Commentary

On the Potential Influence of Radio Waves on the ALPHA-g Experiment

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In the ALPHA-g experiment at CERN, it was found that antihydrogen atoms escaping from a magnetic Penning trap drifted primarily downward. As antihydrogen atoms are electrically neutral, the research team attributed this preference in drift direction to the gravitational force of the Earth. This explanation, in turn, led to the conclusion that matter attracts antimatter. However, it is unclear whether the ALPHA-g experiment team considered the potential for interactions between electrically neutral particles and electromagnetic fields in the radio frequency range, a phenomenon that could warrant further exploration. Unfortunately, the direction of action of these ponderomotive forces is identical for a hydrogen atom and an antihydrogen atom. This article derives the formula of the ponderomotive force from the basic equations of classical physics. Subsequently, it is shown that in unfavorable cases, even comparatively weak radio waves of suitable frequency, such as those generated by electronic devices, could lead to forces that reach or exceed the force of gravity.

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I. Introduction

The ALPHA-g experiment performed at the European Organization for Nuclear Research (CERN) was designed to investigate whether gravity influences matter and antimatter in the same way[1]. The basic aim of the experiment was to release antihydrogen atoms from a trap working with static magnetic fields and to then determine whether the atoms drift upward or downward. In the measurements, the researchers observed that approximately 80% of the antihydrogen atoms escaped downward from the trap, similar to ordinary hydrogen. Thus, the team concluded that Earth's gravitational field attracts antihydrogen atoms and hydrogen atoms in the same way. This article points out, however, that another possible force has not been considered as an alternative explanation.

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Physicists often believe that electrically neutral particles interact only with gravitational fields or, if they have a magnetic moment as in the case of antihydrogen, with static magnetic fields. However, it is less well known that, under certain circumstances, electromagnetic waves can generate a non-negligible force on an electrically neutral particle. Certain conditions must be met for this force to arise. One prerequisite is that the electrically neutral particle must contain components that are electrically charged. With antihydrogen, this requirement is met because the positron has a positive elementary charge, whereas the antiproton has a negative elementary charge.

If such an outwardly electrically neutral particle is in a static electric field, an external force acts on the particle, pulling the two differently charged components of the particle apart, while the internal forces of the particle work against this action. These forces cause the particle to become an electric dipole. For example, hydrogen can be polarized in an electrostatic field, as proven by the Stark effect discovered in 1913[2]. If the external force is sufficiently strong, a hydrogen atom can potentially even be ionized[3].

In harmonically oscillating electric fields, the direction of the electric force at the location of the particle is constantly changing. At very low frequencies, the situation shows little difference from that of an electrostatic field, and the particle is always polarized in such a way that the effect of the external field is weakened. However, a unique frequency exists at which the external and internal forces act synchronously. In classical physics, this phenomenon is called resonance, and the associated frequency is denoted the resonance frequency. A well-known resonance frequency of hydrogen[4] occurs at approximately $f_e = 1.42\,\mathrm{GHz}$, falling within a frequency range that is used intensively for technological purposes.

If the hydrogen atom were a classical system, an alternating electromagnetic field would pull the particle further and further apart with each reversal of polarity and finally tear it apart into its components. In fact, experiments show that a hydrogen atom can be ionized not only by individual photons but also by means of microwave radiation[5][6]. Because the energy of a single photon in the microwave and radio wave range is not sufficient for ionization, theoretical physics explains these effects via multiphoton absorption or tunnel ionization[7].

In theory, alternating electromagnetic fields with frequencies close to the resonance frequency should also be able to accelerate individual atoms if the fields are spatially inhomogeneous. Many electromagnetic transmitters in the gigahertz range satisfy this condition, as they are usually highly localized sources. Examples include clocked microprocessors, switched-mode power supplies, cell phones, Bluetooth transmitters, and routers for wireless local area network (WLAN).

The significance of the spatial inhomogeneity of the electric field strength is that the sum of the two forces on the two components of a polarized particle in a spatially inhomogeneous field is no longer zero. Thus, a small net force is produced in the temporal average, which accelerates the particle despite its electrical neutrality. This force is referred to as the ponderomotive force[8]. Interestingly, this force does not differ between hydrogen atoms and antihydrogen atoms. In most cases, the ponderomotive force is very small for electrically neutral particles. However, this is not always the case, especially if the frequency of the external field is close to the resonance frequency of the particle.

