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Angle Trisection with Growth Rate of The Golden Ratio

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Abstract:

In this paper, we explore the triangulation of angles using the golden ratio and geometric compass principles. By dividing angles into three equal parts based on the golden ratio, we demonstrate a method for solving this geometric problem. Furthermore, we extend our exploration into the third dimension, where a geometric compass can effectively divide a two-dimensional angle into three equal parts. Additionally, our investigation delves into the realm of six-dimensional space-time, where we observe the simultaneous movement of two arms of the compasses based on the golden ratio to achieve the division of every angle into three equal parts. We emphasize the significance of the growth rate associated with the golden ratio in resolving complex mathematical and physical problems. This study provides valuable insights into the application of geometric principles and the golden ratio in solving challenging problems within mathematics and physics.

Keywords: Angle trisection, Golden ratio.

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1. Introduction

Angle trisection with a compass has been a longstanding challenge throughout ancient times. Dividing an angle into three equal parts using only a geometry compass is an unsolvable problem. [1] However, it is worth noting that drawing the angle bisector in several steps can effectively achieve the division of certain angles into three equal parts. While there are various methods and approaches to angle trisection, it remains a complex and intricate task that has puzzled mathematicians and scholars for centuries. There are different ways to Angle trisection [2][3][4].

Probir Roy's proposed method presents a significantly more detailed and comprehensive approach to the subject at hand [5]. With a thorough examination of the intricacies involved, Roy's method offers a comprehensive solution that addresses the multifaceted aspects of the issue. While trigonometry may present certain limitations, the examination of such challenges through the point of view of the theory of general balance in the six dimensions of space and time offers a promising avenue for new insights [6]. This theoretical framework posits that by encompassing not only the traditional three spatial dimensions but also the three dimensions of time, a more comprehensive understanding of complex issues can be attained.

Various methods have been proposed for angle trisection. which can be referred to [7][8][9][10].

The problem of time is not raised in mathematics. And no algebraic equation has a definite answer over time. This means that the objective expression of a simple sum has an infinite answer over time. Accordingly, although Angle Trisection is algebraically unsolvable, it can be solved over time.

Events occur over time. From a physical point of view, when drawing a circle with a compass, the points follow pi. Now, if the radius of the circle is also changed, the limitations of trigonometry can be overcome by creating eccentricity of the ellipse. If the fixed arm of the compass or its angle moves in the desired direction at the same time as the compass rotates around its axis, all trigonometric restrictions for angle trisection will disappear. Accordingly, the rate of change of velocity can be plotted exponentially on both sides of the angle to draw different circles. The purpose of this research is to investigate the issue that it is possible to express a special method geometrically for angle trisection based on the physical point of view.

2. Materials and Methods

The angle can be defined based on the eccentricity of the ellipse. (2.1) Any movement on the expanding sphere is based on eccentricity. Figure1

$$\sqrt{1-\frac{x^2}{r^2}} = sin\left(cos^{-1}\frac{x}{r}\right)\ ,\ cos^{-1}(\frac{x}{r}) = \theta \qquad 2.1$$

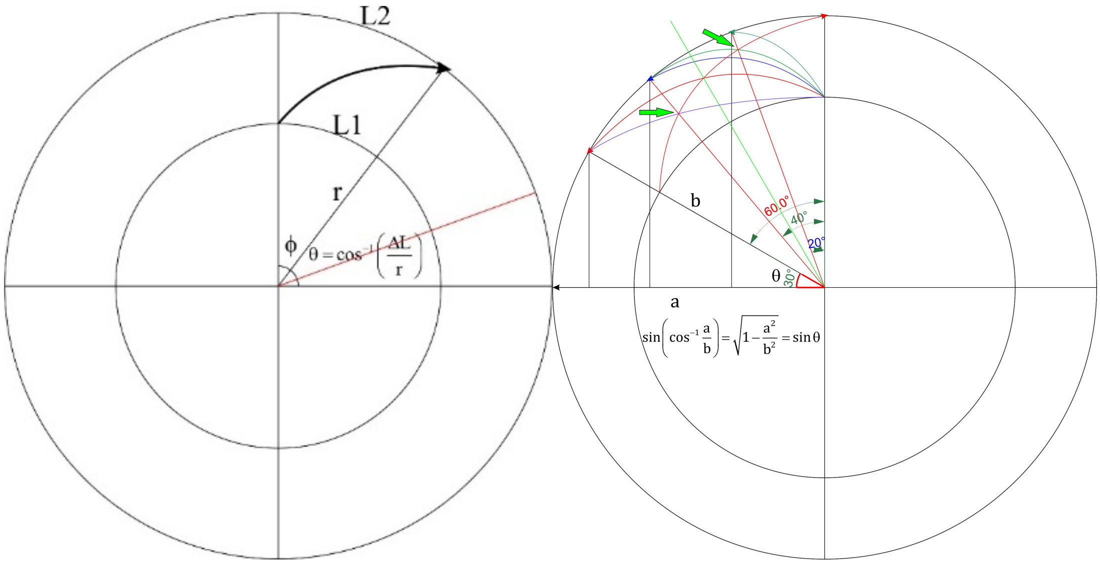

Figure 1: Any movement on an expanding circle is accompanied by the eccentricity of the ellipse.

