Measuring Complexity using Information

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Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.

Abstract

Measuring complexity in multidimensional systems with high degrees of freedom and a variety of types of information, remains an important challenge. The complexity of a system is related to the number and variety of components, the number and type of interactions among them, the degree of redundancy, and the degrees of freedom of the system. Examples show that different disciplines of science converge in complexity measures for low and high dimensional problems. For low dimensional systems, such as coded strings of symbols (text, computer code, DNA, RNA, proteins, music), Shannon's Information Entropy (expected amount of information in an event drawn from a given distribution) and Kolmogorov's Algorithmic Complexity (the length of the shortest algorithm that produces the object as output), are used for quantitative measurements of complexity. For systems with more dimensions (ecosystems, brains, social groupings), network science provides better tools for that purpose. For highly complex multidimensional systems, none of the former methods are useful. Here, information related to complexity can be used in systems, ranging from the subatomic to the ecological, social, mental and to AI. Useful Information **Φ** (Information that produces thermodynamic free energy) can be quantified by measuring the thermodynamic Free Energy and/or useful Work it produces. Complexity can be measured as Total Information **I** of the system, that includes **Φ**, useless information or Noise **N,** and Redundant Information **R.** Measuring one or more of these variables allows quantifying and classifying complexity. Complexity and Information are two windows overlooking the same fundamental phenomenon, broadening out tools to explore the deep structural dynamics of nature at all levels of complexity, including natural and artificial intelligence.

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Keywords: Thermodynamics, Infodynamics, Complexity, Information.

Introduction

The aim of this paper is to create bridges between disciplines that allow researchers to share tools to advance in the building of a unified robust science of complexity. One such tool is the measurement of complexity. Attempts to measure complexity are not new [1][2][3][4]. They are based upon the assumption that complexity is proportional to the number and diversity of elements or symbols of a system and to the relationships between them [5]. As recognized by Wikipedia, the term Complexity is generally used to characterize something with many parts that interact with each other in multiple ways, culminating in a higher order of emergence greater than the sum of its parts. The study of these complex linkages at various scales is the main goal of complex systems theory.

Several approaches to characterizing complexity have been used in different sciences;[6] Among them, relating complexity to information has a long tradition [7][8][9][10]. There is however no unambiguous widely accepted strict definition of complexity. In computer science and mathematics, the Kolmogorov complexity [11] of an object, such as a piece of text, is the length of a shortest computer program that produces the object as output. It is a measure of the computational resources needed to specify the object. In multidimensional systems, this definition is of little use as the Kolmogorov complexity becomes incomputable. Fisher's [12] Information and Shannon's Information Entropy [13] estimate complexity by the information content of a given system, however his method becomes algorithmically incomputable in systems with high dimensions. These methods have large limitations for studying phenomena which emerge from a collection of interacting objects [14], especially if high dimensional systems are being studied.

“The science of complexity is based on a new way of thinking that stands in sharp contrast to the philosophy underlying Newtonian science, which is based on reductionism, determinism, and objective knowledge... Although different philosophers, and in particular the postmodernists, have voiced similar ideas, the paradigm of complexity still needs to be fully assimilated by philosophy” [15]. Fortunately, in natural sciences, these concepts seem to be more assimilated, though in different disciplines they have different flavors.

Examples of Complexity in different disciplines

Discipline	Object	Measured through	Example
Quantum Mechanics	Subatomic particles	Taxonomic complexity	Standard Model
Inorganic Chemistry	Atoms	Taxonomic complexity	Periodic Table
Organic Chemistry	Molecules	Taxonomic complexity	MS-AI
Biology	Organisms	Taxonomic complexity	Tree of Life
Genetics	DNA, RNA, Proteins	Coded strings	Genome
Language, Music	Scripts	Coded strings	Texts, Songs
Computing	Bits, Qbits	Coded strings	Programs
Ecology	Systems, organisms	Networks	Ecological webs
Social sciences	Humans	Networks	Constitutions, Society
Economics	Values Industries	Coded strings Networks	Stock market Economic Complexity
Physics	Mater, energy	Degrees of freedom	Emergence
Infodynamics	Energy, Information	Work achieved	Free Energy
Artificial Intelligence	Models	Work achieved	AI programs