The present article presents theoretical calculations suggesting that under specific circumstances, ponderomotive forces could be non-negligible. While these forces may be effectively shielded in the ALPHA-g experiment, it is worthwhile to assess their potential role as a supplementary consideration.

II. Ponderomotive force

In the following, formulas for the ponderomotive force for electrically charged and electrically neutral particles are derived from the general basic equations of classical physics. The intention of this section is to present the necessary background as compactly as possible.

We start by deriving the ponderomotive force for electrically charged particles, which has some interesting technical applications, such as the free-electron laser[9] and the Paul trap[10]. For this derivation, we assume a spatially inhomogeneous electric field of the form

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_r(\mathbf{r})\,\cos(\omega\,t) \tag{1}$$

with a point charge q in this field. $\mathbf{E}_r(\mathbf{r})$ is taken to be a function that does not depend on the time t but only on the location $\mathbf{r}$. Therefore, the time-dependent component is only present in the oscillation $\cos(\omega\,t)$, where ω is the angular frequency.

The equation of motion of the point charge q with mass m is then

$$m\,\ddot{\mathbf{r}}_q = q\,\mathbf{E}_r(\mathbf{r}_q)\,\cos(\omega\,t). \tag{2}$$

The trajectory

$$\mathbf{r}_q = \mathbf{r}_d + \mathbf{r}_o \tag{3}$$

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of the point charge q consists of a rapidly oscillating part $\mathbf{r}_o$ and a slowly drifting part $\mathbf{r}_d$. If the angular frequency ω is high, the oscillation amplitude $\|\mathbf{r}_o\|$ must be small. Therefore,

$$\mathbf{E}_r(\mathbf{r}_q) \approx \mathbf{E}_r(\mathbf{r}_d) + \nabla \otimes \mathbf{E}_r(\mathbf{r}_d) \cdot \mathbf{r}_o \tag{4}$$

applies approximately, where

$$\nabla \otimes \mathbf{E} := \begin{pmatrix} \frac{\partial}{\partial r_x} E_x & \frac{\partial}{\partial r_y} E_x & \frac{\partial}{\partial r_z} E_x \\ \frac{\partial}{\partial r_x} E_y & \frac{\partial}{\partial r_y} E_y & \frac{\partial}{\partial r_z} E_y \\ \frac{\partial}{\partial r_x} E_z & \frac{\partial}{\partial r_y} E_z & \frac{\partial}{\partial r_z} E_z \end{pmatrix} \tag{5}$$

represents the Jacobian matrix. By inserting Equation (3) and the approximation from Equation (4) into the equation of motion (Equation (2)), we obtain

$$m\,(\ddot{\mathbf{r}}_d + \ddot{\mathbf{r}}_o) \approx q\,(\mathbf{E}_r(\mathbf{r}_d) + \nabla \otimes \mathbf{E}_r(\mathbf{r}_d) \cdot \mathbf{r}_o)\,\cos(\omega\,t). \tag{6}$$

In the next step, we can exploit the fact that the acceleration $\ddot{\mathbf{r}}_d$, which leads to the drift motion, is much smaller than the acceleration $\ddot{\mathbf{r}}_o$. This fact and the small oscillation amplitude of $\mathbf{r}_o$ simplify the equation of motion (Equation (6)) to

$$m\,\ddot{\mathbf{r}}_o \approx q\,\mathbf{E}_r(\mathbf{r}_d)\,\cos(\omega\,t). \tag{7}$$

Because $\mathbf{r}_d$ only changes very slowly compared with $\mathbf{r}_o$, $\mathbf{E}_r(\mathbf{r}_d)$ is essentially a constant in the time period under consideration, which allows the differential equation to be solved, leading to

$$\mathbf{r}_o \approx -\frac{q}{m\,w^2}\,\mathbf{E}_r(\mathbf{r}_d)\,\cos(\omega\,t). \tag{8}$$

By inserting this expression into Equation (6), we obtain

$$\ddot{\mathbf{r}}_d \approx -\frac{q^2}{m^2\,\omega^2} \nabla \otimes \mathbf{E}_r(\mathbf{r}_d) \cdot \mathbf{E}_r(\mathbf{r}_d)\,\cos(\omega\,t)^2 \tag{9}$$

after some rearrangement.