An angle is like the eccentricity of the ellipse in Figure 1. This means that the second side of the angle represents the movement of a specific point in the expanding plane. Although this view is physical. But it expresses the nature of departure from the center of the ellipse in a simpler way.

Procedure A

The arc length is directly related to the radius and is obtained from equation 2.2.

$$L = \frac{\theta}{360}2\pi R \qquad 2.2$$

To bisect an angle, the most common method is to divide the arc opposite the angle into two equal parts. This can be achieved using a compass and a straightedge to construct the angle bisector. Drawing the bisector is done by changing the radius. Accordingly, by changing the radius of the circle, it can be changed to draw the desired angle.2.3

Due to the vagueness of pi, this ratio cannot be obtained accurately. figure 2

$$60^{\circ} \Rightarrow \frac{\pi}{3}R = L \Rightarrow R = \frac{3L}{\pi} \Rightarrow 20^{\circ} \Rightarrow R \cong \frac{9L}{\pi} \Rightarrow 3R \cong \frac{3L}{\pi} \qquad 2.3$$

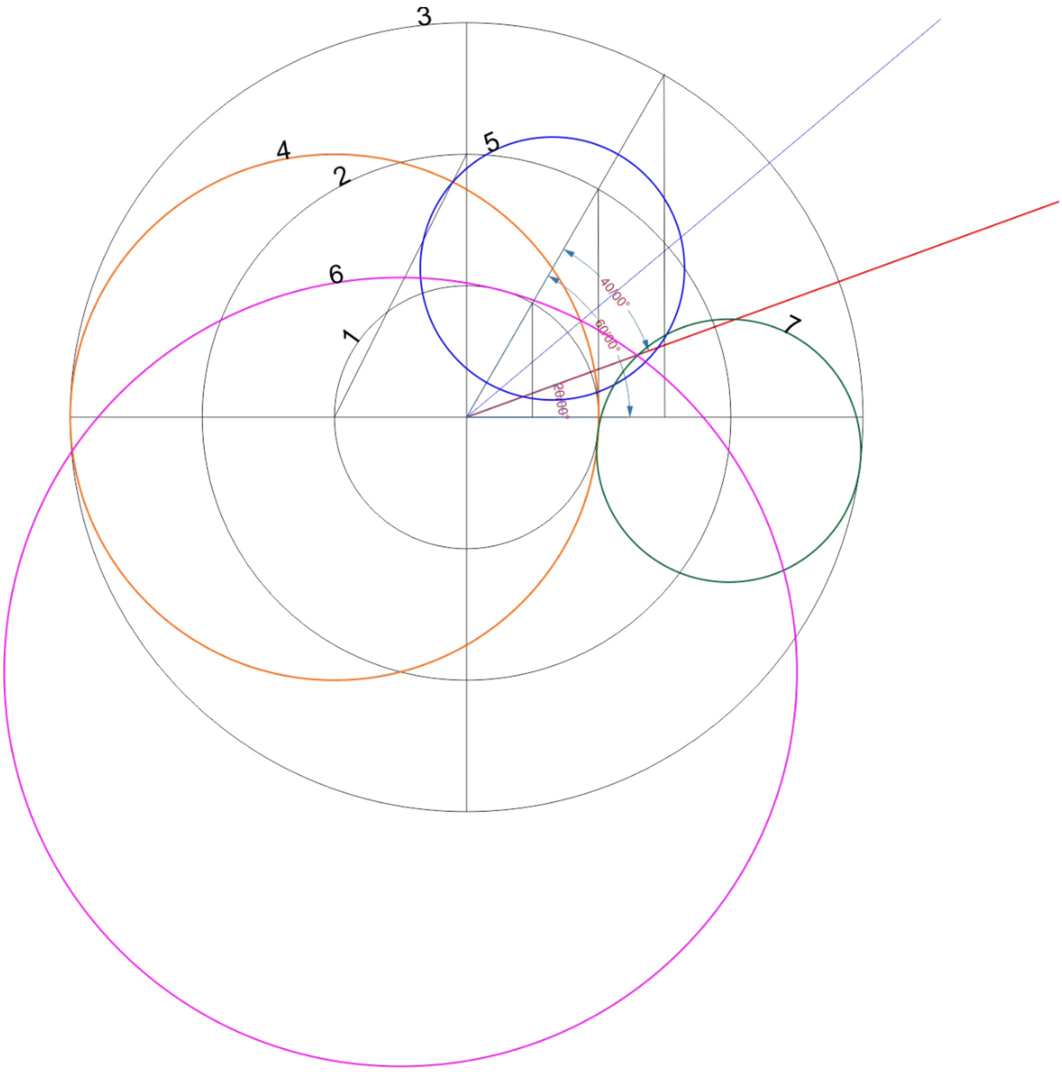

Figure 2: A: Dividing a 60° angle into three parts using radius tripling and the golden ratio.