Taxonomic Complexity: The simplest way of measuring complexity is by counting the parts or components of a system. The examples given in the table include the Standard Model of particle physics. This model is a descriptor of the complexity of the physical world enumerating all known subatomic particles. These 16+ particles explaining 3 fundamental forces of physics allow us to express quantitatively the complexity of the results of interactions between one or more of these subatomic particles. Analogously, the Periodic Table of Chemistry lists 118 known chemical elements. They are ordered in increasing complexity related to their number of protons and electrons. However, the combination of these elements produces an immense universe of possible molecules whose complexity can only superficially estimated by the number of different types of atoms forming the molecule or by the complexity of different manifestations of the molecule [16]. The combinatorial complexity is so great that even artificial intelligence (AI) has limits in its ability to compare their characteristics [17][18]. Similarly, the complexity of life on earth can be graded qualitatively in degrees of complexity in accordance to the complexity of the organism and their position in the universal phylogeny. The variables affecting complexity in living systems, however, have not been tamed so as to produce meaningful measures on complexity. I refer to these types of complexity as Taxonomic Complexity.

Algorithmic Complexity: Another type of complexity can be referred to as information coded in strings of symbols. This is the case of Genetics, Animal and Human Language, Music and Computing. In all these cases, information can be represented as strings of coded characters such as bits, qbits, letters, numbers, nucleotides or amino acids. As the representation uses only one dimension in the sequence, and the coded characters are finite, analytical scalar measures of complexity, even very sophisticated ones, are possible [19]. Following the insight of Shannon, information entropy content can serve as a efficient measure of complexity [20] and can be applied to music [21] literature [22] genomes [23] and computer language [24]. Alternatively, Kolmogorov Complexity continues to be a widely used method[25] for sting codes. Other methods are based on order that eventually can be reduced to information entropy [26], or are based on specific physical properties [27]. Complexity analysis of series of marker data[28] show that time series of values of commodities in financial markets contain complex structures that help understand fundamental characteristics of the markets [29].

Networks: Not all natural phenomena can be reduced to strings of code. For two or more dimensions, network science has developed a series of complexity measures [30]. These measures of "aggregate complexity" are sometimes extension of computer complexity algorithms [31], others are adapted to ecological studies[32][33][34][35] and animal communication [36], and others are used in economics, popularized by the "Index of Economic Complexity"[37]. Similar indices have been developed in other Social Sciences and Law Studies [38]. For example, empirical work based on such indices showed that more complex constitutions may be more harmful to society [39]. Remarkably, ecologists have developed analytics for complexity measures systematically for a long time [40][41][42][43]. Some of these studies open the door for sophisticated multidimensional complexity, ranging from simple networks to vascular branching, artificial, neural, cortical and immunological networks [44].

Multidimensional Complexity: Tackling high-dimensional problems with multidimensional vectors is common practice [45], and expansions such as tensor-based higher-order network alignment [46] are being developed. Analytics of high complexity problems is better at explaining phenomena already known than predicting new ones [47]. It has had a mixed success in understanding highly complex systems. Even for low dimensional topology (4 or less dimensions) pure analytic methods have not been able to solve many issues [48]. At ever larger degrees of freedom, computational mathematical tools do not help in explaining phenomena such as Emergence [49]. Novel dimensions require new kinds of information and new metrics that span over different levels of emergence. Aggregates of atoms form molecules which may form cells that can organize into "organisms" that evolve brains. But brains and computers [50] have properties and forms of storing and managing information that atoms do not have. A different view on complexity is required.

Shannon information, and algorithmic information content can be combined to produce mathematical definitions for complexity that adapt more to our needs relating complexity with information [51]. Particularly useful quantities are the effective complexity, which is roughly the length of a compact description of the identified regularities of an entity, and total information, which is effective complexity plus an information entropy term (not to confuse with thermodynamic entropy as explained below) that measures the information required to describe the random aspects of the entity [52]. Several other attempts to relate complexity in thermodynamic terms have been published [53][54][55][56][57]. They form the basis of Infodynamics [58] which aims to understand complexity in multidimensional systems as an expression of information capable of producing new emergent properties in a given system. Classical thermodynamics refers to Free Energy as the energy that produces useful work, in contrast to Thermodynamic Entropy which refers to the energy that dissipates as heat and does not produce work. Analogously, the amount of free energy produced by a given amount of this information (useful information) may serve as a way to understand complexity. Information that does not maintain or increase free energy of the system can be considered noise. An infodynamic approach allowed to measure quantitatively and empirically the relation between complexity (or useful information) and free energy produced in over a dozen studies [59]. These included: Social complexity and colony size in ants increases as energy consumption per capita decreases; Economic development of countries increases as their scientific development expands; Per capita electricity consumption decreases in cities as their size increases; Countries with a strong Rule of Law have low infant mortality and a high Human Development Index; Countries with many populist words in their constitution underperformed in Human Development relative to those with simple constitutions.