Equation (9) describes a force, as can be realized by multiplying both sides by mass m. Furthermore, the force at the location $\mathbf{r}_d$ always points in the same direction. Although the force oscillates, the term $\cos(\omega\,t)^2$ is always positive and thus cannot influence the direction of the force. Consequently, a resultant force remains in the temporal mean. This force can be obtained by calculating the time average of $\cos(\omega\,t)^2$. By means of

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \cos(\omega\,t)^2\,\mathrm{d}t = \frac{1}{2}, \tag{10}$$

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we find the approximation

$$\ddot{\mathbf{r}}_d \approx -\frac{q^2}{2\,m^2\,\omega^2}\nabla \otimes \mathbf{E}_r(\mathbf{r}_d) \cdot \mathbf{E}_r(\mathbf{r}_d). \tag{11}$$

If we multiply both sides by the mass m, we finally obtain a force $\mathbf{F}_p$, which is referred to as the ponderomotive force. Because all dependencies on $\mathbf{r}_o$ have now disappeared, the parameter in $\mathbf{E}_r(\mathbf{r}_d)$ can also be omitted, and the formula can be written in a compact form as

$$\mathbf{F}_p = -\frac{q^2}{2\,m\,\omega^2}\nabla \otimes \mathbf{E}_r \cdot \mathbf{E}_r. \tag{12}$$

For the case in which $\mathbf{E}_r$ can be expressed as the gradient $\mathbf{E}_r = \nabla\varphi_r$ of a scalar potential φ_r ($\nabla \times \mathbf{E}_r = 0$), we obtain

$$\nabla \otimes \mathbf{E}_r \cdot \mathbf{E}_r = \frac{1}{2}\nabla\left(\mathbf{E}_r \cdot \mathbf{E}_r\right) = \frac{1}{2}\,\nabla E_r^2 \tag{13}$$

because of

$$\left(\nabla \otimes \nabla\varphi_r\right) \cdot \nabla\varphi_r = \frac{1}{2}\nabla\left(\nabla\varphi_r \cdot \nabla\varphi_r\right). \tag{14}$$

This relation simplifies Equation (12) even further, and we obtain

$$\mathbf{F}_p = -\frac{q^2}{4\,m\,\omega^2}\nabla E_r^2. \tag{15}$$

Equation (15) gives the formula for the ponderomotive force on an electrically charged particle. However, a ponderomotive force can also act on an electrically neutral particle under certain conditions. As far as the author is informed, there are no practical applications for the ponderomotive force on electrically neutral particles. This may explain why the ALPHA-g experiment team did not account for this force in their analyses.

To derive the formula of the ponderomotive force for an electrically neutral particle, a bound particle is considered, which consists of a negative charge $-q$ and a positive charge $+q$. The force between the two charges is modeled by a harmonic oscillator with a coupling constant k.

The oscillating, spatially inhomogeneous electric field $\mathbf{E}$ is again given by Equation (1). The equations of motion are therefore

$$m_n\,\ddot{\mathbf{r}}_n = -q\,\mathbf{E}_r(\mathbf{r}_n)\,\cos(\omega\,t) + k\,\left(\mathbf{r}_p - \mathbf{r}_n\right) \tag{16}$$

and

$$m_p\,\ddot{\mathbf{r}}_p = q\,\mathbf{E}_r(\mathbf{r}_p)\,\cos(\omega\,t) + k\,\left(\mathbf{r}_n - \mathbf{r}_p\right). \tag{17}$$

Here, $\mathbf{r}_n$ is the trajectory of the negative charge, and $\mathbf{r}_p$ is the trajectory of the positive charge. m_n and m_p are the corresponding masses.