Procedure B

For a one-dimensional line to be visually present in two dimensions on a page, it requires a two-dimensional structure. The pi number represents simultaneous movement in two dimensions. (2.4)

$$\pi \equiv \frac{\partial^2 x}{\partial t^2} + \frac{\partial^2 y}{\partial t^2} \qquad 2.4$$

One-dimensional space is embedded in two-dimensional space. Points on the surface of the circle follow the geodesic of the surface. As a result, the movement on the surface of the expanding circle causes the eccentricity of the ellipse in other dimensions as well. Figure1

Accordingly, using one radian in two circles with a double ratio, one-third of some specific angles can be determined accurately. Figure2

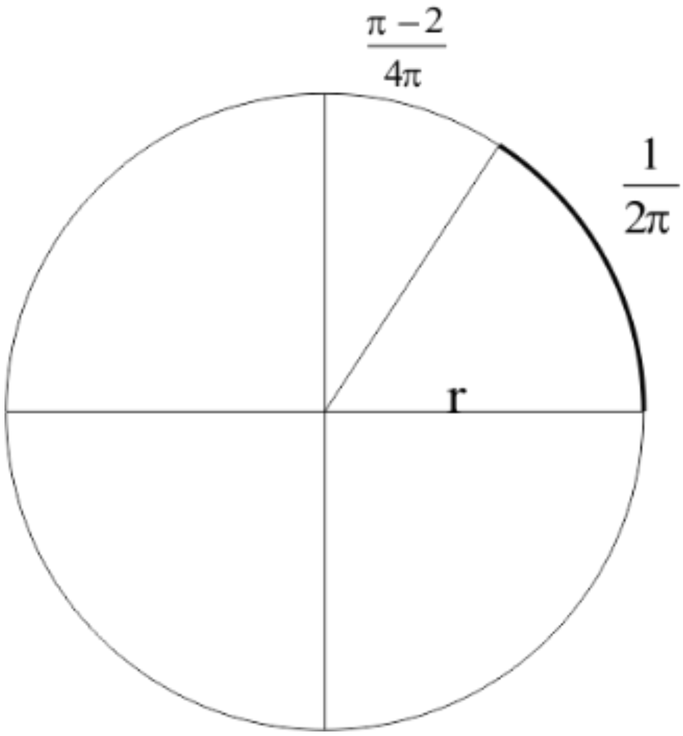

Figure 3: The radius cannot be a quarter of the circumference of the circle. The selected line with compasses geometry segments forms arcs larger than the algebraic results.

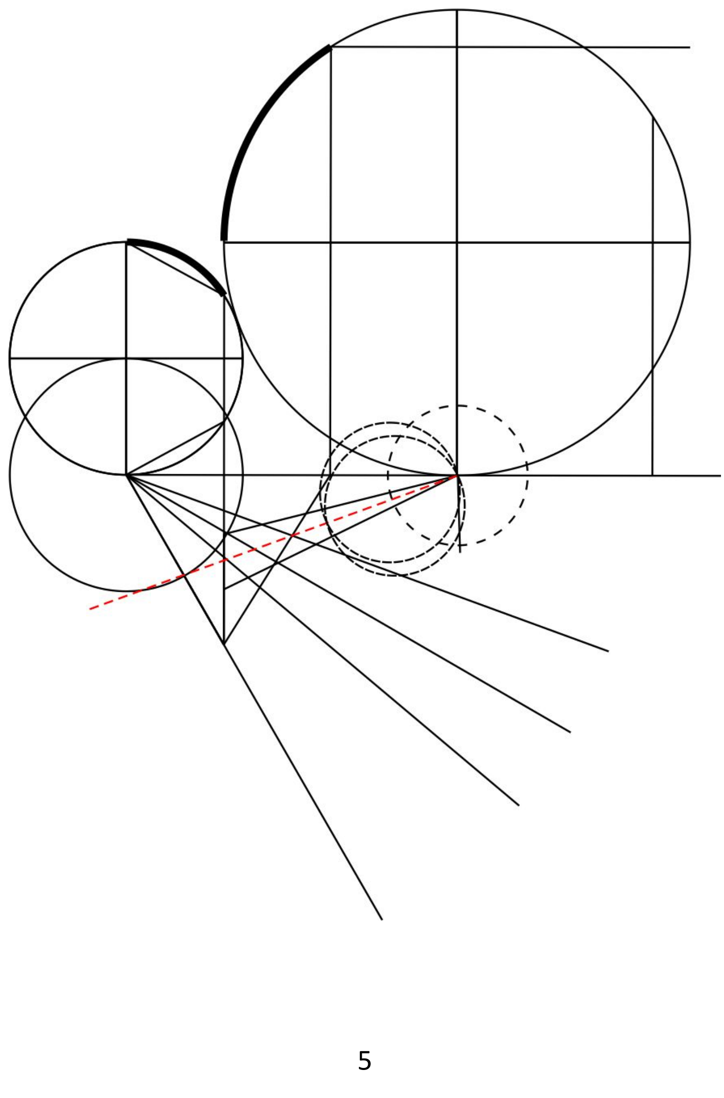

Figure 4: The reflected image of a radian on the side of an angle defines the points associated with doubling the radius and doubling the angle in lower dimensions.

Procedure C

A two-dimensional structural angle consists of two one-dimensional sides. The arc is a two-dimensional structure and the points on the sides of the angle must be selected based on a line. Also, the angle in the plane expands exponentially. 2.5

$e^{i\theta t} = cos\theta t + isin\theta t \Rightarrow re^{i\theta t}$ 2.5

The golden ratio is derived from the doubling ratio in the Fibonacci sequence. (2.6) Accordingly, the golden ratio can be used to divide the angle into three parts, and the remaining part is obtained using the bisector drawing. The direct connection between Pi's golden ratio and Euler's number is evident in the golden spiral.

