Qeios

If knowledge were simpler, we would all be wiser.

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v1 entitled "Simplifying academic knowledge to make cognition more efficient: opportunities, benefits and barriers" posted on SSRN in May 2016. The current version takes account of the recent interest in generative AI due to the release of ChatGPT and Bard, and also the very helpful comments of 16 reviewers on Qeios.com: see Appendix 4 for some notes on my response to these reviews.

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Abstract

A lot of academic knowledge (the sort of expertise that people go to academic institutions to learn about) is complicated and may require considerable time, effort, and expert help to master. University courses, textbooks, and, more recently AI tools like ChatGPT, can help to tackle this problem. However, sometimes another approach may be possible: simplifying academic knowledge so that it is easier to learn, understand and use, without sacrificing its power and usefulness. There are many historical examples of such simplifications, such as the replacement of Roman numerals with the decimal system, and many scientific laws which replace a myriad of disconnected observations with relatively simple general principles. This article explores the idea of simplification of knowledge, how this might apply to current knowledge, and the possibilities this opens up for the future. The benefits, in terms of saving time and enhancement of the power and accessibility of academic knowledge, are potentially enormous. Specialists could reach the frontiers of their discipline quicker and have more time to advance their knowledge. They might also be able to make more, and better, use of other disciplines. Students would need less time for their studies. If we don't start to simplify our increasingly complicated web of knowledge, human progress will slow down or cease as our minds become clogged with unnecessary technicalities, and inevitable over-simplifications take control of our thoughts and actions**.** We need to see the design of elegant, fit-for-purpose, perspectives which help us to make wise judgments as an important task for academia. Then perhaps 10-year-olds will be able to understand quantum mechanics, and the leading edge of science will be beyond anything we can imagine now.

Keywords: *Academic knowledge, Growth of knowledge, Public understanding of science, Simplicity, Simplifying knowledge.*

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Contents

Introduction

It is a well-worn platitude that life is getting more complicated. In the academic world this is reflected in the increasing amount and complexity of the knowledge that academics discover or create, and pass on by teaching, books, articles, and other media. The amount and complexity of this academic knowledge places obvious constraints on what we can achieve, both in academic work and in day-to-day life. Specialists only have a limited amount of time to master their discipline and aspects of other fields of study relevant to their work. The access of lay people to knowledge on, for example, medicine, is limited by the time and background expertise necessary to research the intricacies of the various specialisms.

There are several obvious approaches to this problem: improving learning and educational methods, using technology to enhance the effectiveness of human thought processes via computer technology (including AI which I discuss below) or smart drugs, increasing the time devoted to education, and so on[1]. In this article, I want to explore the possibility of tackling the problem from the other end by simplifying knowledge itself.

The word possibility is important. I am not arguing that simplification is *always* possible, but just that it is sometimes, perhaps often, possible, and it has large potential benefits.

I had difficulty formulating an appropriate title for this article. The word "simple" provides a convenient slogan, but the idea is not a simple one. People typically interact with academic knowledge by learning about it; they may have a deep, or a shallow, or a mistaken, understanding of it; and then they may make use of it, or simply enjoy it, in a variety of ways across a narrow or wide domain. The word "simple" is relevant to all these aspects. It may or may not be simple to learn and understand and use; misunderstandings may or may not be likely. It is important that the simplicity of a piece of knowledge should be judged against the background of its usefulness and the domain in which it can be applied.

The phrases "more appropriate for its context", "better integrated into common sense", "more fit for purpose" perhaps give a more accurate picture of what I have in mind. The simpler version may involve a substantially different set of concepts (e.g. Examples 2 and 3 below) so the process could be described as “conceptual reengineering”. Earlier phrases I wondered about were "empower the ignorant" and "empower the masses". These are both consistent with what I have in mind but suggest that the market for the idea is limited to the ignorant, the lazy and the stupid, which is not at all what I intend. Geniuses should also be able to expand their reach further. I will stick with the word "simple" but please bear in mind that my interpretation of this word is not simple.

It would be nice if I could apply the mantra of trying to simplify things to the idea of simplicity itself and perhaps come up with a neat numerical formula for measuring simplicity. Unfortunately, I can't see how to do it without running into the problem of oversimplification, which is an important issue discussed below.

I have come to the general idea of simplifying knowledge via several interlinked routes:

[1] And, in the long term, more extreme possibilities like genetic engineering, to make us, or our successors, more intelligent.

1. There are historical examples of simplifications which have had enormous and obvious benefits - such as the adoption of the decimal system of numerals, and the evolution of mathematical notation.[2]
2. When teaching (statistics, research methods, quality management, decision analysis and a few other things) it has often occurred to me that the material I am teaching could be simplified while preserving its usefulness. Sometimes the conceptual base could be changed, sometimes the jargon could be made more transparent, sometimes I could ignore conventions decreeing the teaching of approaches that have been superseded, and so on.
3. Simplicity is a key criterion for evaluating, and so creating and choosing, scientific theories. Theories that are too complicated are less useful. As an extreme example, Khamsi (2006) quotes Keith Devlin: "I think that we're now inescapably in an age where the large statements of mathematics are so complex that we may never know for sure whether they're true or false." Obviously, it would help if these statements could be made simpler.
4. Simplicity is an important criterion in the design of technological artifacts, and academic knowledge can be regarded as such an artifact.
5. A person's understanding of almost anything typically has holes in it. Nobody knows everything about how to make a pencil or how a computer works; similarly, my knowledge of the t-test in statistics does not include details about how the t-distribution is derived. Ideas can be simplified by leaving out inessential details. Knowledge is holey and the location and nature of the holes deserve careful consideration.
6. One of my favourite books is Bertrand Russell's (1961) *History of western philosophy*. The subtitle of this book is "and its connection with political and social circumstances from the earliest times to the present day". If he were writing today, I think he would have added technological to his list of types of circumstances. He might also have extended his domain from philosophy to knowledge in general as many scientific ideas start with essentially philosophical questions. Computer technology, for example, provides a massive enhancement of our ability to calculate and search, and the fact that these tasks are now trivial has important implications for what is worth knowing and what details we can safely skip over. Even more obviously, artificial intelligence has implications for the role of human knowledge. Technology must always have had an impact on appropriate forms of knowledge: going back a few years, the invention of writing must have influenced the nature of communities' knowledge, as must the introduction of printing presses. However, the pace of change is now much faster, and the impact of changing technologies is more obvious.
7. If we take an evolutionary perspective on knowledge, the main criterion that new ideas have to satisfy is that they should be useful - either in the biological sense of enhancing the capacity of the organism to survive and breed, or in the cultural sense of their usefulness to individuals and groups - and knowledge is likely to be more useful if it is simple (other things being equal). Unfortunately, incremental evolution may increase complexity (Cohen and Stewart, 1995, 135-8), which suggests the idea that intelligent design for simplicity might be beneficial.

[2] See for example https://en.wikipedia.org/wiki/History_of_mathematical_notation.

8. Finally, simplification is a natural and necessary part of the way human beings make sense of the world. Concepts like "animal" and "rock" enable us to manage our environment without plunging into all the overwhelming complexity of the different types of animals and rocks - unless, of course, circumstances demand more precise categories.

Despite searching on the obvious keywords using Google and Google Scholar, and checking through the first twenty or so hits, I found very little on the general idea of simplifying academic knowledge. In an earlier paper (Wood, 2002a) and short article (Wood, 2002b) I focused on the idea of simplifying knowledge from the educational perspective, and the barriers to the idea that are likely to be imposed by educational systems. In the present article, I want to extend this argument and take a more general perspective.

Perhaps the closest to the general thesis I have in mind are the various campaigns, courses and publications on the theme of simplicity run by Edward de Bono - e.g. de Bono (1998) and the website at https://www.debonogroup.com/services/core-programs/simplicity/. These concern simplicity in various walks of life, but there is little on academic knowledge as such.

There is some relevant material in the writings of scientists and philosophers of science - see, for example, the summaries by Baker (2010) and Fitzpatrick (undated, accessed on 30 September 2015). Simplicity, in the sense of reducing the number of entities or principles posited as far as possible, is widely regarded as a virtue in scientific theories. Baker (2010) gives several similar quotes from scientists including Einstein: "the grand aim of all science…is to cover the greatest possible number of empirical facts by logical deductions from the smallest possible number of hypotheses or axioms."

What these philosophical arguments largely omit is the user's perspective. Although Fitzpatrick undated) says that simpler theories "might also be easier to understand and to work with", this is an afterthought and is not followed up. Simplicity is viewed as a property of the knowledge itself, not of the user's relationship with the knowledge - which is the perspective that is important for my purposes here. Simplicity in the sense of the number of entities of one type or another may still be relevant, but users' familiarity with these entities is likely to be of greater importance. There is a brief discussion of this perspective on simplicity in Cycleback (2010, p. 157), and it is implicit in the penultimate sentence of the preface to the third edition of Simon's (1996) *The sciences of the artificial*: "... I propose that the goal of science is to make the wonderful and the complex understandable and simple - but not less wonderful."