Similar to the case for an electrically charged particle, the solutions consist of a slowly drifting component $\mathbf{r}_d$, which describes the movement of the system's center of gravity, and rapidly oscillating components $\mathbf{r}_{on}$ and $\mathbf{r}_{op}$. Therefore, we can set $\mathbf{r}_n := \mathbf{r}_d + \mathbf{r}_{on}$ and $\mathbf{r}_p := \mathbf{r}_d + \mathbf{r}_{op}$. The drift component $\mathbf{r}_d$ for a bound particle is, of course, identical for both charges and equal to the trajectory of the center of gravity.

Using the approximation in Equation (4), the equations of motion (Equations (16) and (17)) become

$$\begin{aligned} m_n\,(\ddot{\mathbf{r}}_d + \ddot{\mathbf{r}}_{on}) \;\approx\; & k\,(\mathbf{r}_{op} - \mathbf{r}_{on}) \;- \\ & q\,(\mathbf{E}_r(\mathbf{r}_d) + \nabla \otimes \mathbf{E}_r(\mathbf{r}_d) \cdot \mathbf{r}_{on})\,\cos(\omega\,t) \end{aligned} \tag{18}$$

and

$$\begin{aligned} m_p\,(\ddot{\mathbf{r}}_d + \ddot{\mathbf{r}}_{op}) \;\approx\; & k\,(\mathbf{r}_{on} - \mathbf{r}_{op}) \;+ \\ & q\,(\mathbf{E}_r(\mathbf{r}_d) + \nabla \otimes \mathbf{E}_r(\mathbf{r}_d) \cdot \mathbf{r}_{op})\,\cos(\omega\,t). \end{aligned} \tag{19}$$

With the approximations $\|\ddot{\mathbf{r}}_d\| \ll \|\ddot{\mathbf{r}}_{on}\|$, $\|\ddot{\mathbf{r}}_d\| \ll \|\ddot{\mathbf{r}}_{op}\|$, $\|\mathbf{r}_{on}\| \approx 0$, and $\|\mathbf{r}_{op}\| \approx 0$, this expression simplifies to

$$m_n\,\ddot{\mathbf{r}}_{on} \approx -q\,\mathbf{E}_r(\mathbf{r}_d)\,\cos(\omega\,t) + k\,(\mathbf{r}_{op} - \mathbf{r}_{on}) \tag{20}$$

and

$$m_p\,\ddot{\mathbf{r}}_{op} \approx q\,\mathbf{E}_r(\mathbf{r}_d)\,\cos(\omega\,t) + k\,(\mathbf{r}_{on} - \mathbf{r}_{op})\,. \tag{21}$$

This system of differential equations can be solved by assuming that $\mathbf{E}_r(\mathbf{r}_d)$ is essentially only a time-independent constant at the time of consideration. The solutions are

$$\mathbf{r}_{on} = \frac{q\,\mathbf{E}_r(\mathbf{r}_d)\,(\cos(\omega\,t) - \cos(\omega_e\,t))}{m_n\,\left(\omega^2 - \omega_e^2\right)} \tag{22}$$

and

$$\mathbf{r}_{op} = -\frac{q\,\mathbf{E}_r(\mathbf{r}_d)\,(\cos(\omega\,t) - \cos(\omega_e\,t))}{m_p\,\left(\omega^2 - \omega_e^2\right)}, \tag{23}$$

where ω_e represents the resonant angular frequency

$$\omega_e = \sqrt{\frac{k}{m_{red}}} \tag{24}$$

of the bound particle and

$$m_{red} = \frac{m_n\, m_p}{m_n + m_p} \tag{25}$$

represents the reduced mass.

For the center of gravity $\mathbf{r}_d$, the equation

$$\mathbf{r}_d = \frac{m_p\,\mathbf{r}_p + m_n\,\mathbf{r}_n}{m_p + m_n} \tag{26}$$

applies. By differentiating twice, we obtain

$$\ddot{\mathbf{r}}_d = \frac{m_p\,\ddot{\mathbf{r}}_p + m_n\,\ddot{\mathbf{r}}_n}{m_p + m_n}. \tag{27}$$

Substituting the right-hand sides of Equations (16) and (17) yields

$$\ddot{\mathbf{r}}_d = \frac{q}{m_p + m_n}(\mathbf{E}_r(\mathbf{r}_p) - \mathbf{E}_r(\mathbf{r}_n))\,\cos(\omega\,t). \tag{28}$$

