Qeios

Foundations of quantum mechanics revealed by the conservation laws

Richard Oldani
Illinois Institute of Technology
https://orcid.org/0000-0001-6884-5614
email: oldani@juno.com

Abstract

It is often claimed that Einstein is wrong about quantum mechanics. However, when comparisons are made with respect to theoretical foundations rather than experimental results Einstein's theories are found to be superior. Because he did not complete his theory its far-reaching, intuitively motivated derivations were never properly appreciated. It is the only theory based on a relativistically correct foundation, the time evolution of energy states, rather than the stipulation of energy eigenvalues in absolute time. Further development of his arguments based on the momentum reveals that although non-relativistic theory correctly predicts what it is possible to observe (the emissions) with remarkable accuracy it neglects the other half of natural phenomena, that which cannot be observed (the absorptions), thereby violating the conservation of energy. The deficiencies in non-relativistic theory are corrected by introducing Hamilton's principle and deriving relativistic equations of motion. This allows quantum mechanical formalism in abstract space to be replaced by the superposition of particle field geometries in real space. Direct application of the calculus of variations to an electron cycle reveals that the wave function is incomplete because it requires twice the allowable action minimum.

Keywords: Conservation laws; non-relativistic quantum mechanics; relativistic quantum mechanics; Hamilton's principle; energy; momentum; causality

1. Introduction

1.1 The conservation laws

The conservation laws in non-relativistic quantum mechanics are statistical in nature and applied non-locally [1]. "That paradoxical processes must arise in quantum mechanics in connection with conservation laws is to be expected. Indeed, on the one hand, physics is local: Causes and observable effects must be related. On the other hand, measurable dynamical quantities are identified with eigenvalues of operators, and their corresponding eigenfunctions are not, in general, localized."

https://doi.org/10.32388/ROFQKG.4

Classically defined conservation laws, on the other hand, are applied in a straight-forward manner and do not impose limitations upon the accuracy of the individual events. Energy is absorbed when a system heats up and energy is returned to the environment as it cools. The energy of absorption and emission may be examined to any desired level of accuracy no matter small. Thus it is believed that energy exchange holds accurately to the level of individual quanta despite being unobservable at that level. Similarly, the concept of momentum for extremely tiny amounts of energy exchange has a long history in science. It was first proposed by Kepler in 1607 to explain why the tail of Haley's comet faces away from the sun. He also imagined the practical application of momentum in outer space for "sails adapted to the heavenly breezes". Maxwell expanded upon the classical concept of momentum by extending its use to field applications. Like energy, momentum can neither be created nor destroyed, but only changes through the action of forces as described by Newton's laws of motion. We shall assume in the following that classical measurements of momentum may be analyzed as precisely as desired, the same as energy, and that they are also valid at the level of individual quanta.

2. Perspectives of quantum mechanical foundations

2.1 A statistical theory due to Einstein

In Einstein's only attempt at a theory of quantum mechanics he derived Planck's law by statistical methods [2]. There he describes the dynamic equilibrium that exists between the thermal energy absorbed by molecules and its subsequent quantum mechanical emission as black body radiation. Heat energy that is absorbed according to classically defined Maxwell Boltzmann statistics is transformed at the molecular level and emitted according to the Planck radiation law. Sources of energy and momentum at the molecular level are analyzed taking into account viscosity and temperature of the gas, frequency of the radiation, and radiation density.

> "We now turn to the investigation of the motion which our molecules execute under the influence of radiation. In doing this we use a method which is well known from the theory of Brownian movement. . . . . Let a molecule of given kind be in uniform motion with speed **V** along the X-axis of the coordinate system K. We inquire about the momentum transferred on the average from the radiation to the molecule per unit time. To calculate this we must consider the radiation from a coordinate system K′ that is at rest with respect to the given molecule. For we have formulated our hypotheses about emission and absorption only for molecules at rest."

The hypotheses he speaks of are the A and B coefficients. For an atom with energy states $|1\rangle$ and $|2\rangle$ the coefficient B_{12} refers to induced energy and momentum "transferred on average from the radiation to the molecule" in coordinates relative to K during energy absorption. The coefficient A_{21} refers to spontaneous emission and B_{21} refers to induced emission, both measured relative to the nucleus in the coordinates K'. The absorption of a photon causes

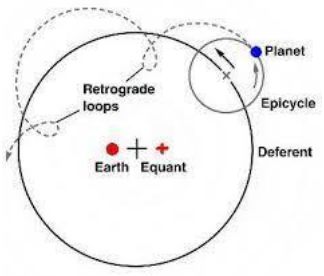
momentum in the direction of propagation, while the *emission* of a photon causes recoil momentum directed in a direction opposite to that of propagation. The sudden reversals and random nature of the impulses cause molecular trajectories to be discontinuous, as observed in Brownian motion.