Another strand of the simplicity literature concerns the design of products of various kinds. Again, most of this is too specific to be relevant for my purposes here, but one exception is the TED talk by Whitesides (2010). He makes the point that there is almost no drive for simplicity in the academic world, whereas in "the real world of people who make things ... there is an intellectual merit to asking: How do we make things as simple as we can, as cheap as we can, as functional as we can and as freely interconnectable as we can?" He also uses the word "stackable" - electronic components are simple in the sense that they are sufficiently reliable and predictable to be assembled into devices like mobile phones, and blocks of stone can be stacked to build a cathedral. If we think of academic concepts and theories as "things", I would say there is enormous merit in asking just the same questions of academic creations.

There are many textbooks in every discipline that aim to communicate their subject matter in as simple a way as possible, as well as research on teaching and learning. Sinatra et al (2014), for example, discuss "conceptual change" as a tactic for "addressing challenges to public understanding of science". The change in question, however, refers to changing the public's conceptual framework, rather than adapting the scientific framework as I am proposing here.

Some examples of how knowledge has been simplified in the past

One of the clearest historical examples of simplification is the adoption of the modern system of numerals in place of earlier systems such as the Roman one: e.g., 2023 instead of MMXX111. The modern system obviously makes arithmetic far easier and caters for arbitrarily large numbers (with the Roman system you need to invent more and more symbols as the numbers get larger). As well as being simpler the modern notation is also far more powerful in terms of what you can do with it.

Similarly, Copernicus's idea that the planets revolve around the sun was far simpler and made it far easier to understand the motions of the planets than systems like the Ptolemaic model (https://en.wikipedia.org/wiki/Geocentric_model) which necessitated an intricate system of wheels within wheels. In both cases, the newer framework has reached the commonsense level, so it is easy for modern readers to appreciate how much more complicated the older frameworks were.

This is not the case with many other areas of science. Take Newton's law of motion: $F=ma$. To those without the appropriate expertise to understand what the symbols stand for, this may not seem remotely simple. However, to those with the necessary expertise who appreciate the power and scope of this equation, its simplicity is staggering. Together with a few other laws from the same stable, it can be used to predict how stones move if thrown, how the planets move around the sun, how much energy you need to ride a bike, and so on. Before Newton there was no general framework for making predictions like these; it would have been necessary to use different rules of thumb in each scenario. It is in this sense that this equation, and others like it, are simple. However, many people do not have the necessary background to appreciate the simplicity of this equation, which raises the question: could this expertise be made even simpler? I think the scope here is limited - but it might be helpful to write the equation in Modified Excel format as suggested below.

More recently, in the late 1940s and 50s, so-called Feynman diagrams were introduced

> “as a bookkeeping device for simplifying lengthy calculations in one area of physics—quantum electrodynamics, or QED, the quantum-mechanical description of electromagnetic forces. ... With the diagrams’ aid, entire new calculational vistas opened for physicists. Theorists learned to calculate things that many had barely dreamed possible before World War II … By using the diagrams to organize the calculational problem, Feynman had thus solved a long-standing puzzle that had stymied the world’s best theoretical physicists for years.” (Kaiser, 2005)

This is an example where I, and I suspect most readers of this article, have not got the necessary background knowledge to appreciate either the problem or its solution. But it is an example of the value of simplicity at the leading edge of research.

Computer software is another interesting area where simplification is very much part of the goal. This has progressed from machine code which involves telling the computer exactly what to do in strings of 0s and 1s, to tools like *App Inventor* (http://appinventor.mit.edu/) which has simplified the process of writing apps (programs) for Android phones to such an extent that their claim that "anyone can build apps with global impact" is almost reasonable (although "anyone" is perhaps too strong).

These are examples of how knowledge has been simplified in the past. The important question, of course, is how present-day academic knowledge might be simplified. This sort of expertise often seems set in stone, unchangeable, but this attitude deserves challenge.

What is knowledge and how does it evolve?

What do I mean by knowledge? To some philosophers, knowledge is "justified true belief", but this is too narrow for my purposes here. Knowledge may concern, for example how to do something, in which case the criterion of truth does not apply. I am concerned with *academic* knowledge which I will define as knowledge of the kind that is learned, taught, discovered, or invented in the academy (colleges and universities and similar institutions). In general, this knowledge is, in some sense, difficult: otherwise, we would not need special institutions to deal with it. The context in which it is actually learned, taught, discovered or invented is irrelevant. Such knowledge may be communicated by teaching, textbooks, research papers, practical examples, YouTube videos, etc. The recipient may be a student or a fellow researcher or any other interested party whether in an academic institution or not. The only restriction is that the recipient should be human: communicating with monkeys or computers is a different ball game which is not my concern here. Knowledge is not the act of communication, or what's in the knower's head, but the content that is communicated, or, more precisely, that is intended to be communicated: as we all know sometimes people misunderstand some academic knowledge, and sometimes they forget essential points. So, the content of textbooks, academic papers etc. *is* the knowledge[3].

There are various words connected with knowledge whose appropriateness depends on the context and type of knowledge we are considering. By what criteria should it be evaluated: truth, correctness, beauty, usefulness, resistance to misinterpretation? Does it have an audience or users? There are obviously many types of knowledge and the terminology appropriate to one type may not be appropriate for other types.

It is helpful to divide - in rough terms - the evolution of knowledge into three stages. The first stage is the initial invention, or discovery, of new ideas - which I will call the *leading edge*. In due course, these new ideas are passed on to other workers in the discipline and taught to students of the discipline. This is the stage at which the standard ideas of the field are absorbed from textbooks, teaching or by some other means. I'll use the phrase *textbook stage* as a convenient label, although textbooks may not be involved in practice. A few people at the textbook stage will go on, after a suitable apprenticeship, to make new innovations at the leading edge. Finally, aspects of the expertise may be picked up, possibly in a distorted or popularized form by the general public: I will

[3] I know this is a little fuzzy, but it's good enough for my purposes. Tacit knowledge - ideas which are not made explicit but may be essential to a full understanding - could be included if this is part of the intention behind, for example, videos and practical examples.

call this third stage the *commonsense stage*. What is the relevance of simplicity to these three stages?

It seems likely that thinkers at the leading edge of their field will strive for simplicity. If the problem is difficult finding a simple way to look at it may be the only way forward. Einstein is said to have recommended keeping things as simple as possible, but not simpler.

However, there may be exceptions to this. If the problem is not difficult, making ideas simple may run the risk of making the pioneers appear rather ordinary. Making ideas difficult may be a way of keeping outsiders out and keeping the club exclusive[4]. And of ensuring the necessity of teachers and teaching institutions. It is tempting to formulate a law of simplicity at the leading edge of a discipline[5]:

> *If the problem is hard, it may be helpful to find a perspective which makes it easier, but if the problem is easy there may be an incentive to make it look hard.*

My main concern in this article is with the textbook stage. Becoming an expert means becoming familiar with the jargon, notation, conventions, and key ideas of the discipline. But there are, I will argue below, often unrecognized opportunities to simplify knowledge at this stage.

Finally, some of this expertise may filter through to the commonsense level, where it may be absorbed without formal education. This is obviously more likely if the knowledge is simple, but there is a danger that some knowledge may be absorbed in a distorted form - I will return to this problem below.

In practice, these three stages are a lot more confused than this summary might imply. There are often several leading edges if the pioneers disagree about the best way forward. And at the textbook stage, the trainee experts may have different backgrounds, motives, and technology from the original pioneers and from other trainee experts, so it may make perfect sense for them to learn different versions of the knowledge. And the distinction between the knowledge itself, and simplifications introduced to help learners, may be difficult to draw. If the simplified version is adequate for the learners' purposes, the simplified version should perhaps be treated as the knowledge itself. According to Kaiser (2005) Schwinger, who invented an alternative to Feynman diagrams (see the Introduction above), "sniffed that Feynman diagrams had 'brought computation to the masses.' The diagrams, he insisted, were a matter at most of 'pedagogy, not physics'". Which, of course, fails to take account of the fact that physics is of little use if nobody can understand it. Bringing computation to the masses sounds like a good thing to me.

AI as an alternative to simplification

There has recently (I am writing in May 2023) been a lot of publicity following the release of ChatGPT (https://openai.com/blog/chatgpt) and Bard (https://bard.google.com/). These "large language

[4] For an example see http://scepticalacademic.blogspot.com/2012/11/peer-regard-for-pot-noodles-critique-of.html.

[5] This is similar to Dawkins's Law of Conservation of Obscurity (see https://www.theguardian.com/books/2016/jun/03/what-is-so-special-about-things-that-never-happened-richard-dawkins-on-fiction-v-science

models" (Bowman, 2023) are widely described as "artificial intelligence" (AI) and can interact with the user in natural language and provide help with, or undertake, a wide range of problems and tasks. If the user does not understand something, she can ask the "chatbot" to explain, just like a patient teacher. The present generation of chatbots do sometimes get things wrong, but in time they are likely to get more reliable.

Doesn't this solve the complicated knowledge problem?

To some extent, yes. Digital technologies, from calculators onwards, are designed to do tasks which require intelligence, and so lessen the burden of dealing with our complicated web of knowledge. Statistical computer packages perform statistical calculations and relieve users of the burden of working through detailed algorithms. Word processors format documents and offer tips on grammar and spelling. Google search will find information about many things far more easily than was possible a few years ago. The present AI tools have taken this process further.

But none of these tools simplify the underlying knowledge. The calculator or the statistical computer package will do the sums, but the human being in charge still needs to understand something about the process and what the answers mean. The AI may take the user through a complicated argument, but the human user still needs to understand the answer and follow the argument in some sense to establish its credibility and relate it to other things. Clearly, the simpler the knowledge is the easier these processes are likely to be.