$\frac{1+\sqrt{5}}{2} = \varphi \quad r = ae^{b\theta}$ 2.6

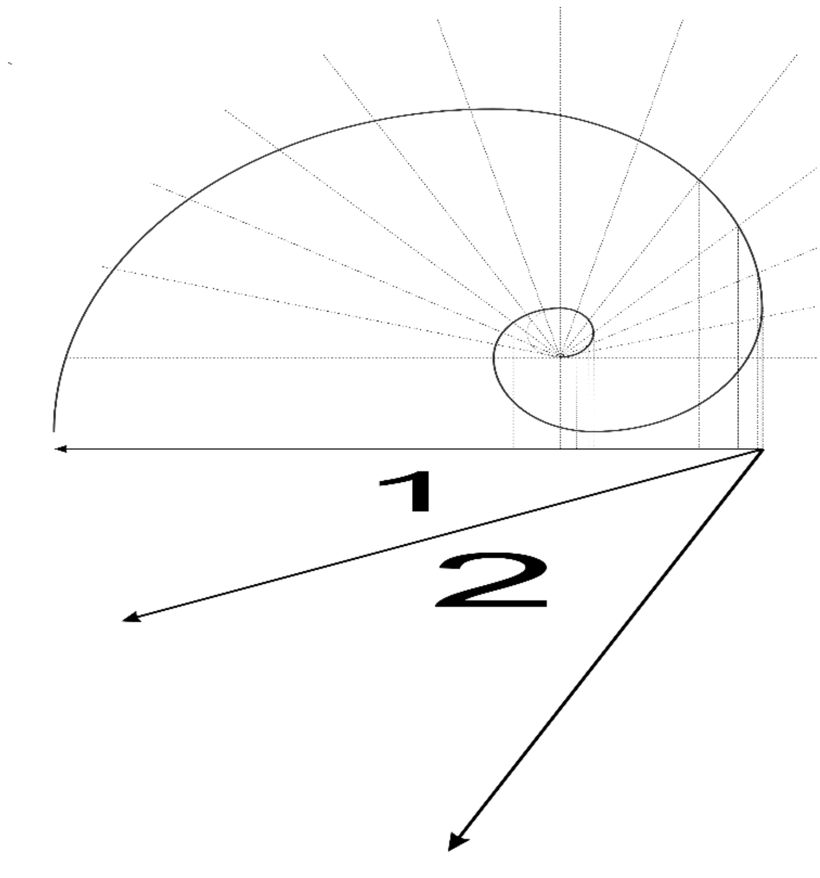

Figure 5: The image of a spiral on a line is a set of points that are used to divide an angle into three sections.

Procedure D

The direct connection between Pi's golden ratio and Euler's number is evident in the golden spiral. The relationship between the three golden ratio pi numbers and Euler's number is related to the movement of a one-dimensional space in a two-dimensional plane in the past, present, and future times. The arc equal to the radius in higher dimensions follows the golden constant growth rate. This issue leads to rotation by maintaining the properties of space in higher dimensions such as the Möbius strip. A Möbius strip transfers properties of lower dimensions to higher dimensions. (2.7)

$$L = \left({}^{\theta}/_{360}\right)2\pi r \quad \theta = 90 \Rightarrow L = \left(\frac{1}{4}\right)2\pi r \Rightarrow \left(\frac{1}{2\pi}\right) = \left({}^{180}/_{\pi}/_{360}\right) = 1Rad$$

$$\left(\frac{90-\frac{180}{\pi}}{360}\right) = \left(\frac{1}{4}\right) - 1Rad = \left(\frac{\pi-2}{4\pi}\right) \Rightarrow \left(\frac{\pi-2}{4\pi}\right) + \left(\frac{1}{2\pi}\right) = \left(\frac{1}{4}\right) \qquad 2.7$$

$$\left(\frac{1}{2}\right)^2 2\pi r \quad \left(\frac{1}{2}\right)^3 4\pi r^2 \quad \left(\frac{1}{2}\right)^4 2\pi^2 r^3 \quad \left(\frac{1}{2}\right)^5 \frac{8}{3}\pi^2 r^4 \quad \left(\frac{1}{2}\right)^6 \pi^3 r^5$$

$$\Rightarrow \left(\frac{1}{2}\right)^6 \pi^3 \cong Ln(\varphi) \Rightarrow r = \varphi^{\frac{2\theta}{\pi}} \quad r = ae^{b\theta}$$

$$(\pi\ Past)(\pi\ present)(\pi\ future) = \pi^3$$

the circle is the image of the movement of the geometric compass in the third dimension in the two-dimensional plane. An expanding circle has three different limits. And over time they approach the area of a sphere. The area of a six-dimensional sphere is five-dimensional space. 2.7

According to the above article and the issue the ellipse is a section of the space-time cone. The perimeter of the ellipse and the desired area five dimensional for the golden point in an angle can be divided the angle into three parts using the golden spiral. (2.8) Figure 6, Figure 7

$$\pi^3 r^5 = a.b \Rightarrow \left(\frac{119.22\times 192.59}{\pi^3}\right)^{\frac{1}{5}}/\pi = 1.1933 \qquad 2.8$$