With the approximation in Equation (4), we obtain

$$\ddot{\mathbf{r}}_d = \frac{q}{m_p + m_n}\nabla\otimes\mathbf{E}_r(\mathbf{r}_d)\cdot(\mathbf{r}_{op} - \mathbf{r}_{on})\,\cos(\omega\,t). \tag{29}$$

Here, the solutions given in Equations (22) and (23) can now be used to obtain

$$\ddot{\mathbf{r}}_d = -\frac{q^2\,\left(\cos(\omega\,t) - \cos(\omega_e\,t)\right)\,\cos(\omega\,t)\,\nabla\otimes\mathbf{E}_r\cdot\mathbf{E}_r}{m_p\,m_n\,\left(\omega^2 - \omega_e^2\right)}, \tag{30}$$

whereby we now only write $\mathbf{E}_r(\mathbf{r}_d)$ as $\mathbf{E}_r$.

In the next step, we again take the average with respect to time in order to remove the fast oscillation of the two charges, which is irrelevant for the movement of the center of gravity. Using $\mathbf{F}_p = \ddot{\mathbf{r}}_d\,(m_n + m_p)$ and

$$\lim_{T\to\infty}\frac{1}{2\,T}\int_{-T}^{T}\left(\cos(\omega\,t) - \cos(\omega_e\,t)\right)\,\cos(\omega\,t)\,\mathrm{d}t = \frac{1}{2}, \tag{31}$$

we eventually obtain

$$\mathbf{F}_p = -\frac{q^2}{2\,m_{red}\,\left(\omega^2 - \omega_e^2\right)}\nabla\otimes\mathbf{E}_r\cdot\mathbf{E}_r \tag{32}$$

for the ponderomotive force of a bound particle. For the case in which $\mathbf{E}_r$ is the gradient of a potential ($\nabla\times\mathbf{E}_r = 0$), this expression can again be simplified and we obtain

$$\mathbf{F}_p = -\frac{q^2\,\nabla\,E_r^2}{4\,m_{red}\,\left(\omega^2 - \omega_e^2\right)} \tag{33}$$

by applying Equation (13).

It is important to point out the term $\omega^2 - \omega_e^2$ in the denominator of the fraction in Equation (33). This term seems to imply that an infinitely strong ponderomotive force acts for $\omega^2 = \omega_e^2$, which is unrealistic. Numerical simulations clearly show that a bound particle resonates at the eigenfrequency and oscillates more and more violently until algorithmic instabilities occur. However, the center of gravity of the bound particle does not change its position. Nevertheless, it is true that the ponderomotive force is particularly strong for angular frequencies ω close to the resonant angular frequency ω_e. The direction of the force depends on the sign of the term $\omega^2 - \omega_e^2$. For eigenfrequencies higher than the field frequency, the ponderomotive force acts in such a way that bound particles are drawn to where the field strength increases.

III. Electric field of a small dipole antenna

In the previous section, it was shown that, under certain circumstances, electromagnetic fields have an effect on electrically neutral particles. In this section, we seek to calculate the electromagnetic field of a small, almost point-shaped transmitter. The aim is to derive a formula that can represent the field of a WLAN router, cell phone, or clocked microprocessor at a distance of 1 m or more. To calculate this formula, we again use the basic laws of classical physics.

One of these fundamental laws relates to the electric field $\mathbf{E}_{sd}$, which is generated by a moving point charge q_s with trajectory $\mathbf{r}_s(t)$ at location $\mathbf{r}_d$. The formula reads

$$\begin{aligned}&\mathbf{E}_{sd}(q_s, \mathbf{r}, \mathbf{v}, \mathbf{a}) = \\ &\frac{q_s}{4\,\pi\,\varepsilon_0}\left(\frac{(\mathbf{r}\,c + r\,\mathbf{v})\left(c^2 - v^2 - \mathbf{r}\cdot\mathbf{a}\right)}{(r\,c + \mathbf{r}\cdot\mathbf{v})^3} + \frac{\mathbf{a}\,r}{(r\,c + \mathbf{r}\cdot\mathbf{v})^2}\right)\end{aligned} \tag{34}$$

