d(x_{n_k+1}, x_{m_k}), d(x_{n_k}, x_{m_k-1}),$$
$$d(x_{n_k+1}, x_{m_k-1}), d(x_{n_k+1}, x_{m_k+1}), \ldots$$

tend to ε^+, as $k \to +\infty$, for some $\varepsilon > 0$.

With the mentioned contractive conditions of a mapping Γ from X to X, it is assumed that they are present for each x, y from X, or for each pair (x, y) from the Cartesian product $X \times X$. One possible weakening of the condition of such results could be, among others, that the contractive condition is fulfilled for all pairs (x, y) belonging to some subset of $X \times X$, or putting in terms of a binary relation defined on a set X, if (x, y) belongs to a given binary relation R on a set X. For details see [3], [6].

Wardowski [22] introduce the notion of F contraction and defined F contraction as follows:

Definition 1 *Let $F : \mathbb{R}_{>0} \to \mathbb{R}$ be a mapping satisfying the following properties:*

F1: F is strictly increasing; that is, for all $a, b \in \mathbb{R}_{>0}$, we have

$$a < b \text{ implies } F(a) < F(b).$$

F2: For each sequence a_n of $\mathbb{R}_{>0}$, we have

$$\lim_{n\to+\infty} a_n = 0 \text{ if and only if } \lim_{n\to+\infty} F(a_n) = -\infty.$$

F3: There exists $\lambda \in (0,1)$ such that $\lim_{r\to 0^+} r^{\lambda} F(r) = 0$.

The set of all functions F satisfying F1-F3 is denoted as $\mathcal{F}$.

I. Altun and M. Olgun, in [4] used the concept of an F contraction in a vector valued metric space in the following way:

Definition 2 *Let $F : \mathbb{R}_{t>0} \to \mathbb{R}_t$ be a function which satisfies the following conditions:*

F1: F is strictly increasing, that is, for all $a = (a_i)_{i=1}^{t}, b = (b_i)_{i=1}^{t} \in \mathbb{R}_{t>0}$, where

$$a < b \text{ implies } F(a) < F(b),$$

F2: For each sequence $\{a_n\} = \left(a_1^{(n)}, a_2^{(n)}, ..., a_t^{(n)}\right)$ of $\mathbb{R}_{t>0}$, we have

$$\lim_{n\to+\infty} a_i^{(n)} = 0 \text{ if and only if } \lim_{n\to+\infty} b_i^{(n)} = -\infty$$

for every $i = 1, 2, ..., t$, where $F\left[\left(a_1^{(n)}, a_2^{(n)}, ..., a_t^{(n)}\right)\right] = \left(b_1^{(n)}, b_2^{(n)}, ..., b_t^{(n)}\right)$.

F3: There exists $\lambda \in (0,1)$ such that $\lim_{a_i\to} a_i^{\lambda} b_i = 0$, for all $i = 1, 2, ..., t$, where $F[(a_1, a_2, ..., a_t)] = (b_1, b_2, ..., b_t)$.

Here, $\mathbb{R}_{t>0}$ is the set of all $t \times 1$ real matrices with positive entries. Then, the set of all functions F satisfying F1-F3 is denoted as $\mathcal{F}^t$.

2 Main results

In this part of the paper, we will give strict critical remarks on the results published in [6]. Namely, we will first show that Theorems 4 and 5 from [6] are equivalent. To demonstrate this, we will first consider the mapping of F from $(0, +\infty)^m$ to $(-\infty, +\infty.)^m$. We used it similarly in recent work [12]. We notice that the mapping F can be specified by its coordinates $F_1, ..., F_m$ which are actually mappings from $(0, +\infty)$ to $(-\infty, +\infty)$.

Now, we recall both Definitions 15 and 16 as well as both Theorems 4 and 5 from [6].