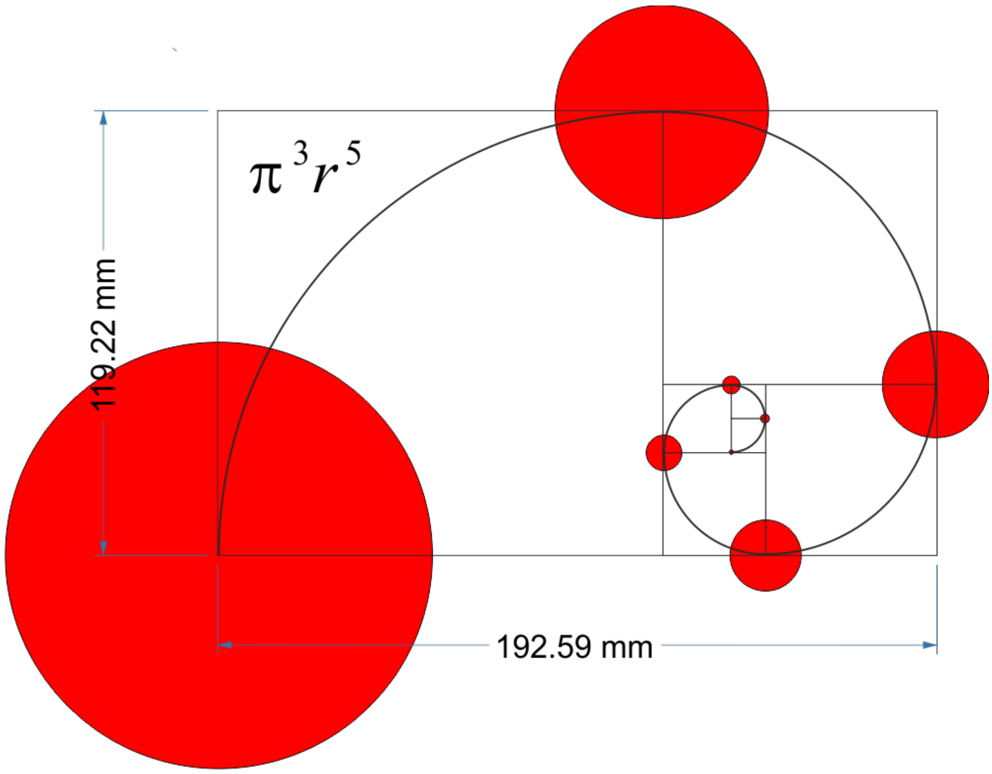

Figure 6: The growth ratio of the radius of the circles in the image and the growth ratio of the golden spiral over time are directly related to each other.