and can be found by solving Maxwell's equations[11]. In Equation (34), $\mathbf{r}$ is the distance vector

$$\mathbf{r} := \mathbf{r}_d - \mathbf{r}_s(\tau). \tag{35}$$

The parameter τ is a certain moment in the past, which can be calculated by using

$$\tau := t - \frac{\|\mathbf{r}_d - \mathbf{r}_s(\tau)\|}{c}. \tag{36}$$

In turn, the parameters $\mathbf{v}$ and $\mathbf{a}$ are defined by

$$\mathbf{v} := \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}\tau} \tag{37}$$

and

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$$\mathbf{a} := \frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}\tau^2}. \tag{38}$$

Equation (34), which follows directly from Maxwell's equations, allows us to calculate the field generated by a very small antenna located at the coordinate origin in a rather simple way. To obtain the field of the antenna, we simply add the field of a stationary positive point charge q to the field of an oppositely charged point charge $-q$ that moves with a trajectory $\mathbf{r}_s(\tau)$. For the antenna model to be reasonable, however, the condition

$$\|\mathbf{r}_d - \mathbf{r}_s(\tau)\| \approx r_d \tag{39}$$

must be satisfied for all times τ, i.e., the two charges must remain close together.

Using Equation (39), it is possible to simplify Equation (36) to

$$\tau \approx t - \frac{r_d}{c}. \tag{40}$$

From this, Equation (35) becomes

$$\mathbf{r} = \mathbf{r}_d - \mathbf{r}_s(t - r_d/c), \tag{41}$$

and Equations (37) and (38) become

$$\mathbf{v} = -\dot{\mathbf{r}}_s(t - r_d/c) \tag{42}$$

and

$$\mathbf{a} = -\ddot{\mathbf{r}}_s(t - r_d/c). \tag{43}$$

The resulting field $\mathbf{E}_a$ of the antenna is then

$$\mathbf{E}_a = \mathbf{E}_{sd}(q, \mathbf{r}_d, \mathbf{0}, \mathbf{0}) + \mathbf{E}_{sd}(-q, \mathbf{r}, \mathbf{v}, \mathbf{a}). \tag{44}$$

By plotting this field, one can verify that it corresponds to the well-known field of the Hertzian dipole[12].

To carry out simple calculations, this formula should be simplified even further. For this purpose, one can exploit the very small magnitude of $\mathbf{r}_s$, which allows us to express $\mathbf{E}_a$ by a first-order Taylor series. After a skipped calculation, we obtain

$$\begin{aligned}\mathbf{E}_a \approx -\frac{q\left(\frac{\mathbf{r}_d}{r_d} \times \left(\frac{\mathbf{r}_d}{r_d} \times \ddot{\mathbf{r}}_s\left(t - \frac{r_d}{c}\right)\right)\right)}{4\,\pi\,\varepsilon_0\,c^2\,r_d} + \\ \frac{q\left(r_d^2\,\dot{\mathbf{r}}_s\left(t - \frac{r_d}{c}\right) - 3\mathbf{r}_d\left(\mathbf{r}_d \cdot \dot{\mathbf{r}}_s\left(t - \frac{r_d}{c}\right)\right)\right)}{4\,\pi\,\varepsilon_0\,c\,r_d^4} + \\ \frac{q\left(r_d^2\,\mathbf{r}_s\left(t - \frac{r_d}{c}\right) - 3\mathbf{r}_d\left(\mathbf{r}_d \cdot \mathbf{r}_s\left(t - \frac{r_d}{c}\right)\right)\right)}{4\,\pi\,\varepsilon_0\,r_d^5},\end{aligned} \tag{45}$$

i.e., the electric field of the Hertzian dipole including the near field. This expression can likewise be simplified even further if we assume that we are not in the immediate vicinity of the antenna, but that the distance is sufficiently great for the last two terms to be irrelevant. In this case, we can focus on the far field of the Hertzian dipole and obtain

$$\mathbf{E}_a \approx -\frac{q\left(\frac{\mathbf{r}_d}{r_d} \times \left(\frac{\mathbf{r}_d}{r_d} \times \ddot{\mathbf{r}}_s\left(t - \frac{r_d}{c}\right)\right)\right)}{4\,\pi\,\varepsilon_0\,c^2\,r_d}. \tag{46}$$

The sign and scaling factor vary in the specialist literature, owing to the fact that one can also allow the positive charge or both charges to oscillate. However, these details are irrelevant for the following considerations.