Infodynamics

Infodynamics applies the logic of thermodynamics used in understanding the dynamics of energy, but applies it to understand information dynamics. It distinguishes between Thermodynamic and Information Entropy [60]: The latter relates to uncertainty in outcomes, while thermodynamic entropy pertains to energy distribution in physical systems. But the production of Thermodynamic Free energy is related to information [61].

Let's define Information Complexity **I** as the total amount of information in a system, and Useful Information **Φ** as the one producing Free Energy [62] **F** and thus work [63]. Free Energy and Work are thermodynamic concepts so that

$$\mathbf{F = E - S/T}$$

Where **E** is total energy [64], **S** the thermodynamic entropy, and **T** is the temperature in degrees Kelvin [65].

If **Φ** is useful information or the information that accounts for **F**, **I** the total information accounting for its complexity, and **N** useless information or noise, then

$\mathbf{\Phi} = \mathbf{I - N}$ and $\mathbf{F = E - S/T}$ then $\mathbf{\Phi} = \boldsymbol{k}\mathbf{(E - S/T)}$

were $\boldsymbol{k}$ is a function or constant relating **F** with **Φ**

Here we have a tool to measure **Φ** quantitatively and empirically, where Infodynamic Complexity (Total Information **I)** can be measured as

$$\mathbf{I = \Phi + N + R},$$

where **R** is redundant information which is important for conserving information in time.

This result relates the amount of information to the amount of work that can be produced by a system. This approach allows handling complex systems, including living organisms and ecosystems, and might be appropriate to tackle problems of quantification of information and useful work in Artificial Intelligence.

The quirk in complex irreversible processes is to differentiate between different kinds of information and the means to assess them. Separating "Useful Information" that produces "Free Energy" available to produce "Useful Work" from other kinds, such as redundant, useless information or noise is not a trivial exercise. For example, literary critics consider that the artistic value or usefulness of The Faust from Wolfgang von Goethe [66] is much greater than his publications on vision and colors [67], but in both cases, total information I as computed using Information Entropy criteria[68] is similar. The difference between both works is the balance between **Φ** and **N** of the text, which will depend on the useful energy we expect to produce. Either measured as **F** to produce scientific advance in the sciences of vision, or **F** in eliciting emotions as a literary-artistic achievement.

The cost of information is only calculable indirectly. It might relate to the cost of engraving a substrate, reading it, storing it, communicating it, transmitting it through a medium, or the cost of acquiring it. For example, complexity of Large Language models in Artificial Intelligence, and thus of its information content, is often measured using the size of the databases used to train the models, or the size of the stored information. This method does not distinguish between noise, redundancy and useless data. Redundancy is relatively easy to measure by comparing the encrypted information with itself. Separating noise from useful information is possible by measuring its effect on the production of work. Large Language models can then be compared by their efficiency in producing useful information rather than by the energy expended in creating them, or by the size of the encrypted information they contain.

Not all information is equal in achieving useful work. Two empirical examples illustrate this. The Atlas of Economic Complexity [69] and the relation between economic development and different scientific activity in a country[70]. The first example shows that complexity and information underlying different industries have different effects on the economic development of a country. The second example shows that the development of different scientific fields have different impacts on the economic development of a nation. Thus, Useful Information **Φ** differs in its complexity and its effects on Free Energy **F**. Only empirical studies will throw more light on these issues.

This approach might also resolve the shortcomings of the Landauer principle that quantifies the thermodynamic cost of the recording/erasure of one bit of information [71]. This cost must take into account the transitions **Φ →N** and **N →Φ** which can be studied experimentally.

Discussion

Classical Physics works with four fundamental physical dimensions. Acknowledging Godel's insight[72], it is impossible to build a 4-d model of all atoms in the world with a human brain, or with any human build contraption, that has 4 or less dimensions. The challenge thus is how to analyze features that emerge from the interactions of systems more complex than those grasped by 4-d models, such as values, fitness, power, intelligence and synergy. Each of these features can be treated as a novel dimension rather than as the outcome of the 4-d interactions of zillions of subatomic particles. The task of the science of complexity is to define relevant new dimensions.