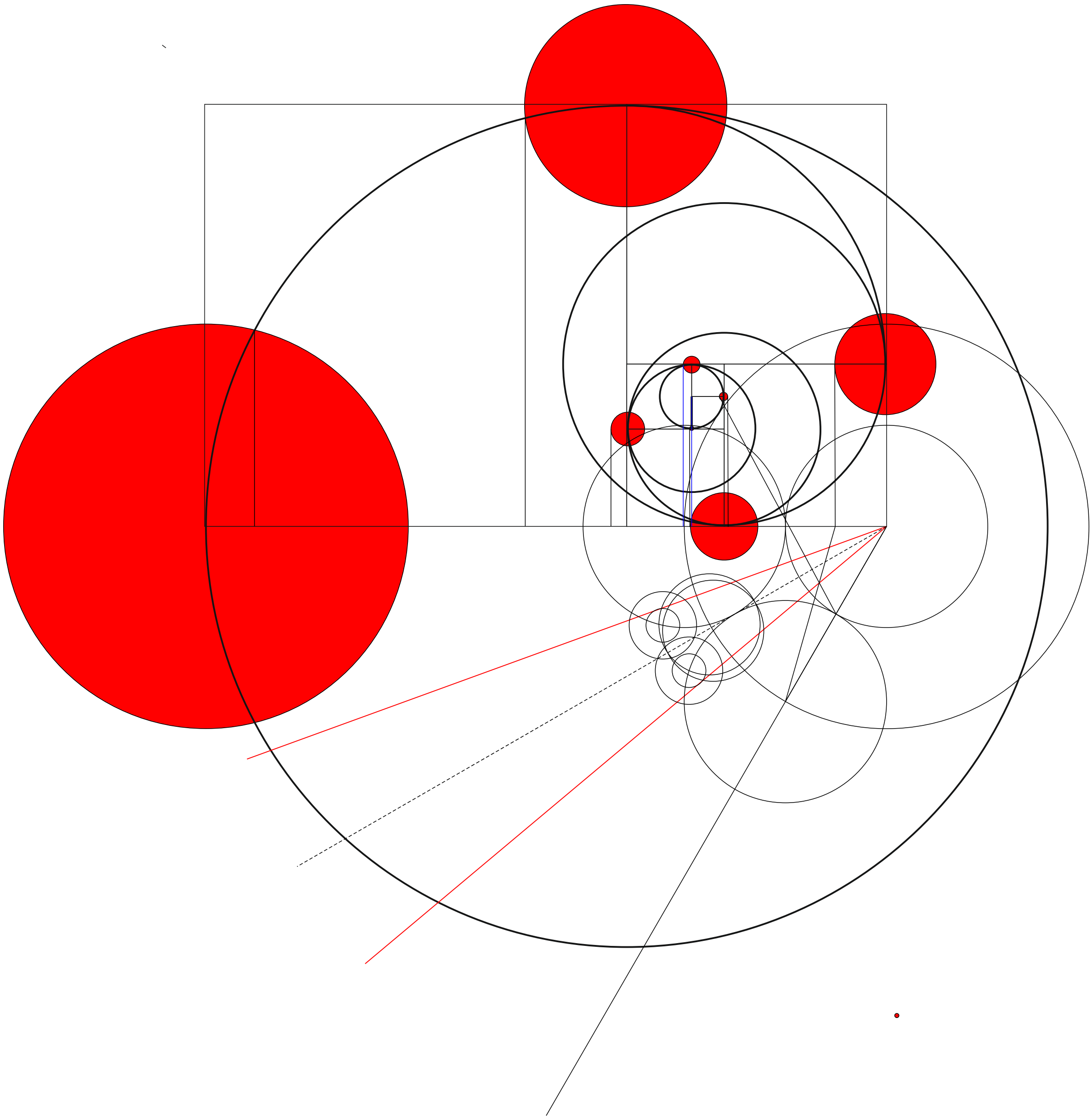

Figure 7: all angles intersect the boundary of the circles along the arc. The growth ratio of the circles can determine the number of angle divisions.

Procedure E

According to the eccentricity of the ellipse, the golden constant growth rate, and the usual methods, a more comprehensive and simple geometric method can be expressed.

Figure 7: by increasing the radius by three times and forming an ellipse based on the Fibonacci sequence and drawing the diameter of certain arcs, the angle can be divided into three equal parts.

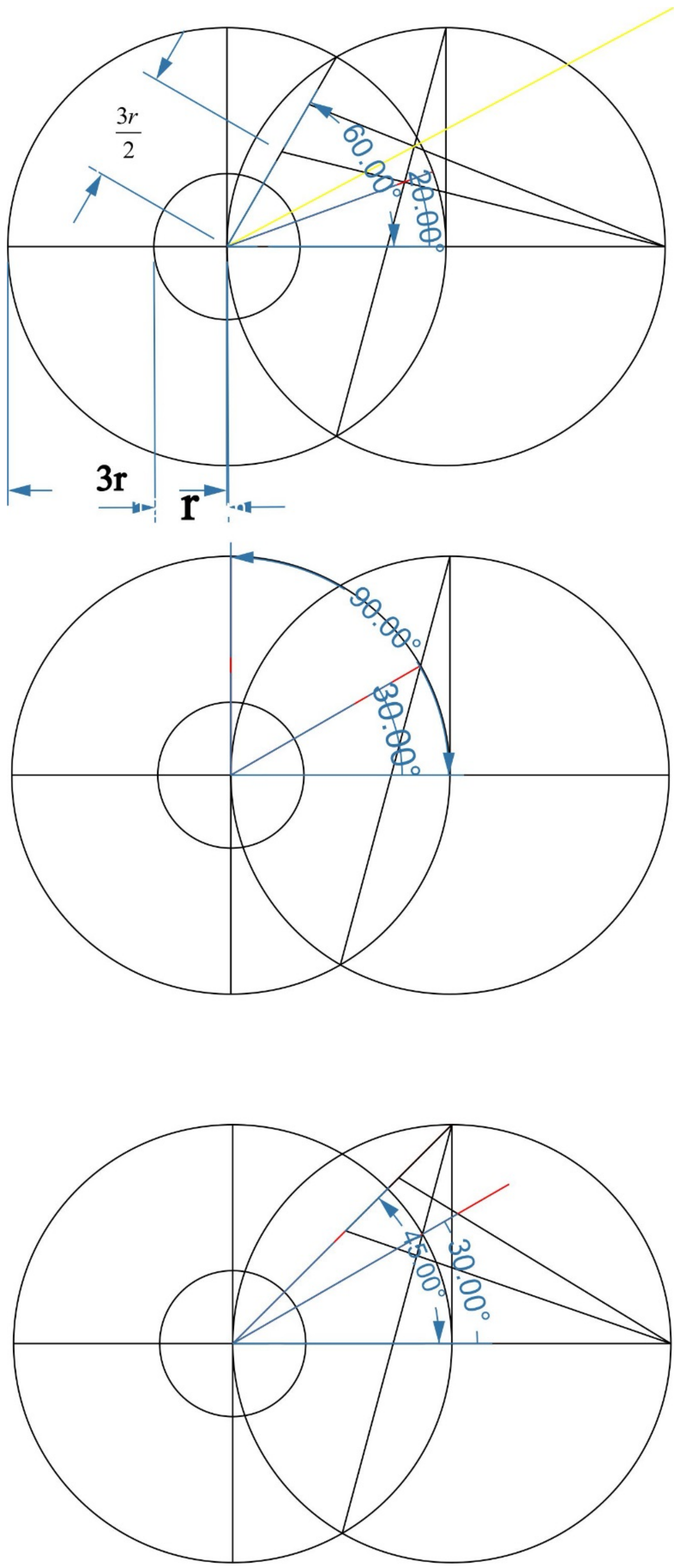

Figure 8: based on tripling the circle and creating bifocals to form an ellipse, the angle is divided into three equal parts based on the departure from the center of the ellipse.