Two coordinate systems are used to describe the equations of motion, a classical system K in laboratory coordinates that describe how energy evolves according to Maxwell Boltzmann statistics and a quantum mechanical system K' relative to the nucleus that describes how energy is emitted according to Planck statistics. The molecular level exchange of energy and momentum between K and K' is governed by the conservation laws so that transformations of coordinates are not possible. In other words, it is impossible to describe the momentum and kinetic energy of molecules in K using atomic coordinates in K' and vice versa. It is hypothesized therefore that physically independent equations of motion in K and K' will be necessary.

Molecular impulses transfer energy to the oscillator which then radiates the energy by emitting a photon thereby demonstrating the close causal relationship that exists between momentum and energy. Einstein explains the mechanism and in a later passage the theory behind it.

> "If a radiation beam with a well-defined direction does work on a Planck resonator [quantum oscillator], the corresponding energy is taken from the beam. According to the law of conservation of momentum, this energy transfer corresponds also to a momentum transfer from the beam to the resonator."
>
> "In general one restricts oneself to a discussion of the energy exchange, without taking the momentum exchange into account. One feels easily justified in this, because the smallness of the impulses transmitted by the radiation field implies that these can almost always be neglected in practice, when compared with other effects causing the motion. For a theoretical discussion, however, such small effects should be considered on a completely equal footing with more conspicuous effects of a radiative energy transfer, since energy and momentum are linked in the closest possible way."

Energy is measurable as a magnitude and easily incorporated into the equations of motion. Momentum, on the other hand, is difficult to measure at the microscopic level so we tend to underestimate its contribution to the equations of motion. The advantage of momentum is that it can provide information about a system's time evolution. In the Ptolemaic planetary system, for example, equations of motion for retrograde loops are in clear violation of the conservation of momentum (see figure). If astronomers had understood this they would have rejected theories that suggest massive objects can reverse motion in empty space. Geocentrism was replaced after a thousand years, not due to the conservation laws, but for conceptual reasons.



Ptolemaic system

Einstein is more concerned with the momentum of energy exchanges than with the eigenvalues of energy states as in non-relativistic models. In doing so he places emphasis upon the continuous time evolution of states rather than upon their discrete measurement in absolute time. Excitation to a higher energy state is due to a continuous transformation of heat energy, while emission is spontaneous and "corresponds to that of a radioactive reaction". The time duration between absorption and emission is not zero, "but only that this time should be negligible compared with the times which the molecule spends in states". Thus a physical separation exists between absorption and emission, and the laws of motion on each side of the discontinuity will differ. By examining momentum exchange we obtain a picture of the time evolution of energy states that is continuous and contrasts with the non-relativistic picture in absolute time [3].

2.2 Heisenberg's non-relativistic matrix model

Matrix mechanics evolved from long-standing attempts to describe dispersion phenomena, the continuous change in the angle of refraction of different frequencies of light by a prism or other medium. Although light disperses continuously across the entire spectrum, at certain specific frequencies characteristic of the medium, it is completely absorbed forming lines. Heisenberg discovered complex sets of mathematical rules describing the relationships of the observed frequencies and intensities of spectral lines. This allowed him to formulate a theory of quantum mechanics reconciling the continuity of radiation fields with the discrete energy states of an atom by expressing electron transitions in the form of a matrix [4].

$$\sum_k (p_{nk} q_{km} - q_{nk} p_{km}) = \begin{matrix} i\hbar \text{ for } n=m \\ 0 \text{ for } n \neq m \end{matrix} \qquad 1)$$

Although p in the above equation refers to momentum it is not the continuous, classically described momentum in K used by Einstein, rather it is the discrete momentum of a photon designated by two indices and emitted in K'. The diagonal elements of the matrix $m=n$ refer to possible energy states of a quantum system. Because the theory only concerns observables,

such as emission frequencies and transition probabilities, it is formulated exclusively in coordinates relative to K'.

Off-diagonal elements are assigned a value of zero because the energy of an absorption offsets the energy of an emission except for a difference in phase. However exchanges of momentum in K and K' originating with these same absorptions and emissions are ignored because they are unobservable. Although the collisions cannot be observed individually they are evident statistically in measurements of gas temperature. Heisenberg did not take these microscopic interactions into account because he believed that quantum mechanics should be "founded exclusively upon relationships between quantities which in principle are observable". This caused him to focus exclusively on emissions, the intensity and frequency of the spectral lines, as measured in K'.