Examples of how current knowledge might be simplified

The three examples below are intended to illustrate the principle that simplification is possible and may be useful, not to establish how widespread this possibility is. To simplify something usefully requires a thorough understanding of the area (as de Bono, 1998, p. 283 points out), so my examples are all in areas I know something about - which obviously restricts the range of examples I can discuss. Whether a proposal is a genuine simplification, taking account of the audience and the context, is at the root an empirical question, so these illustrations should be regarded as unproven hypotheses.

I chose these examples to discuss in detail because they are relevant to many different contexts, my suggestions and how they compare with the standard approach are relatively easy to explain without assuming too much technical background, and because these case studies illustrate several tactics for simplifying knowledge which are listed in the subsections below. Each of the three examples involves a change to the currently accepted approach to the problem in question. There are some references to some more of my attempts at simplification in Appendix 3.

I am only discussing three examples in detail, but I think there will be many similar examples in other areas, so the combined effect of simplifications across the whole web of knowledge should be substantial.

Example 1: Modified Excel notation for mathematical formulae

The formula below is an example of standard mathematical notation. The function *f(x)* is the probability density of a normal distribution at the value *x*, but this is irrelevant here. I'm just concerned with the appropriateness of this symbolic representation of a mathematical relation.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

To a mathematician, this is beautifully concise. However, people without a mathematical background are likely to have a few issues. What is e and how does it relate to the symbols above and to the right of it? What do the symbols mean? And how can we type it into a computer so that we can work out what it comes to in specific situations? My suggested simplification to help with problems like these is a *modified Excel notation*. The only modification in this particular example is the use of three different types of brackets which help to clarify how the brackets match up. The equivalent formula is (*npd* stands for normal probability density, and *sd* for standard deviation):

npd = {1/[sd*SQRT(2*pi)]}*EXP{-0.5*[(x-mean)/sd]^2}

The key points are that the formula is a sequence of symbols that can be typed on an ordinary keyboard which means that it can easily be made compatible with computer software; it is possible to search for the meanings of function names like EXP and operations like ^ with a search engine; and the variables (for which I've used lower case letters) and function names (upper case) are designed to be recognizable words or abbreviations.

This idea could be extended to cater for a wider range of mathematical concepts – integrals, vectors and so on. For example,

$\int_1^3 2x\, dx$

could be written as

INTEGRAL([3, 1] 2*x, x)

where INTEGRAL is a new function which would need to be carefully defined. (It is obviously helpful if function names are as short as possible, but the obvious abbreviations could have been confused with INT or INTEGER.) There is a more detailed discussion of this idea and its pros and cons in Appendix 1.

Kramer (2023) puts the case for another style of notation: "Iconic Mathematics: Math Designed to Suit the Mind". Icons are "more concrete than symbols, and often illustrate their own meaning, are their own mnemonic devices, and hint at their own intended use" which he suggests "ought to suit the very people traditional mathematics leaves behind." People with different psychological profiles may find different notation systems more amenable. Perhaps my suggestion above of a modified Excel notation indicates that my psychological preference is for words rather than symbols or icons?

My aim here is not to claim that a specific proposal is ideal, but simply to point out that it is possible to invent new notation systems for mathematical concepts, and this may sometimes be a useful tactic. Mathematics is not the symbols, but the concepts they refer to. The integral symbol (∫), for example, was introduced by Gottfried Leibnitz (1646-1716) more than 300 years ago when the world was a very different place. Perhaps it is time to consider alternatives which are more convenient with current technology, or more in line with the psychological profile of particular groups of people?

Example 2: Exponential growth

I used to teach reliability theory to M.Sc. students. One of the key formulae is the reliability function for the exponential distribution[6]:

$$R(t) = e^{-\lambda t}$$

or in Excel format (λ is the failure rate which I'll call fr, and t is the length of a time interval):

R(t) = EXP(-fr*t)

This formula involves the *exponential function,* e^x: it was easy to explain to the students how to work it out with a calculator or the spreadsheet Excel. Calculators typically have a button for e^x, and Excel has a built-in function EXP. What was not so easy was to explain what the function meant and where it came from. Historically (according to Wikipedia) the concept of *e* was introduced by Jacob Bernoulli in 1683, and the symbol *e* was first used by Leonhard Euler in 1727 or 1728. It has mathematically interesting and powerful connections to logarithms, calculus, infinite series, and imaginary numbers, as well as its role in modeling exponential growth.

Most of my students, however, had either never met all this mathematics, or had long since forgotten it and had no wish to revisit it. So, I was forced to present the calculator or computer as a black box: you put some numbers in and you get the answer out, but you really don't know what's inside the box.

Does treating the exponential function as a black box matter? Yes, it does, because although you can get the answer from a black box, you will probably not understand in any detail what the answer means or why the method works. You'll be far less likely to notice mistakes, and you're much less likely to see any flaws in the answer, or take the method further, or use it in new situations. You're also more likely to forget the whole thing completely. If you really understand the mathematics behind it, the formula above is little more than common sense. Thorough understanding really does have lots of advantages.

The exponential function, e^x, is helpful for analysing growth or decline - such as compound interest, or the growth of populations of people or bacteria under certain conditions, or the decline in the numbers of unreliable components that are still operational. It crops up in many mathematical formulae. My suggestion is that if this mathematics were to be reformulated with the *Compound Growth Multiplier* (CGM) as its core concept instead of e^x, its meaning and rationale would be far clearer to people like my MSc students, and it could be completely disentangled from theories of calculus, logarithms, infinite series and so on.

To see how this would work, let's take a simple example. If you invest some money at an interest rate of 2% per year, over a period of 40 years it will generate interest and at the end of 40 years the total amount of money will have grown by a factor of 80% (40x2%). This is simple interest, so we can call 80% the *Simple Growth Multiplier* (SGM). However, if the interest is added to the investment throughout the 40 years so that you get interest on the interest – known as compound interest – the *Compound Growth Multiplier* would be 122.6% (see Appendix 2), which is considerably more than

[6] See, for example, https://accendoreliability.com/using-the-exponential-distribution-reliability-function/.

the 80% you would get from simple interest. How we can work out CGM from SGM is explained in Appendix 2: logarithms, infinite series, calculus and so on do not appear in this story. The answer can be looked up in a table like Table 1, or worked out from the mathematics which could easily be built into Excel, or other software, as a standard function.

Table 1: Compound Growth Multipliers (CGM) from Simple Growth Multipliers (SGM)

SGM	CGM	SGM	CGM	SGM	CGM	SGM	CGM
-1	-0.632	0	0.000	1	1.718	2	6.389
-0.8	-0.551	0.2	0.221	1.2	2.320	2.2	8.025
-0.6	-0.451	0.4	0.492	1.4	3.055	2.4	10.023
-0.4	-0.330	0.6	0.822	1.6	3.953	2.6	12.464
-0.2	-0.181	0.8	1.226	1.8	5.050	2.8	15.445

These figures refer to the growth not the total at the end of the time period. And, obviously, to convert to percentages they should be multiplied by 100: e.g. the bottom row in the second column means that an SGM of 80% corresponds to a CGM of 122.6%.

In a similar way the reliability formula would be:

R(t) = 1+CGM(-fr*t)

This formula is an obvious application of CGM (given an understanding of the scenario being modelled), not a mysterious formula to be learned and used with little understanding.

Using SGM and CGM to model growth instead of e^x [EXP(X)] is simply a reformulation of the traditional approach using different core concepts. The relationships between the two sets of core concepts are:

CGM(sgm) =EXP(sgm)-1, SGM(cgm) = LN(cgm+1)
EXP(X) = CGM(X)+1, LN(X) = SGM(X-1)

(LN is the Excel formula for the natural logarithm, which is one of the concepts the SGM and CGM framework manages to avoid.)

Example 3: Statistics: the strength of evidence for a hypothesis

Statistical concepts like correlation coefficients, standard deviations, regression analyses, significance tests and so on, are widely used but often with little understanding of their background or rationale. Mistakes, misunderstandings, and general bafflement are common. It is an area crying out for appropriate simplifications: there are references to some of my attempts in this area in Appendix 3 and the References at the end of this article. One particularly problematic area is significance testing (p values), which is the subject of this subsection.

The p-value, or significance level, is a very widely used way of deciding if the evidence for a hypothesis stacks up. Low p values indicate that the chance explanation, based on a “null hypothesis”, is unlikely, so the hypothesis of interest must be right. In most contexts the mathematics behind the calculation is complicated, but this is usually done by a computer package so is not an issue for the researcher.

The difficulty is, however, that p values may be, and very often are, misinterpreted. This problem is very widely acknowledged (see, for example, Wasserstein and Lazar, 2016), but just as widely ignored. In many domains of study p values (otherwise known as significance levels) are still the gold standard for assessing statistical hypotheses. In other areas, the problems are acknowledged, and journals will not accept articles citing p values, but the difficulty is that there is no generally accepted alternative.

The first, obvious, remedy would be to use more transparent jargon, or at least to avoid misleading jargon. The word "significant" in ordinary English means large or important, whereas the statistical meaning is (roughly) "unlikely to have arisen by chance". There is a strong case for avoiding the term "significant". Wood (1984) suggests that the p value could be described as the *plausibility* of the null (chance) hypothesis, which has the advantage of giving a rough idea of the meaning: low p values suggest that the chance hypothesis is not plausible.