IV. Ponderomotive force of a dipole antenna in the far field

Equations (33) and (46) can now be used to calculate the force exerted by an electromagnetic transmitter on a hydrogen or antihydrogen atom. A number of electronic devices, such as WLAN routers or cell phones, can be considered as transmitters. These devices emit almost perfect sine waves of a certain frequency f in the gigahertz range. The antennas are relatively small, and the wavelength is so short that the far-field approximation in Equation (46) can be used at a distance of just 1 m.

For this reason, we only use the approximation in Equation (46) and assume that the negative point charge moves along the trajectory

$$\mathbf{r}_s(\tau) = \mathbf{e}\,A\,\cos(\omega\,\tau). \tag{47}$$

Here, A is a very small spatial displacement significantly smaller than the diameter of an atom, $\omega = 2\,\pi\,f$ is the angular frequency, and $\mathbf{e}$ is a direction vector that represents the orientation of the dipole antenna. Inserting this expression into Equation (46) gives the corresponding electric field

$$\mathbf{E}_a = \frac{A\,q\,\omega^2\,\left(\frac{\mathbf{r}_d}{r_d} \times \left(\frac{\mathbf{r}_d}{r_d} \times \mathbf{e}\right)\right)\,\cos\left(\omega\left(t - \frac{r_d}{c}\right)\right)}{4\,\pi\,\varepsilon_0\,c^2\,r_d}. \tag{48}$$

The numerical value of $A \cdot q$ is an unknown quantity. However, the electric field strength generated at a defined distance is often known. Therefore, the partially unknown parameters can be summarized in a single parameter U_n, and this voltage can be selected in such a way that the measured field strengths correspond to the model, leading to

$$\mathbf{E}_a = \frac{U_n}{r_d}\left(\frac{\mathbf{r}_d}{r_d} \times \left(\frac{\mathbf{r}_d}{r_d} \times \mathbf{e}\right)\right)\,\cos\left(\omega\left(t - \frac{r_d}{c}\right)\right). \tag{49}$$

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From this, we can now calculate the ponderomotive force in Equation (33). First, we can read from Equation (49) that the amplitude of the field strength is

$$\mathbf{E}_r = \frac{U_n}{r_d} \left(\frac{\mathbf{r}_d}{r_d} \times \left(\frac{\mathbf{r}_d}{r_d} \times \mathbf{e} \right) \right). \tag{50}$$

In the next step, we verify that $\nabla \times \mathbf{E}_r = \mathbf{0}$. Thus, we can use Equation (33), and after calculating and inserting the term $\nabla (\mathbf{E}_r \cdot \mathbf{E}_r)$, we obtain the ponderomotive force

$$\mathbf{F}_p = \frac{q^2 \, U_n^2 \, \left(\mathbf{r}_d \, \left(r_d^2 - 2 \, (\mathbf{e} \cdot \mathbf{r}_d)^2 \right) + \mathbf{e} \, r_d^2 \, (\mathbf{e} \cdot \mathbf{r}_d) \right)}{2 \, m_{red} \, \left(\omega^2 - \omega_e^2 \right) \, r_d^6}, \tag{51}$$

i.e., the force exerted by a small electromagnetic transmitter on an electrically neutral bound particle that consists internally of two charges $+q$ and $-q$, with reduced mass m_{red}.

For the special case in which the dipole antenna is aligned in the z-direction, $\mathbf{e} = \mathbf{e}_z$, the ponderomotive force is strongest in the x-y plane. Therefore, we can set $\mathbf{r}_d = \mathbf{e}_x \, r_d$, and Equation (51) becomes

$$\mathbf{F}_p = \frac{q^2 \, U_n^2 \, \mathbf{e}_x}{2 \, m_{red} \, \left(\omega^2 - \omega_e^2 \right) \, r_d^3}. \tag{52}$$

As one can easily see, the force is the same for hydrogen and antihydrogen and has an attractive or repulsive effect depending on the sign of $\omega^2 - \omega_e^2$. It is also clear that the force for $\omega \approx \omega_e$ can be strong.