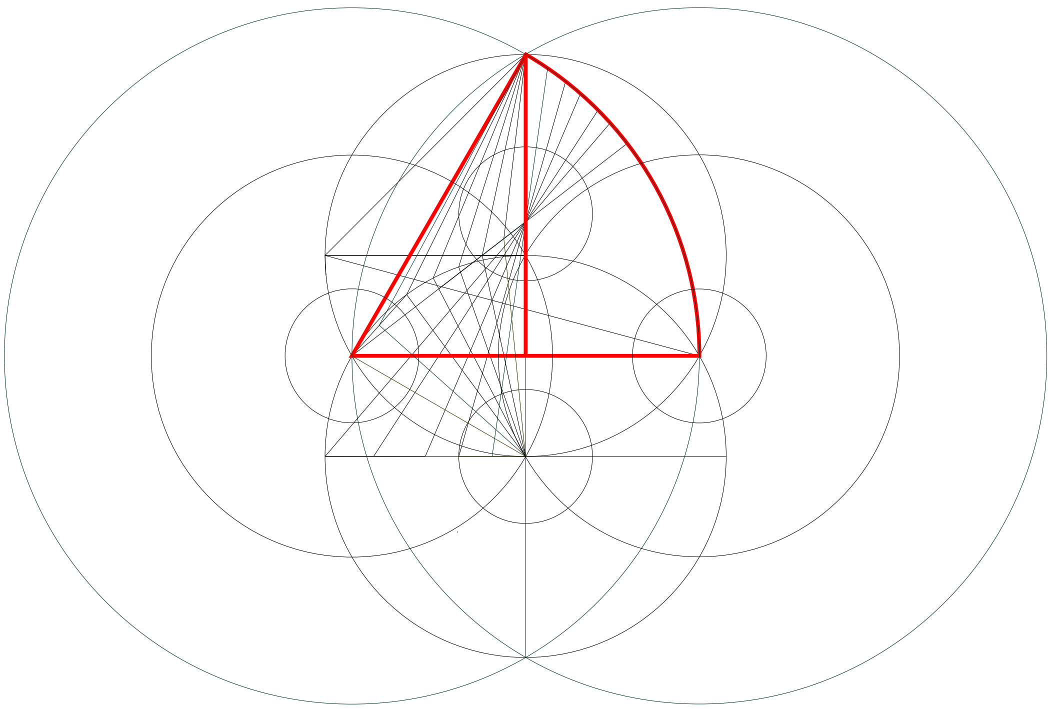

Figure 9: The golden ratio plays a fundamental role in angle division.

Procedure F

In higher dimensions, an angle is like two planes. Each of these plates passes through a sphere. Figure 10

A sphere can only be divided into two equal parts by a plane. And a sphere cannot be divided into three equal parts. The sphere is like a geometric compass over time.

The geometric compass has two movements, one around its axis and the other angle change, only by changing the angle during movement, it can divide an angle into three parts based on the doubling ratio of the golden constant. (2.9) Figure 11, Figure 12

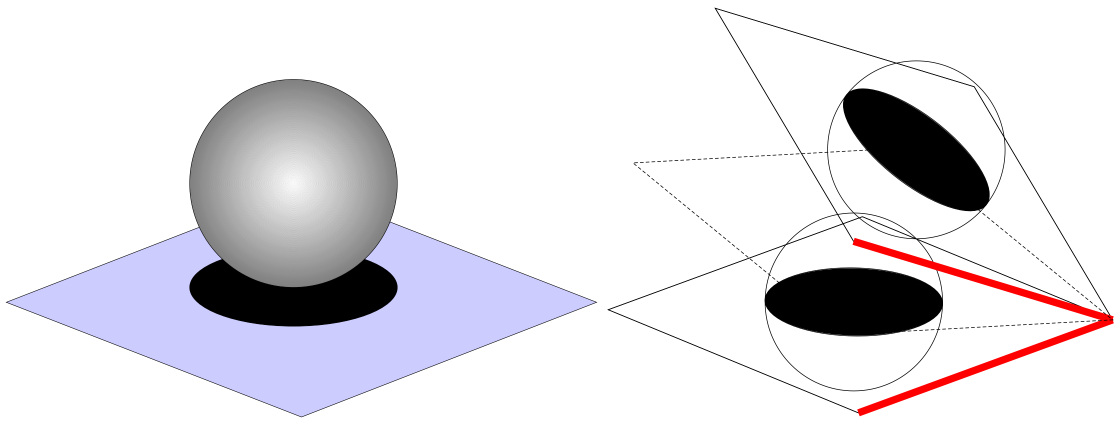

Figure 10: An angle is like two planes from a higher-dimensional perspective. Just as a plane cannot divide a sphere into three equal parts. A geometric compass cannot divide an angle into three equal parts.

$$\frac{V_1+\sqrt{V^2{}_1+V^2{}_2}}{V_2}=\varphi\,,\ \frac{\theta}{V_2}\equiv\frac{V_1+\sqrt{V^2{}_1+V^2{}_2}}{V_2} \qquad 2.9$$