Describing energy by means of emission processes alone is a violation of energy conservation. The meaning of energy is not embodied only by observables in the form of emissions, or only in absorptions, for due to the conservation laws energy can neither be created nor destroyed. It is correctly described as a transformation from an absorption process to an emission process.

2.3 Einstein vs. Feynman

Einstein's statistically derived quantum theory of radiation (2.1) evolves continuously in time. Heisenberg's matrix mechanical model performs measurements of the physical parameters frequency and spectral line intensity relative to absolute time. Should a non-linear gravitational field intensity be imposed the physical variables of statistical theories derived in continuous time will adapt consistently. However non-relativistic theories are only valid in inertial systems for absolute time. The question how to incorporate gravitational fields into non-relativistic theory is highly contentious and many theories are devoted to it. Einstein's statistically based theory, ignored because it is not experimentally verifiable, satisfies the conditions of relativity theory due to first principles.

Feynman's sum-over-paths approach describes particle dynamics without *explicit* use of a wave function thereby removing the conceptual difficulties of collapse [5]. Predictions are made by summing the probability amplitudes of all possible paths and squaring the result to obtain the probability of an event. The absorption energy in K is experimentally determined so only paths of probability amplitudes in K' (the emissions) are considered. All the paths in Feynman's path integral formulation contribute to the probability amplitude for an event so they are regarded as *possible* trajectories, but they are not like anything anyone has ever experienced for they are unrestricted by the conservation laws. They may form loops, extend to infinity, go backwards in time, or exceed the speed of light; and the use of "all possible

paths" is an indication that boundary conditions extend to infinity. Because all paths and all histories are possible the path integral approach to non-relativistic quantum mechanics necessarily violates the conservation laws.

3. Relativistic quantum mechanics

3.1 The relativistic absorption of energy by a classical system

To describe particle paths in a way that allows the local application of conservation laws we apply Hamilton's principle. It assumes that a particle, in this case the electron, begins at a particular position P_1 and point in time t_1 and travels at a certain velocity to arrive at a position P_2 after a given amount of time [6]. The tentative paths are not restricted in any way. Not only can we choose any path between the points but the particle's motion in time may change freely as well. All that we require is that motions begin at the observed time t_1 and end at the observed time t_2. The time integral of the energy extended over the entire motion between P_1 and P_2 is called "the action". The quantity that we use as a measure of the action S is the time integral of the difference between the kinetic and potential energies (T-V).

$$S=\int_{P_1}^{P_2}\int_{t_1}^{t_2}(T-V)dt=0 \qquad 2)$$

Each of the paths between the same two end-points P_1 and P_2 will have a different action. The principle of least action asserts that the particle path actually taken is the one for which the action is a minimum. For constraints on a classical system of fixed end positions P_1 and P_2 and fixed time travel t_1 and t_2 the action minimum is equal to zero.

3.2 The relativistic absorption of energy by a quantum system

Following Einstein's lead (see 2.1) we wish to describe the energy that is absorbed from K. Even the simplest atomic system, the hydrogen atom, includes an infinite number of paths that connect the electron's initial and final points, from the ground state to an excited state. Hamilton's principle expresses the meaning of the entire set of differential equations describing the paths. It calls for minimizing a single physical quantity, the action, in order to obtain the path actually taken. The principle of least action gives a relativistically correct description for it is independent of any particular system of coordinates and it is invariant for all coordinate transformations in a manner that is also consistent with general relativity.

Hamilton's principle is valid for systems upon which work is performed; that is, non-conservative systems. Radiating atoms that absorb and emit energy are *non-conservative systems*. Work is performed on them causing energy to be taken from the environment and transferred to the system. Neither Heisenberg's nor Feynman's methods include energy absorption so they treat the atom incorrectly as a *conservative system*. Therefore to properly describe the energy absorption of an atom we utilize the symmetry of an atomic system and apply Hamilton's principle in generalized coordinates. The transition of an electron from the

ground state to an excited state is characterized in generalized coordinates with six dimensions, three to describe its position on the electron shells R_1 and R_2, and three to describe its trajectory. We can think of the Schrödinger equation as a collection of all possible paths of an electron between electron shells, each of which has a different probability of happening and describes a path with a different action. The actual path of the electron, the one chosen by nature, is the one whose action is minimum.