Complexity and Information are closely linked. Complexity may be defined as the amount of knowledge that is required to describe a system. More complex systems require more information for its description. For example, a simple system can be described with a few simple properties, such as its size, shape, and color. A more complex system requires much more information in addition to its patterns and representations. Describing a human requires describing its physical features, its personality, its memories, motivations, its thoughts, etc. Lets call it a multidimensional complexity

In biological evolution, natural selection forces genomes to behave as a natural ‘‘Maxwell Demon,’’ within a fixed environment. This drives genomic complexity to increase [73]. Yet not all increase in complexity leads to an increase in useful information as shown above with the example of the usefulness of constitutions. Nor do organisms that have longer DNA chains in their genome have always a higher complexity than others with less DNA. The Australian lungfish has a genome 14 times larger than the human genome! That means that not all information is equivalent as much information might be redundant, irrelevant or useless noise. Working with Useful Information Φ is a way to overcome this limitation.

Many problems relating complexity with Infodynamicsremain unresolved. For example:

- Crystals in inorganic chemistry are believed to minimize thermodynamic entropy and information entropy of the system. Non-symmetric heterogeneous crystals such as DNA, RNA or protein crystals do not. We lack descriptive tools to better understand these differences.
- The depth of resolution of details of a system influences its metrics of information and complexity. It is not the same to count organisms in a system than to count cells, atoms or subatomic particles in the same system. Solving a problem requires the appropriate resolution of details of the system. Defining the appropriate resolution for studying specific problems is a challenge.
- Structural Information is always present and not evident when exploring a problem that does not include the dimensions defining the border condition of the system appropriately. For example, what are the structural features that can be considered as relevant information when analyzing the free energy gained by the total energy released by an explosive inside a cannon? What are the relevant processes that allow algorithms with better intelligence? Trial and error is often the only way to figure this out, making empirical studies indispensable.
- The probabilistic nature of information, uncertainty, meta-information, the working of evolution of emotions are among a multitude of areas that need further explorations.

When equating Complexity with Information, we have to keep in mind that different types of information exist, and thus different kinds of complexity. Information can be structural, enformation, intropy, entangled, encrypted, redundant, synergic, noise, meta-information (information about information), and others, as recognized by Infodyamics [74].

Conclusions

This paper is part of a series of 5 articles published in Qeios recently[75]. They led to the conclusion that the most relevant type of information for understanding natural sciences, engineering, biology, ethics, psychology and other disciplines is what Infodynamics calls Useful Information **Φ**: the one that allows systems to produce Free Energy that produces Useful Work. Here I relate these findings with concepts of complexity that range from simple complexity measured in bits, to evolutionary products such as human brains, social dynamic complexity and AI. The future challenge is to tackle measurements of complexity empirical, such as engineers do. This is made possible by Infodynamics, and will increase our understanding of the world of complexity.

Understanding the basics of thermodynamics equips engineers with the necessary tools to analyze energy systems and enhance their efficiency. It provides a framework for solving complex problems in energy management, system optimization, and sustainability, which are central concerns in today's engineering challenges. Analogously, understanding the basic working of information and its relation to the production of free thermodynamic, biological or cultural energy, will allow us to better understand general intelligence and emergence of new knowledge. This understanding is at the root of any progress in complex systems science, emergent intelligences, and biological evolution [76]. Having measures of complexity that allow different systems to be compared by applying a common metric is fundamental in stabilizing the foundations of Infodynamics. Measures of complexity and types of information allow different systems to be compared to each other by applying a common metric. This is especially meaningful for systems that are structurally or functionally related. Differences in complexity among such related systems may reveal features of their organization that increase our understanding of complexity and information [77].

Complexity and Information are two windows overlooking the same fundamental phenomenon. Exploring the intricate interplay between complexity and information opens new vistas to unresolved problems helping in revealing their underlying mechanisms. broadening out tools to better understand nature. It seems we are just at the beginning of a long search.

Acknowledgments

No funding was received. All data is available on the web. No competing interests. I thank Caglar Uyulan, Reza Abasi, John Cioffi and Michael Nosonovsky for comments on the previous version of this paper.

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