However, a more serious problem with *p* values is that they do not tell you what you probably want to know - which is how likely the hypothesis of interest is, as opposed to how plausible the null (chance) hypothesis is. One suggestion is to cite confidence intervals instead of p values (Gardner and Altman, 1986). Wood (2019) has suggested building on this concept to calculate confidence levels for hypotheses which could be called "tentative probabilities"[7]. Then, instead of qualifying the conclusion that "patients treated by female surgeons were slightly less likely to die within 30 days" with the statement "$p = 0.04$" (Wallis et al., 2017), we could qualify it with by writing "confidence level, or tentative probability = 98%". Unlike the p-value, this gives the reader a direct assessment of how likely the hypothesis of interest is to be true. "Confidence" is a standard statistical term with a meaning similar, but not identical, to probability. The advantage of the term "tentative probability" is that it makes it clear that we should not have too much confidence in the idea because it depends on assumptions that may not be fully satisfied.

Tactics for simplifying knowledge

The three examples above illustrate several tactics which may be useful for simplifying knowledge: these are outlined in the six subsections below. The word "consider" below is intended to emphasize that these are just tentative suggestions.

Consider alternatives to the historical development of the subject

There is a tendency for the way a subject developed historically to become entrenched as the one truth, as the only viable possibility. Modern mathematical notation (Example 1 above), and the theory of the exponential function (Example 2) were developed over 250 years ago and the modern approach, which is mirrored in textbooks and other teaching materials, largely follows the historical development. The widespread use of significance tests and *p* values is probably largely due to the

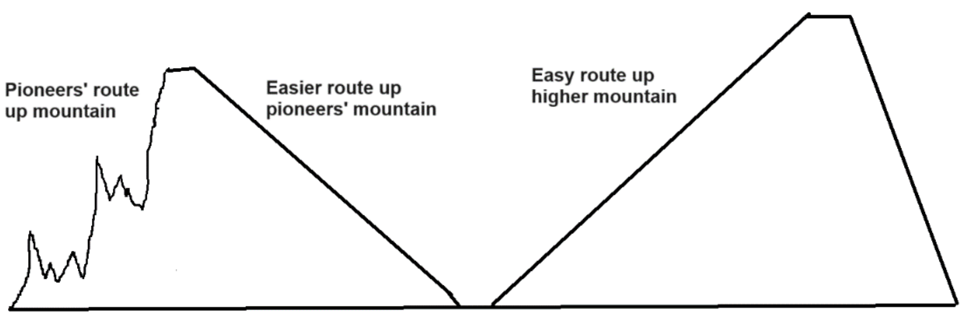
[7] I published this proposal in a research methods journal. The reaction of a reviewer for a statistical journal was withering - the problem being that confidence is considered a completely different concept from probability, which in my view shows a distinct lack of imagination.

influence of Ronald Fisher in the 1920s, but textbooks and courses still follow his lead despite the problems discussed above. The three examples above demonstrate that there may be alternatives which are more appropriate for particular circumstances and potential users.

Normally the training of experts - whether formally by courses in schools, colleges, and universities, or informally by picking up the basics of a discipline by reading textbooks or research papers, follows the path taken by the innovators on the leading edge. The hierarchy of concepts used, the jargon, and the methods of research and argument, are likely to mirror the historical development of the discipline. When they reach the leading edge, the typical expert understands thoroughly all the steps taken to reach the pinnacle, so this may well seem like the only possible route up.

However, this may not be the only route up. Having got to the top the view may reveal easier routes up, or even better mountains to climb. The followers may also be able to avoid some of the problems of the pioneers by using the ropes left by pioneers: the followers may be able to jump over a few hurdles without worrying about what lies in the chasm below. In short, there might be alternative, easier, routes which the followers could take. However, this may not be obvious to those who have reached the leading edge for whom the route they took may seem the only way up. This is likely to apply to senior academics whose work provides the basis of academic curricula: they are unlikely to be receptive to the idea of simplifying academic knowledge. And this attitude is likely to be reflected in textbooks which are likely to follow the pioneers' route on the left of Fig. 1 rather than the two easier routes.

Fig 1. Three routes up two mountains



But ... you may object that trainee experts should familiarize themselves with the "proper" version of the expertise. Anything else is giving in to the temptation to cut corners. To which the answer is: what's wrong with cutting corners?

Consider changing jargon and notation

Researchers at the leading edge of a discipline often need to invent names for concepts, and notation systems to facilitate communication. For workers at the leading edge of the field, the precise nature of this jargon is probably not a big issue because they will absorb it by repeated exposure. For newcomers, on the other hand, jargon is often a problem. Sometimes it is overly complicated, sometimes it has no relation to the audience's frame of reference, and sometimes jargon may be likely to mislead. This is an issue in all three of the examples above. The first concerns mathematical notation, the second suggests using concepts with obvious names and interpretations instead of the traditional exponential function and natural logarithms, and the third concerns the concept of statistical significance which is a disaster in the hands of people who don't understand what it means.

Consider the conclusion[8] that "orthoptists in the GSC [German-speaking countries] preferred using spectacles plus occlusion as their first-choice treatment, significantly more than their UK counterparts who preferred spectacles only as their first choice ($p<0.001$)" (Tan et al, 2003). Interpreted as ordinary English, the word "significantly" implies that the difference between the GSC and the UK is a large one. Statistically, however, this is not the meaning at all: the word significantly means that the results obtained in the study were unlikely to have arisen if there were no systematic differences between the GSC and the UK[9] - a convoluted concept that makes the misinterpretation of the word "significantly" almost inevitable. It implies nothing whatsoever about the size or importance of the difference.

The design of transparent jargon to avoid such problems should be a flourishing area of research - which, to the best of my knowledge, it isn't. Names invented by leading-edge researchers have a tendency to stick when more user-friendly alternatives might be helpful. Often concepts are named after their originator, or some key person in the history of the field. Bayesian statistics, for example, is named after Thomas Bayes, an eighteenth-century clergyman who first came up with the theorem on which this approach to statistics is based (although it was not published until after his death). A name like Updating Probability Methods would give a better impression of the nature of the Bayesian approach which involves updating probabilities as you acquire more data.

Consider changing core concepts

My suggestion in Example 2 above was to express the theory of exponential growth in terms of CGM and SGM instead of the traditional e^x and LN(x). This involves using *core concepts* which are more clearly and directly related to the concerns of the students. Similarly, in Example 3, confidence levels or tentative probabilities may be a more useful core concept than p values or significance levels.

Reduce pre-requisites as far as possible

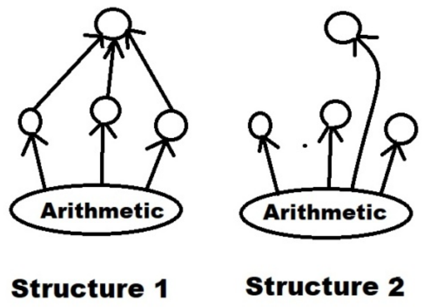
One of the advantages of the suggested alternative to the exponential function (Example 2 above) is that the only prerequisite for understanding it is basic arithmetic, in contrast to the way I remember learning about it which involved calculus, power series and logarithms. The principle here is to have as few prerequisites as possible so that different bits of the web of knowledge can grasped independently of each other. Figure 2 below shows two different structures for a domain of knowledge: the circles represent bits of knowledge, and the arrows represent the fact that the circles at the start of the arrow must be grasped before the ones at the end. Obviously, Structure 2 has the advantage that people can just choose the bits they are interested in, whereas Structure 1 requires knowledge of all the circles to get to the top.

The title of a book by George Hrabovsky and Leonard Susskind (2014) - *The Theoretical Minimum: What You Need to Know to Start Doing Physics* – sums up the issue neatly. The pre-requisites – what you need to know to understand the knowledge in question – should be as few as possible.

[8] I chose this example more or less at random: the idea of statistical significance is routinely used in many different areas of science.

[9] Studies with a large sample size can yield a highly significant result (low *p* value) even if the difference between the two groups is tiny.

Figure 2: Different pre-requisite structures



Consider using a different approach to the problem

Example 2 above (the exponential function) is just a reformulation of standard knowledge. Everything in the standard version can be translated *exactly* to the new version and vice versa. With Example 3, on the other hand, I proposed a new approach to the problem by estimating a confidence level for the hypothesis of interest instead of a p value for the null hypothesis. The new version tackles the same problem, but it provides different information using different concepts. It is a minor example of a paradigm shift.

Consider focusing on general ideas

Obviously, if you've made the effort to get to grips with a new bit of knowledge, the more you can do with it the better. Traditional statistics textbooks, for example, tend to cover a whole catalogue of different statistical tests – these are recipes for working out p values or significance levels (Example 3 above). However, the important thing is the general principle (see, for example, Wood, 2003, Chapter 8); once you've understood this you should then be in a position to look up the recipe for your particular situation or use the appropriate menu option in a computer package.

Further issues about simplifying knowledge

The subsections below contain a brief discussion of other possibilities and concerns.

Simplicity is not simple: there may be conflicting criteria

Appendix 1 lists three reasons for preferring my modified Excel notation for mathematical formulae (Example 1), and three reasons for preferring the conventional version. Similarly, the choice between p values and alternatives in Example 3 is not a simple one. P values are simple to work out (with a computer package) and cite and appear to have a straightforward interpretation, but this interpretation may be seriously misleading. In Example 2 I think the arguments are more clearly on the side of my proposal, but the inertia of habit and history means I can't see it catching on. There will always be pros and cons which need careful consideration.