Energy is absorbed by atoms from a radiation field in K due to momentum exchange. The path taken by the electron initiates its motion at a fixed point on the equipotential surface R_1 of the ground state at time t_1, proceeds along a path r, and upon arriving at the excited state R_2 adopts the circular path $2\pi r$ of an orbital thereby assuming orbital angular momentum. The action minimum S[r(t)] for a path between stationary points on R_1 and R_2 yields the reduced Planck's constant ħ.

$$S[r(t)] = \int_{R_1}^{R_2}\int_{t_1}^{t_2} (T - V)dt = \hbar \qquad 3)$$

The action, S[r(t)], is a functional that describes the absorption process in four dimensions. It has as its argument an infinite number of functions, the possible electron trajectories r(t). The path of the electron that is actually followed, has an action minimum not equal to zero as in eqn 2) of classical mechanics, but ħ.

3.3 Relativistic emission of energy by an atomic system

At the relativistic or high end of the energy spectrum in quantum field theory, particles are treated as excited states of the more fundamental underlying quantum fields. As Nobel laureate Frank Wilczek noted [7], "In quantum field theory, the primary elements of reality are not individual particles, but underlying fields." Particles are described by an action functional $S[\psi(x_i)]$ where the fields $\psi(x_i)$ of particles are defined throughout space. Each of the elementary particles has a field and the action depends on all of the fields. We will follow these same practices in order to extend the field interpretation to the low energy emissions currently described by non-relativistic theory.

The region of space-time that is of interest is located in K' (see 2.1) and lies between the two states of an atomic oscillator. Within the space-time region between the excited and ground states we define a Lagrangian density of the fields and their first derivatives $£(\phi_i, \phi_{i,\mu})$ which allows for a complete accounting of the energy interactions, where ϕ_i is the current density and $\phi_{i,\mu}$ is the electromagnetic field strength. The action integral for a quantum oscillator with an outer electron that occupies either of two allowable energy states may now be formulated, with emission initiating from the excited state $R_2 = (x_2,y_2,z_2)$ at time t_2 and finalizing at the ground state $R_1 = (x_1,y_1,z_1)$ at time t_1. Applying Hamilton's principle we require the integral of the Lagrangian density over the region of space-time between the excited and ground states to be a minimum for all small variations of the coordinates inside

the region, where the action minimum for an arbitrary quantum system is defined in angular measure to be the reduced Planck's constant ħ.

$$S[\phi_i(t)] = \int_{R_2}^{R_1}\int_{t_2}^{t_1} \pounds(\phi_i\,\phi_{i,\mu})d^3x dt = \hbar \qquad 4)$$

The energy states |2> and |1> coincide with the electron shells and determine invariant field boundaries where fields vanish. The change in action yields a relativistic formulation of emission that is invariant, the same for all observers. The action $S[\phi_i(t)]$ is a functional, a function of the values of coordinates on the *discrete* boundaries of the space-time surfaces R_2 and R_1 which are in turn functions of the *continuous* space-time variables of the fields within the surface. The field boundaries are uniquely fixed in four dimensions by the volume d^3x and the time interval t_2-t_1 causing the photon to be described as a four-dimensional localization of fields. Experimental verification can be found by examining the behavior of an electron cyclotron [8]. Thus the initial and final points of an electron's path can be confirmed experimentally with nearly perfect accuracy.

3.4 Comparison of the relativistic and non-relativistic models

The principal difference between relativistic and non-relativistic models is in their underlying physical assumptions. In relativistic theory the emission of energy is described with a Lagrangian (T-V) as a four-dimensional localization of fields $\pounds(\phi_i, \phi_{i,\mu})$, while in non-relativistic theory it is described with a Hamiltonian (T+V) as the potential of a point electron. To demonstrate their equivalence we will show how the two methods relate physically. In non-relativistic theory a particular type of wave function or "spinor", is used to describe energy emission. It defies explanation in ordinary space-time for it acts like an ordinary vector for infinitesimal rotations, but transforms to its negative for complete rotations and requires two complete rotations to return to its original state. To simultaneously describe translations and rotations in a single expression is an impossibility in ordinary space-time. Thus attempts to visualize spinor behavior make use of imaginary geometries such as the Möbius strip.