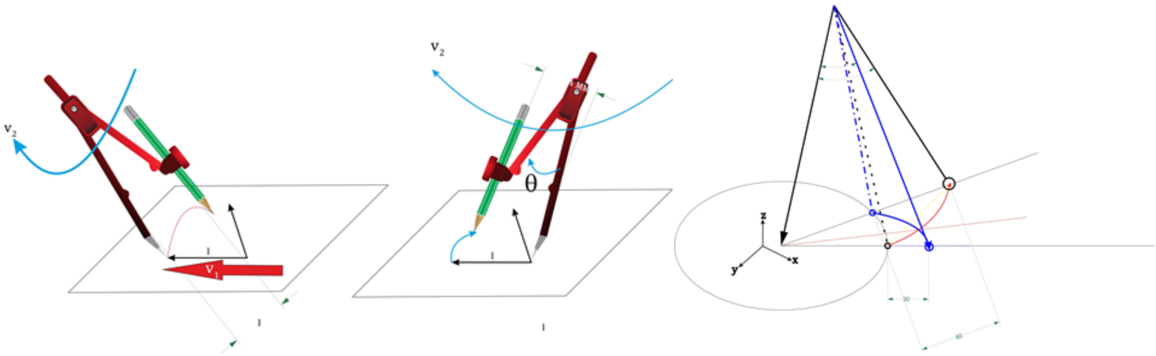

Figure 11: At the same time as the compass rotates, the needle also moves on the line of the angle. At the secondary level, the speed of one arm is twice as much. As the other arm. It is also possible to create golden arcs by changing the angle of the geometry compass to the rotation speed of a compass.

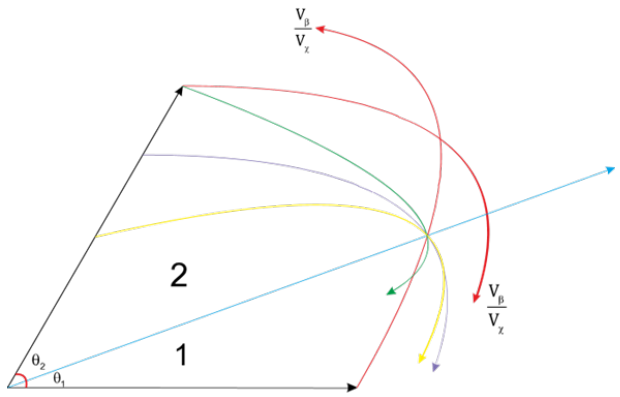

Figure 12: Based on this, by changing the speed and different ratios, there are infinite ways to divide the angle.

3. Results and discussion

As a result of this research, complex problems become very simple in higher dimensions. The angle is expanded in the two-dimensional plane. And a compass moves in the third dimension. As a result, it can easily divide an angle into two equal parts by using the pi number. As the length of the sides of the angle increases, the points chosen to draw the arc should also change. Drawing an ellipse instead of a circle with a compass based on the eccentricity of the ellipse is the basic way to avoid the impossibility of angle trisection. To divide the angle into three equal parts, the compass must have an additional movement in the fourth dimension. As a result, if we consider the fourth dimension as time. The compass should have two different movements over time. Simple movements in higher dimensions look very complex from the perspective of lower dimensions. The eccentricity of an ellipse in higher dimensions is expressed by the sine of an angle**.**

Examining events over time brings different ways to solve unsolved problems in mathematics, geometry, and physics. However, it seems that the nature of the problem has changed. Algebraic operations change over time. There is no certainty for physics and mathematical facts to remain constant over time. Changing the laws of Euclidean geometry in curved spaces shows the reality of time. No equation has certainty over time. And all the facts in the world are dependent on the passing of time**.**

it is possible to express the speed changes using the reflection of the circular radius in the third dimension on the sides of the angle. The golden ratio is the expression of this change of perspective. Selecting the points that represent the change of length based on the angle. Like the length of the shadow of a vertical, which changes with the change of the radiation angle. Although the methods proposed in this research are very briefly stated, each of them can be a research field for finding a comprehensive solution for angle trisection. Events have a different structure over time. They do not follow known mathematical rules. As a result of this research, there is an attitude about time that all objects have a quantum behaviour during time. A person can pass through one gate in the first moment and pass through the second gate in the next moment. But in general, it passes through both gates over time. Material wavelength is very small for macroscopic objects, but this wavelength changes when speed increases and time dilation. Over time, macroscopic objects behave quantumly. As a result of this attitude, the expression of mathematics with time is necessary to have a unified theory. And also angle trisection has expressed the necessity of this vision for more than a thousand years. This research can lay the groundwork for the geometrical expression of speed, acceleration, force, etc. using the basis of the golden constant, Pi number, and Euler's number. time mathematics is not physicking, but rather a mathematical and geometrical interpretation of the passage of time. The passage of time is the most important problem of experimental sciences. There are only dimensions between reality and imagination**.**

Natural numbers and fundamental natural constants are related to each other.[11] The growth of plants, rain, biological molecules, the growth and metabolism of life organs, the structure of the universe, and chemical reactions can express this relationship**.**

Based on the evaluation of the growth of different systems over time, it is possible to understand the relationship between the three numbers pi, phi, and Euler's number with each other. Buffon's needle problem proves the relationship of pi in the past, present, and future times, with Euler's number and the golden ratio in the six-dimensional space-time. The method presented in this paper can be used to prove many complex problems by examining events over time. Complex problems such as Hodge's conjecture, Riemann's hypothesis, P versus NP, and ... can also be examined from this point of view.

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