People may have different purposes and agendas

Different people may have different motives for learning about something. For example, the pure mathematician, and someone who wants to use a mathematical theory for a particular purpose, may be best served by different types and levels of simplification.

Experts and universities won't like new simple versions

Any change to the established order may meet with resistance just because that is what people are familiar with, courses are set up to teach it, and so on. Any changes will just feel wrong. If it works, don't change it, is often excellent advice, but it ignores the fact that occasionally change might help. So-called "normal science" is often a good idea, but sometimes a "paradigm shift" may be beneficial[10].

My first articles on the general topic of simplifying knowledge (Wood, 2002a and 2002b) were inspired by the title of a *Telegraph* leader column: "Maths should be hard"[11]. My point is exactly the opposite: Maths should be as easy as possible. However, this is often seen in the educational world as letting "standards" slip, or “dumbing down”. The business model of colleges and universities and many experts implicitly depends on keeping knowledge hard so that the need for their services is maintained. If knowledge were too easy, university courses and expensive experts may no longer be necessary. Or they would need to adapt their game to work at a level which is genuinely hard.

Peer reviewers are unlikely to approve new simple versions

The peer review system could have been invented to discourage change. Almost by definition, the peer reviewers used by journals to vet articles are steeped in conventional ways of looking at their subject and so may not be receptive to alternatives. (I am fairly sure that peer reviewers would be unlikely to endorse any of the suggestions in Examples 1-3 above.) There is a strong case for supplementing peer reviews with reviewers from outside the discipline (Wood, 2021).

Transparency is good

How much insight does knowledge give users into its justification and where it might be used? The compound growth framework (Example 2 and Appendix 2) seems far more likely to provide such insight than a superficial understanding of exponential functions and natural logarithms and calculus. Such insights are important for several reasons: users are less likely to make errors and are more likely to be able to adapt the framework to different problems, misinterpretations are less likely, and ideas are less likely to be forgotten. Similarly, the idea of tentative probabilities (Example 3 above) should make far more sense to the relatively uninitiated than the idea of p values.

Simpler methods may be as powerful or more powerful

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[10] https://en.wikipedia.org/wiki/Normal_science

[11] https://www.telegraph.co.uk/comment/telegraph-view/3571087/Maths-should-be-hard.html

Simple should not be assumed to mean inferior. The alternative framework suggested for the exponential function in Example 2 above can easily be converted to the standard functions and vice versa. In the history of science, new simple methods are often much more powerful - for example, the decimal system for numerals and Copernicus's and Newton's innovations discussed in the Introduction above.

Changes in technology will affect the best approach

This is entirely obvious, and I have mentioned it briefly in the Introduction. Computers and calculators mean that the simplest way of doing arithmetic no longer needs long multiplication, log tables and slide rules. Artificial intelligence and Internet search engines have made some human knowledge redundant, but the interface between these technologies and human knowledge is worth very careful consideration.

Simulation and trial and error may be useful

What proportion of three-child families comprise three girls? For readers familiar with probability theory the answer is obviously one in eight (1/8 or 12.5%). But if you aren't familiar with probability theory you could *simulate* the situation by tossing three coins lots of times and checking how many times you get three heads (letting a head represent a girl): the answer will be about one in eight.

Now imagine you know how to multiply but not divide and you want to know what you need to multiply 6 by to get 42. Not knowing about division, you might start by guessing the answer is 5 and work out 5 times 6 and get 30. Obviously too small so try, say, 8. Too big. With luck, you will eventually home in on 7.

These are elementary examples of tricks mathematicians and statisticians use when the problem is too difficult to come up with a neat formula - which does actually happen a lot[12]. From the perspective of someone without the necessary mathematical background, these methods have two big advantages. First, they get the answer without using any extra technical concepts. Second, they are generally more transparent because you can see how they work.

I have called methods like these *crunchy methods* because you crunch through problems without using clever mathematical trickery (Wood, 2001). In practice, in more advanced domains, computers are a must for implementing methods like these (e.g., Simon, 1992; Wood, 2004 and 2005).

Just-in-time knowledge: defibrillators, recipes, Alexa and ChatGPT

Defibrillators, or, strictly, automated external defibrillators (AEDs), are designed to diagnose and treat cardiac arrests. Further, they are designed to be used by people without training: the device speaks to the person operating it and tells them what to do. This person is thus told what they need to know when they need to know it. It is just-in-time learning which is similar in principle to just-in-time manufacturing, the system whereby components arrive as they are needed thus avoiding the need to carry extensive stocks.

There are many more elementary examples. Any set of instructions that aims to deliver wisdom only when it is needed uses this principle: for example, recipes for cooking can be followed without an

[12] e.g. Integrals are often evaluated by guessing the answer and then differentiating to see if you are right.

extensive preparatory course. And Alexa and AI-powered chatbots like ChatGPT can, in theory, answer questions and deliver wisdom as and when it's needed.

The trade-off between learning well in advance *just-in-case* the knowledge is needed, and learning *just-in-time* when needed, always was an important consideration. Just-in-time learning avoids the problems of forgetting, of learning lots of things that are never needed and perhaps failing to learn things that do turn out to be necessary. On the other hand, the just-in-time approach may lead to a shallower understanding. However, with the expanding amount of human knowledge, and smarter tools to implement it, the balance seems likely to swing increasingly in favour of the just-in-time approach.

Keep it short

I have three books on my desk as I write this. Richard Dawkins' The God Delusion is 400 pages long. I've dipped into it, and it looks interesting, and I'm sure God is a delusion, but I am not prepared to read 400 pages on the topic. It's far too long. On the other hand, Bertrand Russell's History of Western Philosophy, and the third book on my desk - a maths textbook - are OK because these are collections of relatively brief expositions of particular topics. The bible is also OK for a similar reason.

Many books are too long. In the days when most people read things on physical pages, the book was a convenient size to sell and read. Now, of course, the advance of technology means that we can be more flexible over the length of documents. A similar argument applies to many (by no means all) academic articles which would benefit from more focus on the main argument and less on reviewing the field: interested readers can easily be given links to suitable review articles. But, as always, habits last well past their best-before date, and authors who want to make their mark feel the need to expand their efforts to fill a decent-sized book or article.

Suppose Richard Dawkins had reduced the God Delusion to 50 pages. Then I would have the time to read it and 8 similar works in the time it would have taken me to read the 400-page tome. This principle is the basis of the app and website https://www.blinkist.com/ which provides summaries of books with the slogan "More knowledge in less time." Wikipedia is also an important source of summaries of ideas for people who do not have time or inclination to consult the original. But, of course, there are dangers in taking this principle too far.

Novels are an interesting case. I think many of these are too long, but there is also a very reasonable argument that the point of many novels is to get the reader absorbed in the story which takes time, so long may be good.

Words, symbols, icons, graphs, pictures, stories, videos, etc.

It is important to remember that there are many different means of communicating knowledge: words, symbols, icons, graphs, videos, etc. Within each type of medium, such as words on a page or screen, there may be different genres: philosophical arguments, stories, and so on. Some things we take for granted now, like time series line graphs, are comparatively recent inventions and would have bewildered audiences a couple of hundred years ago[13]. Kramer (2022 and 2023) discusses the use of icons (e.g. the concept of “future” might be “the icon of a clock surrounded by a clockwise

[13] https://www.newyorker.com/magazine/2021/06/21/when-graphs-are-a-matter-of-life-and-death

arrow") and points out that "on smartphones and computers, writing icons can now be faster than writing alphabetic words" (Kramer, 2023, p. 1).

In the future, new possibilities are likely to surface and become accepted as technology progresses and habits change, and the need to make knowledge more accessible becomes more urgent. The options are expanding rapidly, and which is best obviously depends on the particular context.

What's NOT worth knowing

Nobody knows everything about anything. Possibly a slight exaggeration depending on the precise meaning you attach to "everything about", but as a general principle it's important to think about what's *not* worth knowing. I know a fair amount about statistics, but my understanding of how the normal distribution formula is derived is hazy, to put it mildly. What's more, I don't care; I don't think it's worth the investment in time and energy to find out.

The rise of AI (artificial intelligence) adds another dimension to this problem. It seems likely that as AI advances, we will entrust more and more to computer systems with humans needing to know less and less about how they work. This raises many obvious opportunities, but also the danger that AI systems may not serve the purposes for which they were designed[14].

In a sense this is a continuation of a trend that has been going on for the last 3000 years. In this period human brain size has been decreasing which, according to DeSilva et al (2021), may be the result of "the externalization of knowledge and advantages of group-level decision-making due in part to the advent of social systems of distributed cognition and the storage and sharing of information." According to this hypothesis, now that we have computers, books and large communities of other people to help us, we no longer need to think as hard and can make do with smaller brains. Or, of course, we can do more with the brains we have if we externalise as many tasks as possible.