A simpler explanation is possible by using physical arguments from the relativistic model. Rather than treat quantum mechanical energy emission as an event carried out by a single particle we use equations 3) and 4) to describe it with field sources, an electron of field ϕ_i and a photon of field $\phi_{i,\mu}$, during two distinct physical processes, absorption and emission. The field ϕ_i of an electron has spin described by Pauli matrices oriented in three-dimensional space and the field $\phi_{i,\mu}$ of a photon is oriented due to polarization in four-dimensional space-time by Maxwell's equations. We interpret the spinor therefore as a superposition of two field geometries that undergo continuous change as they cycle through the physical processes of excitation, localization, and emission. There are two paths possible, spin-up or spin-down, and two rotations are necessary to complete an entire photon emission, where rotations represent changes in phase of the photon's electromagnetic field. Thus non-relativistic spinors

are rotations in abstract "space-time", which we interpret relativistically as the time evolution of *real* particle field geometries. An electron does not absorb energy when it is excited, rather excitation causes field boundaries to be erected that localize energy within the atom to create a photon. In view of these experiments and the above physical arguments we may visualize the wave function as the combined action of a particle and an associated localized field.

4. Discussion

To determine the true evolution of a quantum system we use Hamilton's principle and take advantage of the symmetries of the system to specify generalized coordinates describing the electron shells R_2 and R_1. Then the actual path the electron follows between the initial and final stationary points of a complete cycle between t_1 and t_2 is the one which minimizes the action. However, the true path is not determinable through experimental means. We have no better way to describe the path of an electron other than to calculate the probability of its arrival at a given detector by using the Schrödinger equation. The path with the greatest probability does not necessarily minimize the action. Only paths having action minimum $\hbar$ are realized. Electron excitation is one path and decay is another. The fact that two transitions, or paths, cause one result, a photon emission, is to a large extent what makes quantum theory so different from classical theory.

The Schrödinger wave equation describes the absorption-emission process by using two rotations of the wave function. Each rotation of 2π represents the change in phase of a photon's field with action minimum $\hbar$. Thus it includes an incoming quantum of action $\hbar$ in K given by 3) and an outgoing quantum of action $\hbar$ in K' given by 4). In other words, the emission of energy is a combination of two physical processes, each of which obeys the uncertainty principle and is determined by an action minimum. Wave functions describing the changes in state of an electron cycle, or of any fermion for that matter, require twice the action minimum. They cannot describe the true path of the electron for they represent two independent paths.

5. Conclusion

The significance of Einstein's 1917 paper, "On the quantum theory of radiation", is now clear. He begins by noting that when atoms absorb energy from a radiation field in K, external to the atom, they are excited to a higher energy state by momentum transfer that occurs in the direction of propagation (see 2.1). He argues that this constitutes a causal process, "To this transfer of energy there also corresponds a momentum transfer from radiation bundle to resonator, by momentum conservation." And when the atoms decay, an equal amount of energy is emitted, relative to K', causing recoil momentum in a direction opposite to that of propagation. Two reference frames are necessary so two statistical equations of motion are required to describe black body radiation, Maxwell Boltzmann

statistics in K and Planck statistics in K'. To describe individual events two equations of motion are required due to the same arguments; the time integral of a Lagrangian (Eqn 3) in K and the time integral of a Lagrangian density (Eqn 4) in K'. When applied to a quantum oscillator consecutively in time the equations provide a complete description of energy absorption, transformation, and photon emission. It has also been used to interpret the time evolution of energy states for chaotic systems.

References

[1] Aharanov, Y., Popescu, S., Rohrlich, D. "On the conservation laws in quantum mechanics" *PNAS* **118** (1) e1921529118 (2020) doi: https://doi.org/10.1073/pnas.1921529118

[2] Einstein, A. "Quantum theory of radiation" *Phys Z* **18**, 121 (1917). W.

[3] Oldani, R. "Time evolution of energy states" (2023) DOI: 10.20944/preprints202305.0952.v5

[4] Heisenberg W. "Quantum mechanical re-interpretation of kinematic and mechanical relations", *Z Phys* **33**, 879 (1925) in B.L. van der Waerden (ed.), *Sources of Quantum Mechanics*, (Dover, 1968).

[5] Feynman, R. "A space-time approach to non-relativistic quantum mechanics" *Rev Mod Phys* **20**, 367-387 (1948).

[6] Lanczos, C. *The Variational Principles of Mechanics* 4th edn. (Dover, 1970)

[7] Wilczek, F. "Mass Without Mass I: Most of Matter," *Physics Today* **52** (11), 11-13 (1999).

[8] Brown, L.S.; Gabrielse, G. (1986). "Geonium theory: physics of a single electron or ion in a Penning trap"*Reviews of Modern Physics*. **58** (1): 233–311. doi:10.1103/RevModPhys.58.233.