Equation (52) can now be used to estimate the force exerted, for example, by a WLAN router on a hydrogen atom at a distance of $5\,\mathrm{m}$. WLAN routers use different frequencies depending on the standard. A typical frequency is $f = 2.4\,\mathrm{GHz}$. As already mentioned, the eigenfrequency f_e of hydrogen is $1.42\,\mathrm{GHz}$. WLAN routers generate a field strength ranging from approximately $0.7\,\mathrm{V/m}$ to more than $3\,\mathrm{V/m}$ at a distance of 1 m[13][14][15]. Thus, we use the parameters $U_n = 1\,\mathrm{V}$, $\omega = 2\,\pi\,f$, and $\omega_e = 2\,\pi\,f_e$. The reduced mass m_{red} corresponds to approximately the mass of an electron. Because the hydrogen atom consists of an electron and a proton and the antihydrogen atom consists of an antiproton and a positron, $q^2 = e^2$. If these parameters are inserted, a force of $F_p \approx 7.61 \cdot 10^{-31}\,\mathrm{N}$ is obtained.

The gravitational force of the Earth on a hydrogen atom can be estimated by using the formula $F_g = g\,m_p$, where g is the free-fall acceleration and m_p is the mass of a proton. Inserting these parameters gives $F_g \approx 1.64 \cdot 10^{-26}\,\mathrm{N}$. This force significantly exceeds the value of F_p. Therefore, the force exerted by a WLAN router at a distance of 5 m would not present an issue.

However, the situation changes if the transmitter is nearby and the transmission frequency is closer to the resonance frequency of hydrogen. For example, if we assume that the transmitter is transmitting at a frequency of $f = 1.41\,\mathrm{GHz}$ and is only 1 m away, then $F_p \approx 1.26 \cdot 10^{-26}\,\mathrm{N}$, which approaches the force of gravity. If the transmitter is even closer or the field strength is greater than the relatively small field strength of $1\,\mathrm{V/m}$, the ponderomotive force can be stronger than the gravitational force. Importantly, the direction of the ponderomotive force is identical for both an antihydrogen atom and a hydrogen atom.

V. Conclusions

This article has shown that ponderomotive forces should not be ignored in the ALPHA-g experiment. It is likely that the apparatus used in the experiment is relatively well shielded against external electromagnetic fields in the radio and microwave range because of its metallic structure. However, such fields can penetrate through glass windows or electrical cables. A problem arises here, as the fields inside would be relatively inhomogeneous. In addition, standing waves could occur. The resulting ponderomotive forces would then have an equally attractive or repulsive effect on hydrogen and antihydrogen atoms. Moreover, because the ponderomotive forces can theoretically be stronger than the weak force of gravity, the ponderomotive forces could, under certain conditions, influence the observed drift direction of antihydrogen atoms, potentially complicating the interpretation of gravitational effects if not adequately ruled out.

For this reason, it is necessary to prove in the ALPHA-g experiment that there are no or almost no alternating electromagnetic fields inside the cylinder and that no individual frequencies are particularly prominent in the spectrogram. One potential avenue for future validation could involve controlled exposure of antihydrogen to radio-frequency fields under experimental conditions, to assess the magnitude and directionality of any resulting ponderomotive effects. It is certainly possible that no interactions with radio waves occur, because the $1.42\,\mathrm{GHz}$ resonance in hydrogen relates to magnetic coupling with the electron spin. Therefore, the considerations made in this article based on the electric field strength might not reflect the situation correctly. Furthermore, hydrogen atoms are not classical systems.

This article is not intended to dispute the findings of the ALPHA-g experiment but rather to offer a constructive criticism of the experimental interpretation, grounded in classical physics. The intention is to stimulate scientific discussion by presenting a theoretical mechanism that may warrant consideration in future experimental validations. This work is written with the goal of fostering open scientific debate and encouraging feedback from other physicists and engineers, particularly those with expertise in atomic and antimatter systems. Only through such discourse can we ensure that fundamental questions, such as how gravity acts on antimatter, are approached with the fullest scientific rigor.

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