Sometimes knowledge can be simplified simply by ignoring anything that adds no value. Whitesides (2010) quotes Antoine de Saint-Exupery: "You know you've achieved perfection in design, not when you have nothing more to add, but when you have nothing more to take away." When the elusive Higgs boson particle was finally detected by researchers at CERN the result was described as having "a statistical significance of five standard deviations (5 sigma) above background expectations. The probability of the background alone fluctuating up by this amount or more is about one in three million" (from the CERN website in April 2015). The essential figure here is one in three million[15]: the 5 standard deviations are very difficult to make sense of and add nothing at all to the argument or the conclusions. Obviously, this is a trivial point, but the danger is that the audience's attention will be deflected from the Higgs boson to the essentially pointless issue of the meaning of "5 sigma".

Don't forget the aesthetic dimension

In the science of physics, beauty is sometimes seen as a guide to truth. If it's beautiful it's more likely to be true according to some physicists, although others will say that the universe is messy so a good physical theory should reflect this. But regardless of one's view about this, it does seem to me undeniable that beautiful theories are likely to have the advantage of being more pleasurable to

[14] See, for example, https://en.wikipedia.org/wiki/AI_alignment.
[15] This is a good example of where statistical significance and p values do make sense. One in three million is a p value.

develop and use. Ideas that are fun and inspiring will almost inevitably be absorbed more efficiently and used more productively than those that are seen as dull and boring. If the discipline of statistics is seen as boring and ugly, people will make little effort to master it. If, on the other hand, it can be made fascinating and elegant everyone will be better off.

Truth is meaningless, usefulness is everything, and simpler is more useful

Finally, some ideas from cognitive psychology and evolutionary theory. If we accept the theory of evolution, how our minds work must have been shaped by natural selection. Donald Hoffman (2015) takes this argument further and claims that organisms that see the world as it really is will be at a disadvantage compared with organisms that evolved perspectives based solely on evolutionary fitness. Given that we are the product of evolution this means that what we see as the truth about reality is likely to be the product of evolution: it is a convenient illusion which may have little connection with reality[16]. And given that our theories about reality are mere illusions and not copies of reality in any sense, we might as well make use of the freedom this gives us to be creative and make these illusions as appealing, and as simple to comprehend and manipulate as possible.

The problem of over-simplification

Obviously, the simpler knowledge is the better, provided the simple version is at least as good as the complicated version. However, it is possible that sometimes the simple version may be a lot worse: it may be a very rough approximation, misleading, useless, or simply wrong.

Simple statements like "The Swiss are much happier than Nigerians", or "Exposing yourself to strong sunlight is a bad idea" or "There are two types of social research - quantitative and qualitative", or "Britain could derive a huge amount of energy from renewable sources" are - for many purposes - oversimplified and potentially very misleading. Some Swiss are doubtless much more miserable than some Nigerians: the generalization just refers to averages. People who don't expose themselves to any sunlight run the risk of vitamin D deficiency. If you think there are only two types of social research you will probably identify the analysis of subjective feelings with the qualitative pole, ignoring the fact that there is a whole industry based on assessing subjective feelings on 1-5 scales analyzed with quantitative statistics (Wood and Welch, 2010). And it is not enough to know that the potential of renewables is "huge"; we also "need to know how it compares with another 'huge,' namely our huge consumption" (MacKay, 2008). The use of adjectives like "huge" oversimplifies the problem: "to make such comparisons, we need numbers, not adjectives."

Politicians need neat soundbites like "we must balance the budget" because "the economy is like a household", which of course it isn't. This is an oversimplification, but any more nuanced view would be unlikely to have as much impact on the electorate. Which is a problem.

In 2008 the world economy plunged into a crisis triggered, according to Silver (2013), largely by over-simplified, and horribly inaccurate, measures of risk. Banks and other financial institutions buy and

[16] I have said these illusions *may* have little connection with reality whereas Hoffman seems to make the stronger claim that they *will* have little connection with reality. His work is largely based on computer simulations involving perception. To me his basic conclusion seems reasonable, but whether one can reasonably generalise from his simulations must depend on the contexts he is simulating. And there is also the issue that his conclusions presumably apply to his conclusions which themselves must be just illusions. It's not clear to me what the term "reality" means in this world view.

sell securities like mortgages, and they obviously need an estimate of the risk of losing their money on these transactions. In practice, estimating these levels of risk is a complicated task, so they simplified the problem by using the assessments of rating agencies - these organizations categorize risk into bands. AAA, for example, is intended to mean a probability of default of 0.12%, but during the crisis, 28% of these securities defaulted. This led to losses far greater than expected, and to the necessity to bail the banks out and all the other consequences of the crisis.

Even organizations whose job was to manage financial risk felt the need to use simplified measures provided by a third party. And the reason that these risk assessments were wrong was that the rating agencies over-simplified the task of making these assessments. They made an assumption - that the component risks were independent of each other - which was a long way from the truth, leading to a massive underestimation of the risk of default. This was an elementary mistake, but without it, the task of assessing risk becomes difficult and the answers become vaguer, so the temptation to make the simplifying assumption was probably overwhelming. Perhaps with a more thorough understanding of how the probability theory works these mistakes would have been less likely.

There are many ways in which we can oversimplify things, but it is worth mentioning one that seems to appeal to the academic mindset: over-extending or over-simplifying appealing concepts. One such example is the idea of statistical significance (Example 3 above). And, as should be apparent from what I have said above, the concept of simplicity itself is far more multifaceted and problematic than my use of it as a slogan might make it appear. Many subdivisions into two or three categories (simple/complex, qualitative/quantitative, left-wing/right-wing, heavy drinker/light drinker/teetotaler) run a serious, and often unacknowledged, risk of over-simplification. Simplification is a powerful and seductive tool of thought, but we need to be very careful.

Over-simplification is certainly a problem. But the cause is often that a more appropriate understanding is too complicated to make sense of, so we may have little option but to go for the crude and unhelpful over-simplification. Complicated knowledge may push us into dumbing down. The remedy is dumbing up: sensible and thoughtful simplification.

Finally, there is the concept of simplicity itself. A realistic assessment of how simple a piece of knowledge is needed to take many factors into account: the audience and what they know already, what the knowledge will be used for, what technology will be available to assist in the learning and use of the knowledge, whether the knowledge facilitates a deep understanding of its meaning, rationale and limitations, how broad the domain of application is, and so on. Oversimplifying the concept of simplicity runs the risk of ignoring some of these dimensions.

How could we get hard evidence that simplifications are beneficial?

Throughout my teaching career (in colleges, universities, and short courses for businesses) I have often thought that what I was teaching could be simplified in a way that would preserve the usefulness of the original but would be easier for my students to understand and use. I am not talking about the presentation of the ideas but the ideas themselves. Sometimes the simplifications just involved ignoring unhelpful ideas and focusing on the simple and useful ones: this was easy to incorporate in teaching notes (e.g. https://soths.files.wordpress.com/2019/01/RmNotes.pdf).

However, sometimes this was not possible because the simplifications I was envisaging went beyond focusing on some aspects of established knowledge rather than others. Here I would have been inventing essentially new knowledge which was designed to be simpler or more appropriate for the purposes of the users, than the standard fare. The students were expecting to be taught one set of concepts, and I wasn't in a position to say, "I'm teaching you this instead because I think it'll be more useful". I may have been wrong, and certainly did not feel justified in imposing my ideas on unsuspecting students. The wisdom of this decision is perhaps reinforced by the fact that the better alternative perspectives tended to become clearer after I had stopped teaching these topics, and I was less concerned about the short-term teaching problem and more interested in mulling over the general problem of how human cognition could be improved.

What was needed was research on simplifying knowledge which might, or might not, have confirmed my hypotheses: ideally a trial to see if a simpler version of something really was better than the standard version, along the lines of clinical trials to evaluate new medical treatments. But the problems of organizing such a trial are obviously massive. What measures would be used to evaluate the supposedly simpler knowledge? The trial subjects would need to agree to the trial, and there would be a problem with learning something non-standard that was not consistent with other knowledge: in Example 2 the trial subjects would inevitably come across versions of Excel with the functions EXP and LN but not CGM and SGM, and in Example 3 they would be a disadvantage when they inevitably come across references to p values. The fact that knowledge is an inter-connected web makes it difficult to change just one bit.

But I hope someone will attempt such a trial at some stage.

Conclusions: can we put quantum mechanics in the primary school curriculum?

Conventionally, academic knowledge is viewed as a given: it should not be fiddled with to suit the requirements of users and the context of use. I think this is a mistake: it should be designed, or redesigned, to make it as simple as possible for the context of use. We have looked at a few examples where knowledge might usefully be simplified. These are just a few areas I know something about: there are likely to be many similar opportunities across the whole spectrum of human knowledge. It is in areas which seem trivial to experts, but difficult for novices, that the biggest benefits of simplification are likely to lie. The difficulty is that, in most areas of academic knowledge, there is no tradition of trying to design simpler (or more appropriate for particular contexts or audiences) versions of well-known concepts, techniques or theories.

The benefits brought by such simplifications are potentially enormous. Let’s imagine that some difficult knowledge can be simplified by a factor of about a quarter. (I think in many cases this could be nearer 50% or even more.) This might mean that people spend 25% less time learning about it, or using it, or that the simplification means that they can master 25% more than they would otherwise have been able to, or that they make 25% fewer mistakes, or that 25% more people are able to master it. Over the whole spectrum of difficult knowledge, this has the potential to make an enormous difference. Imagine that students around the world could spend 25% less time on their studies. The rest of the time could be spent taking their studies further or doing something completely different. And researchers would arrive at the frontiers of their discipline sooner, or with a better understanding of other relevant areas, both of which should lead to faster progress.