Definition 3 *[6] Let* (X,σ) *be a vector-valued metric space and* Γ *be a self-mapping on* X*. If there exist* $F\in\mathcal{F}^t$ *and* $\xi=\left(\xi^{(i)}\right)_{i=1}^{t}\in\mathbb{R}_{t>0}$ *such that*

$$\xi+F\left[\sigma\left(\Gamma p,\Gamma q\right)\right]\leq F\left[\sigma\left(p,q\right)\right], \text{ for all } p,q\in X,\sigma\left(\Gamma p,\Gamma q\right)>0,$$

then Γ *is called a Perov-type* F *contraction.*

Definition 4 *[6] Let* (X,σ) *be a vector-valued metric space equipped with a binary relation* R*. Then, a self-mapping* Γ *on* X*, is called a theoretic-order Perov-type* F *contraction if there exist* $\xi=\left(\xi^{(i)}\right)_{i=1}^{t}\in\mathbb{R}_{t>0}$ *and* $F\in\mathcal{F}^t$*such that*

$$\xi+F\left[\sigma\left(\Gamma p,\Gamma q\right)\right]\leq F\left[\sigma\left(p,q\right)\right], \text{ for all } (p,q)\in R \text{ with } \sigma\left(\Gamma p,\Gamma q\right)>0.$$

Theorem 3 *[6] Let* (X,d) *be a complete metric space equipped with a binary relation* R *and* Γ *be a self-mapping. Suppose the following:*

The pair $(R:\Gamma)$ *is a compound structure (Definition 17 of [6]);*
For all $(p,q)\in R$ *with* $d\left(\Gamma p,\Gamma q\right)>0$ *such that*

$$\xi+F\left(d\left(\Gamma p,\Gamma q\right)\right)\leq F\left(d\left(p,q\right)\right)$$

where $\xi>0$ *and* $F\in\mathcal{F}$*, then* Γ *has a fixed point;*

Furthermore, if $C_R\left(p,q\right)\neq\emptyset$*, (Definition 10 of [6]) for all* $p,q\in X$*,then* Γ *has a unique fixed point.*

Theorem 4 *[6] Let* (X,σ) *be a complete vector-valued metric space equipped with a binary relation* R *and* Γ *be a theoretic-order Perov-type contraction such that the pair* $(R:\Gamma)$ *is a compound structure. Then* Γ *has a fixed point. Moreover,* Γ *has a unique fixed point if* $C_R\left(p,q\right)\neq\emptyset$*, for all* $p,q\in X$*.*

Before giving more significant results in this paper, we first state two very important remarks on the proof of Theorem 4 from [6]. Namely, the proof that the defined Picard sequence is a Cauchy one as well as the uniqueness of a possible fixed point are clearly wrong. Namely, the authors use the fact that convergent order sum is equal to zero, which is incorrect. See pages 9 and 10 in [6]. The error in the Cauchyness proof is easy to fix. First of all, it follows from the obtained condition that $\sigma\left(x_n,x_m\right)\leq\sum\limits_{j=n}^{m-1}\sigma\left(c_j,c_{j+1}\right)\to 0$ as $n\to+\infty$ because the series $\sum\limits_{j=1}^{+\infty}\frac{1}{j^{\frac{1}{\lambda}}}$ is convergent and is equal to $\zeta(\frac{1}{\lambda})$, which is larger than 1 for $0<\lambda<1$. Similarly, the error in the proof of the uniqueness of a fixed point is eliminated.

Now we will show that actually Theorem 5 from [6] is not new, but is equivalent to Theorem 4 also from [6]. We have previously had that many fixed point results in the framework of cone metric spaces are equivalent to the corresponding ones in ordinary metric spaces (see [2]). It is obvious that from Theorem

5 follows Theorem 4. Since every generalized metric d i.e., valued metric d is given by its coordinates $p_i, i = 1, 2, ..., n$ and that at least one pseudo-metric p_i is a real metric. From there, we have the corresponding contractive condition in the metric space (X, p_i) and this gives us the result according to Theorem 4 (for more details, see paper [12]).

Since we proved that Theorems 4 and 5 from [6] are equivalent but if the function F satisfies all three properties in both scalar and vector form. Let us also note that when we consider the general metric space, i.e., valued metric space, we can only go up to $n = 2$, which is essentially the same if $n > 2$. It is only about technical things and complicated writing at first glance. For details see [12]. If we add to the relation R on the set X the property of its transitivity, then it can be proved that Theorems 4 and 5 (of course in the new formulation) are equivalent if the mapping F satisfies only the property of strict growth.