Why isn't this happening already? The main reason is probably that nobody seems to have pursued the idea as a general principle, perhaps because of the assumption that academic knowledge expresses the truth, and the truth is fixed and not adjustable to suit the circumstances. Or because the only people in a position to produce a useful simplification are the experts in the domain who are unlikely to see the need - for them the expertise is trivial and an essential part of the discipline. Like all paradigm shifts, the most likely champions are outsiders, but outsiders may not have a sufficiently deep understanding to create a viable alternative.

But whatever the reason, the idea of simplifying knowledge does not seem to be on anybody's agenda. This lack of interest is reinforced by the complete absence of academic journals on the theme of simplification. To get published, an academic needs to produce something complicated; a simple version of an existing idea is rarely viewed favorably by the gatekeepers of academia. There is also the conservative influence of the education system, and the attitude of teachers at all levels that "standards" should be maintained, that knowledge should not be "dumbed down", and that, in the words of the *Daily Telegraph* leader column "maths should be hard".

I would like to see simplification on the agenda. Two projects, for example, appeal to me. The version of statistics taught in the academic curriculum is a major source of confusion and incomprehension in urgent need of reform. And I would love to understand quantum mechanics at a deeper level than I do, but I think I need to wait for a version more in tune with my aptitudes and the time I have available. Such a version is probably right outside the possibilities envisaged by current physicists but, eventually, I suspect there will be a version of quantum mechanics which is appropriate for primary school children.

If we don't start to simplify our increasingly complicated web of knowledge, human progress will slow down or cease as our minds become clogged with unnecessary technicalities, and inevitable over-simplifications take control of our thoughts and actions**.** We need to see the design of elegant, fit-for-purpose, perspectives as an important task for academia.

Obviously, there can be no general recipe for achieving this: the best I can do is to suggest some general tactics which may be helpful. These are listed in the sections above on Tactics for simplifying knowledge and Further issues about simplifying knowledge.

I look forward to hearing some more suggestions from readers of this article.

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Appendix 1: Modified Excel notation for mathematical formulae

The formula below is an example of standard mathematical notation.

Conventional formula (1)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

The Excel version of the formula for the right-hand side of this equation is:

Excel formula (2)

=(1/(sd*SQRT(2*PI())))*EXP(-0.5*((x-mean)/sd)^2)

If you aren't familiar with Excel formulae: the symbols -, *, /, ^ are used for subtracting, multiplying, dividing, and raising to the power of, and EXP, SQRT and PI are built-in functions for the exponential function e^x and π (=3.142...). The brackets after these functions contain the value for which the function is evaluated – a long expression in the case of EXP but nothing at all in the case of PI.

The Greek letters μ and σ in the formula above stand for mean and sd (standard deviation) so I have used these in the formulae. For this to work in Excel, you need to either replace x, sd and mean with appropriate numbers, or tell Excel which cells to get the data from (using Formulas – Name manager in my version of Excel). I have used lower-case letters for variables to distinguish these from the upper-case function names (EXP and SQRT in our example).

If, for example, x=8, mean=10 and sd=2, then 0.120985 will appear in the cell you type the formula into. In terms of the original conventional formula, this is the value of the function *f* evaluated where *x*=8. You can get the same answer by using the built-in Excel function NORMDIST.

A computer will understand this easily, which of course is its main advantage. From the point of view of a human being understanding how the formula "works", it's a bit of a mess. The main problem is the brackets: it's difficult to see at a glance how they pair up. We can make this clearer by using the three different kinds of brackets on the computer keyboard. In the formula below I've done this and also used a bigger font for {} and [] (not completely necessary but it helps), and replaced PI() with pi which is not a built-in function and needs to be linked to its value (3.142 or PI()) using the Excel Name Manager.

Modified Excel formula (3)

={1/[sd*SQRT(2*pi)]}*EXP{-0.5*[(x-mean)/sd]^2}

How do these formulae compare? There are several considerations here. I'll start with those favouring the conventional formula.

1. The conventional formula is what mathematicians are used to. It has developed over the centuries as mathematics has developed. Notation is an important aspect of mathematics, both for communicating with others and for working things out. Experience shows the notation in Formula 1 works well. Which isn't, of course, to say that an alternative couldn't work as well or better.

2. The conventional formula is more concise than the Modified Excel version. The right-hand side of the conventional formula has 18 symbols (letters, brackets or operation symbols); the Modified Excel formula has 46. This should mean that, if you are familiar with the conventions, Formula 1 should be easier to use in the sense of understanding its structure and manipulating it in the various ways mathematicians are familiar with, and it's quicker to write down – this is not a trivial point if you are writing out an argument which involves a long sequence of formulae.
3. The conventional formula makes use of the two dimensions of the computer screen or a piece of paper by means of the fraction line and the use of superscripts for exponents, whereas the Excel versions are a linear string of symbols. I think this may make it easier for the mathematically literate to grasp what Formula 1 means because the structure may be clearer.

Against these points, there are reasons for preferring the Modified Excel formula.

4. Unlike the conventional formula, the Excel formulae can be keyed into a computer as a sequence of symbols on the keyboard which makes it much easier to work with on a computer – both for computing values and for writing the formula out. For doing computations, to get back to an Excel readable formula from the Modified Excel formula, you just need to use search and replace to swap the brackets back.
5. There are two reasons why the Excel formulae have more symbols. The first is that juxtaposition is used instead of some symbols. In the Excel formulae * is used for multiplication; there is no equivalent symbol on Formula 1 for multiplication because this is indicated by placing symbols together: 2π instead of 2*π. Similarly, exponents (raising to a power) in the conventional formula are indicated by superscripts but in Excel by an extra symbol ^, and the fraction line is replaced by a division symbol (/) and brackets which is a bit more cumbersome. However, the extra symbols have two advantages for the relative newcomer. If you aren't sure what ^ or * mean, you can easily google it, which is far more difficult with the juxtaposition system. There is also an ambiguity in the conventional notation: *f(x)* means the function *f* evaluated at *x*, not *f*(x)* as you might assume from what I've just written. This is an ambiguity mathematicians would not notice, but computer languages (like Excel), of course, must be unambiguous. In the Excel versions brackets adjacent to a word indicate a function: for example, SQRT(2*pi).
6. The other reason the conventional formula is shorter is that the Excel formulae use words or abbreviations in place of single letters or other symbols: mean instead of μ, SQRT instead of √, etc. Words, or abbreviations, obviously make it easier to interpret the formula if you aren't familiar with the symbols[17]: if necessary, you can use a search engine to get help. The conventional notation must stick to single letters because otherwise "mean" might be confused with "m*e*a*n". Thinking of new symbols can be a problem. Using Greek letters obviously helps but this may make it difficult for people who don't speak Greek (like me: I know some Greek letters but others I have to think of in terms like "vertical squiggle" which complicates the process of making sense of a formula.) Using words or abbreviations resolves this problem, but at the cost of making formulae longer.

[17] A lot of computer and smartphone software uses many symbols which I don't understand. I wish they would use words or abbreviations.

How these pros and cons balance up is, of course, a matter of judgment taking the context into account. The system could be extended to incorporate additional mathematical concepts by creating additional functions or interpretations for symbols on the standard keyboard. The key point is that the formula must be a sequence of symbols that can be typed on an ordinary keyboard so that it is (or could easily be made) compatible with computer software, and it is possible to search for the meanings of function names and operators with a search engine.

My feeling is that, in the long term, something like my modified Excel notation may be more appropriate even for professional mathematicians, but the influence of habit and history is so strong that I can't see many people agreeing with me in the near future.

Appendix 2: A reformulation of the exponential function

What I want to propose here is a way of reformulating the exponential function so that it is more appropriate to people like my M.Sc. students who want to understand thoroughly, but don't want to get bogged down in calculus and logarithms. To do this I'll use a slightly easier example and come back to reliability later.

Let’s imagine that a population (of human beings, bacteria, rabbits or whatever) grows at 2% per year. How big will the population be after 40 years?

At first sight, the answer is that the population will have grown by 80% (2% times 40 years which is 80% or 0.8) so at the end of 40 years it will be 180% (1+0.8 or 1.8) of its size at the start. This, however, ignores the fact that each year the population will be a bit bigger, so 2% of the population will get bigger and bigger, so the population will grow faster.

This is like the difference between simple interest and compound interest. If you invest some money with 2% per year simple interest for 40 years, at the end of the 40 years your interest will be 80% of what you invested so you will end up with 180% of, or 1.8 times, your initial investment. Compound interest is a little trickier to work out. After one year your investment will have grown by 2% to 1.02 times your initial investment. After the second year, it will have grown to 1.02 times what it was at the end of the first year so it will be 1.02*1.02 (1.02x1.02) or 1.0404 times your initial investment. This is a little more than the 4% growth you'd have got with simple interest because you're getting interest on the interest. If you repeat this argument for each of the 40 years you will find that the final investment is 1.02^40 (or 1.02^{40} in conventional notation) times the initial investment which comes to 220.804% of (or 2.20804 times) the initial investment. Which, for obvious reasons, is a bit more than the 1.8 (180%) multiplier you get from simple interest.

The population growth question is obviously like compound interest except for one thing. After, say six months the population will have grown by 1%, so we could apply the compound interest method with 80 intervals in which the growth is 1%. This suggests the final population is multiplied by a factor of 1.01^80 or 2.216715. Which is a little bit more than you should expect. But why use six-month intervals? If we used a tenth of a year and a growth rate of 0.2%, the final answer would be 1.002^400 or 2.223764. Again, a little more but not much. You'll find if you take smaller and smaller intervals that the answer settles down to about 2.226 and never gets bigger than this[18]. And this, of course, is the answer you get from the exponential function: $e^{0.8}$ or EXP(0.8) = 2.226.

That's the theory. The function e^x gives us an easy way of working out compound growth problems like this so that we don't need to go through this rigmarole each time we need to work out something about compound growth.

In practice, the key application of e^x is compound growth (the reliability formula is based on the compounding idea), so it makes sense to link e^x to this and keep it separate from calculus, logarithms, and infinite series. However, there are four problems with e^x from this perspective:

1. The symbol e^x and the name exponential function. The link between these and the idea of compound growth is a little hazy.
2. The name of the reverse function for going back from the compound to the simple case is $\log_e$ (LN in Excel): again, this would seem arbitrary and meaningless to someone without a mathematical background.
3. How the two functions - (1) and (2) above - are explained. An explanation which does not involve logarithms and calculus would help our beginners.
4. The fact that you start with a number representing growth (0.8) but you finish with a number representing the total of what you started with plus the growth (2.226).

I'll start with the last point. The difficulty is that as well as the difference in terms of simple or compound growth, 0.8 and 2.226 also differ in that the first represents the increase or growth whereas 2.226 represents the total including what you started with. This can only be a distraction. For my proposed reformulation we need to decide whether we are going to go for the increase or the total. I'll go for the increase or growth because I think this makes things a little easier. The growth in the simple case is 0.8, and in the compound case is 1.226 (2.226-1). They are both, of course, multipliers: you need to know what population (or investment) you start with, and then you multiply by 0.8 for the simple case and 1.226 for the compound case. I'll refer to 0.8 as a Simple Growth Multiplier (SGM) and 1.226 as a Compound Growth Multiplier (CGM).

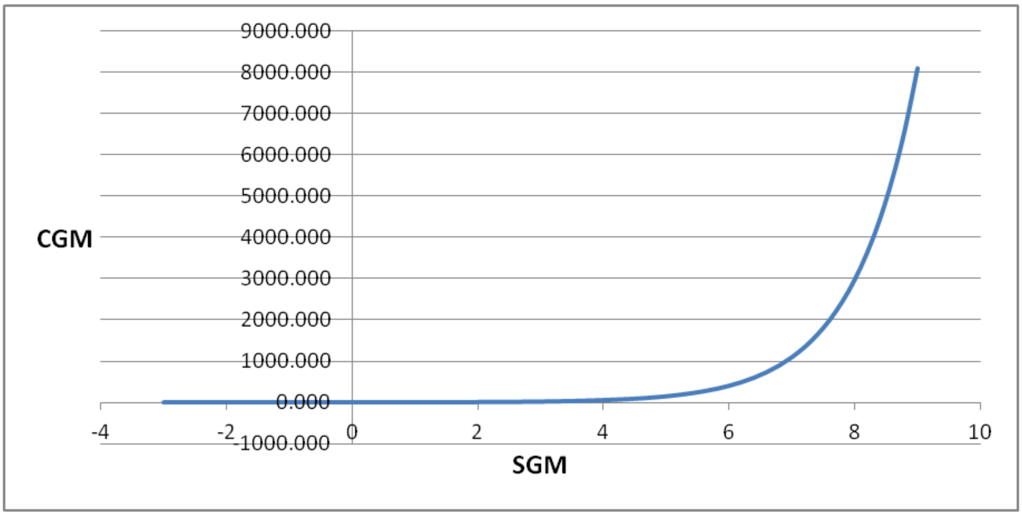
I've explained how we can start with an SGM of 0.8 and calculate the corresponding CGM (1.226). We could obviously do this for other SGMs and then make a table like Table 1 in the main text above.

Making the reverse table where we start off with a value of CGM and then work out the corresponding value of SGM is a little harder work because it involves trial and error. Imagine we start with CGM=1. Looking at the table SGM obviously has to be between 0.6 and 0.8. So, we might try 0.7, look at the resulting CGM, and possibly then try again until we've got it close enough. Hard work without a computer, but trivial with a computer.

We can also represent the relationship between SGM and CGM by a graph. People tend not to realise just how large the compound multiplier can be - the graph below illustrates this nicely.

Figure 3. Relationship between SGM and CGM

[18] This is known as a limit. Another, more intuitive, example of a limit is the sum of the infinite series 1+1/2+1/4+1/8+... which gets closer and closer to 2 but never quite gets there.



We could build these two functions into a spreadsheet or calculator or other software. So, for example:

CGM(0.8) = 1.226
SGM(1.226) = 0.8

These names now refer directly and obviously to what the functions represent (which deals with Problems 1 and 2 above). The procedure for going from SGM to CGM is longwinded but the rationale is relatively straightforward and does not involve any extraneous higher mathematics (Problem 3 above). And SGM and CGM only differ in terms of compounding which deals with Problem 4.

If the growth is compound, any mention of simple growth, and SGM, is hypothetical. In the original example, SGM was 0.8: this is what would happen *if the growth were simple*. But it isn't; if it was, we wouldn't be interested in CGM. The only way we could get around this problem of having to think in terms of hypothetical quantities is by bringing in the time period (40) and the growth rate (0.02) which would make it a function with three variables which immediately makes everything more complicated. Tables and graphs then wouldn't work. I'll stick with the hypothetical SGM.

Let's now imagine a parallel universe in which Excel has the functions SGM and CGM built in but not EXP and LN. The names of the functions now link directly to their interpretations, and the explanation is long-winded but reasonably straightforward for people who understand the rules of arithmetic. Formulae involving e^x in our parallel universe would be reformulated in terms of SGM and CGM. So, for example, the formula (modified Excel version) for the probability density of the normal distribution (see above) becomes[19]:

={1/[sd*SQRT(2*pi)]}*{CGM{-0.5*[(x-mean)/sd]^2}+1}

[19] This formula needs four levels of nested brackets, so I have used a larger ({}) as the extra symbol. In practice, three levels of nested brackets is probably the maximum which can be easily grasped, so it would probably be better to simplify the equation by defining the standard normal variable as snv=(x-mean)/sd (a concept which would be familiar to anyone using the normal distribution equation) and using this in the equation which would reduce the number of nested brackets.

Similarly, the reliability formula about becomes:

$$R(t) = 1+CGM(-fr*t)$$

This could either be presented as a formula to be used but not understood like the conventional formula above. Or it could easily be explained in terms of compound growth[20].

Let's take another example to see how this works. According to Wikipedia, the world population is increasing by about 1.1% each year, which means that over 800 years SGM will be 800x1.1%=880% or 8.8. From the graph, CGM will be over 6000 (6633 to be more precise using the figures the graph is based on). This means that the population will be over 6000 times its present level of 7.8 billion. If, of course, the population keeps growing at 1.1% per year.

These concepts also apply to negative growth, when populations are declining. If you make up some scenarios with negative growth, you should see the figures in the table make sense[21]. (The scale of the graph means you can't really see the negative bit here.)

This reformulation is just that: it is just as powerful as the traditional version, and it is easy to go from one to the other using these relationships:

CGM(SGM) =EXP(SGM)-1, SGM(CGM) = LN(CGM+1)
EXP(X) = CGM(X)+1, LN(X) = SGM(X-1)

Appendix 3: References to some of my attempts at simplifying knowledge

In addition to the list below, all my works listed in the references above are directly relevant to the idea of simplifying knowledge with the exception of Wood (2021).

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[20] This formula applies when you have a constant failure rate so, under the simple assumption the number failing in time t will be fr*t times the original number. The failures mean that the total will be decreasing so the growth is negative: -fr*t. The reliability is then simply the number surviving under the compounding assumption which is 1+CGM. Looking at it this way brings the key assumption of a constant failure rate to the fore.

[21] The reliability example is such an example of a population which declines as components (or whatever) fail.

Appendix 4: Comments on my response to reviewers on Qeios.com

I'd like to thank all the reviewers of the first draft of this paper on Qeios who gave me much food for thought. I have corrected a numerical mistake[22], and revised and reorganized the paper to take account of the reviewers' comments, and some further thoughts of my own. I have added a section entitled Tactics for simplifying knowledge, with six subsections, which I hope will provide some more structured suggestions.

One reviewer asked "How can I simplify quantum chromodynamics? I don't know." Which, of course, is the point. I don't know either, but somebody, someday, may come up with the answer – which must be a good thing! If it was easy, it would have been done already because, as I acknowledged in my "law of simplicity at the leading edge of a discipline", when even the experts acknowledge that something is difficult, they are likely to look for ways to make it easier.

I would agree that the article is more an opinion piece or a blog than an evidence-based argument, but new ideas must start somewhere. I have added a section on the difficulties of obtaining hard evidence for the benefits of simplification.

And finally, I would acknowledge that the range of examples I discuss in detail is limited, but I have no choice because of my limited expertise. I have added a brief discussion of Feynman Diagrams to the section on past examples of simplification. I would love it if readers were to add examples of their own – perhaps as a comment on this article.

[22] In the original article I referred to a value for CGM of 220% when I should have written 120%.