In the continuation of the work, the function F will be strictly increasing, i.e., for every $a = (a_i)_{i=1}^{m}, b = (b_i)_{i=1}^{m} \in \mathbb{R}^m$, whenever $a < b$ then $F(a) < F(b)$. In order not to use the properties F2 and F3 as in [6] we will assume that the relation R given on the non-empty set X is transitive.

Let us formulate and prove the following two results:

Theorem 5 *Let (X, d) be a complete metric space equipped with a transitive binary relation R and Γ be a self-mapping. Suppose the following:*

The pair (R, Γ) is a compound structure;
For all $(p, q) \in R$ with $d(\Gamma p, \Gamma q) > 0$ such that

$$\zeta + F(d(\Gamma p, \Gamma q)) \leq F(d(p, q))$$

where $\zeta \in \mathbb{R}_{>0}$ and $F \in \mathcal{F}$, then Γ has a fixed point;

Furthermore, if $C_R(p, q) \neq \emptyset$, for all $p, q \in X$, then Γ has a unique fixed point.

Proof. In our approach to the proof of the formulated theorem, the function F participates only by its strict growth. Instead of the properties F2. and F3. of the function F, we assumed the transitivity of the relation R. In this way, we have a hybrid correction of Theorem 4 from [6]. The transitivity of the relation R and the strict growth of the function F allows us to prove the Cauchyness of the constructed Picard sequence by applying Lemmas 1. and 2. in their proof of Theorem 4. Due to the assumption of a compound structure (R, Γ), one can construct the Picard sequence $\{c_n\}$ as

$$c_0, c_1 = \Gamma c_0, \ldots, c_n = \Gamma c_{n-1} = \Gamma^n c_0, \ldots$$

where c_0 is the starting point that exists. For the obtained sequence, we have that the adjacent members are in the relation R, but due to the assumption of transitivity, every two members of it are in the relation R. This, first with contractive conditions, gives that the series $d(c_n, c_{n+1})$ is decreasing and due to the property of the strictly increasing function F, it is obtained that it tends to zero as $n \to +\infty$. Now if the sequence $\{c_n\}$ is not Cauchy using Lemma 2 and taking $p = x_{n_k}, q = x_{m_k}$ we get that it follows

$$\zeta + F\left(d\left(\Gamma x_{n_k}, \Gamma x_{m_k}\right)\right) \leq F\left(d\left(x_{n_k}, x_{m_k}\right)\right).$$

Again according to the important property of the strictly increasing function F we get

$$\zeta + F\left(\varepsilon^{+}\right) \leq F\left(\varepsilon^{+}\right)$$

which is a contradiction with $\zeta > 0$. That the mapping Γ has a fixed point and that with an additional condition it is unique is shown as in paper [6]. □

Remark 1 *By adding transitivity to a given relation R, the previous Theorem is a significant generalization of Theorem 4. from [6]. Namely, under the new assumption in the proof we do not need the properties F2. and F3. which many authors still use ignoring our recent results published for example in [7] (also see: [8], [9], [12], [13], [17], [18], [20], [21], [23] and [24]).*

Theorem 6 *Let (X, σ) be a complete vector-valued metric space equipped with a transitive binary relation R and Γ be a theoretic-order Perov-type F contraction such that the pair (R, Γ) is a compound structure. Then Γ has a fixed point.*

Moreover, Γ has a unique fixed point if $C_R(p,q) \neq \emptyset$, for all $p, q \in X$.

Proof. We see that the proof of this Theorem is almost identical to the previous one. namely, if $m = 1$, we have the previous Theorem. And if $m > 1$ then using J. Jachymski, J. Klima famous result [10] we have to give the contractive condition becomes

$$\zeta_i + F_i\left(p_i\left(\Gamma x_{n_k}, \Gamma x_{m_k}\right)\right) \leq F_i\left(p_i\left(x_{n_k}, x_{m_k}\right)\right),\ i = 1, 2, ..., m.$$

As also according to J. Jachymski, J. Klima at least one of pseudo-metric say p_{i_0} is usual metric then based on the previous Theorem applied to the metric space (X, p_{i_0}) we have a result, i.e., a proof of the assertion of the formulated Theorem. □

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