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Foundations of quantum mechanics revealed by the conservation laws

Richard Oldani
Illinois Institute of Technology
https://orcid.org/0000-0001-6884-5614
email: oldani@juno.com

Abstract

It is often claimed that Einstein is wrong about quantum mechanics. However, when comparisons are made with respect to theoretical foundations rather than experimental results Einstein's theories are found to be superior. Because he did not complete his theory its far-reaching, intuitively motivated derivations were never properly appreciated despite the fact that his is the only theory based on a relativistically correct foundation. The development of his arguments reveals that although non-relativistic theory correctly predicts what it is possible to observe (the emissions) with remarkable accuracy it neglects the other half of natural phenomena, that which cannot be observed (the absorptions), thereby violating the conservation of energy. The deficiencies in non-relativistic theory are corrected by applying Hamilton's principle and deriving relativistic equations of motion. This allows quantum mechanical formalism in abstract space to be replaced by the superposition of particle field geometries in real space. Direct application of the calculus of variations to an electron cycle reveals that the wave function represents a non-conservative force and it is incomplete because it requires twice the allowable action minimum.

Keywords: Conservation laws; non-relativistic quantum mechanics; relativistic quantum mechanics; Hamilton's principle; energy; momentum; causality

1. Introduction

1.1 The conservation laws

The question of what is observable in scientific inquiry is a sticking point that has a long history in theoretical physics. On one side is Bohr and the majority of physicists. "It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature." On the other side of the dispute, Einstein believed that, "On principle, it is quite wrong to try founding a theory on observable magnitudes alone." The conservation laws in quantum mechanics are statistical in nature and unobservable locally [1]. "That paradoxical processes must arise in quantum mechanics in connection with conservation laws is to be expected. Indeed, on the one hand, physics is local: Causes and observable effects must be related. On the other hand, measurable dynamical quantities are identified with

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eigenvalues of operators, and their corresponding eigenfunctions are not, in general, localized."

The accuracy of classically defined conservation laws is not determined by whether individual events can be measured. Energy is absorbed when a system heats up and energy is returned to the environment as it cools. The energy of absorption and emission may be examined to any desired level of accuracy no matter small. Thus it is believed that energy exchange holds accurately to the level of individual quanta despite being ordinarily unobservable at that level. Similarly, the concept of momentum for extremely tiny amounts of energy exchange has a long history in science. It was first proposed by Kepler in 1607 to explain why the tail of Haley's comet faces away from the sun. He even imagined the practical application of momentum in outer space for "sails adapted to the heavenly breezes". Maxwell expanded upon the classical concept of momentum by extending its use to field applications. Like energy, momentum can neither be created nor destroyed, and it changes locally through the action of forces as described by Newton's laws of motion. We shall assume in the following that measurements of momentum may be analyzed as precisely as desired, the same as energy, and that they are valid locally at the level of individual quanta.

2. Perspectives of quantum mechanical foundations

2.1 A statistical theory due to Einstein

In Einstein's only attempt at a theory of quantum mechanics he derived Planck's law by statistical methods [2]. There he describes the dynamic equilibrium that exists between the thermal energy absorbed by molecules and its subsequent quantum mechanical emission as black body radiation. Heat energy is absorbed by molecules according to classically defined Maxwell Boltzmann statistics, is transformed at the molecular level, and emitted according to the Planck radiation law. He analyzes the sources of energy and momentum at the molecular level taking into account the viscosity and temperature of the gas, frequency of the radiation, and radiation density; and incorporates them into equations of motion.

> "We now turn to the investigation of the motion which our molecules execute under the influence of radiation. In doing this we use a method which is well known from the theory of Brownian movement. . . . . Let a molecule of given kind be in uniform motion with speed **V** along the X-axis of the coordinate system K. We inquire about the momentum transferred on the average from the radiation to the molecule per unit time. To calculate this we must consider the radiation from a coordinate system K′ that is at rest with respect to the given molecule. For we have formulated our hypotheses about emission and absorption only for molecules at rest."

The hypotheses he speaks of are the A and B coefficients. For an atom with energy states $|1\rangle$ and $|2\rangle$ the coefficient B_{12} refers to induced energy and momentum "transferred on average from the radiation to the molecule" during energy absorption in coordinates relative to K. The coefficient A_{21} refers to spontaneous emission and B_{21} refers to induced emission, both measured relative to the nucleus in the coordinates K'. The absorption of a photon causes

momentum in the direction of propagation, while the *emission* of a photon causes recoil momentum directed in a direction opposite to that of propagation. The sudden reversals and random nature of the impulses cause molecular trajectories to be discontinuous, as observed in Brownian motion.

Two physically independent coordinate systems are used to describe the equations of motion, a classical system K in laboratory coordinates that describe how energy evolves according to Maxwell Boltzmann statistics and a quantum mechanical system K' relative to the nucleus that describes how energy is emitted according to Planck statistics. The exchange of energy and momentum between K and K' is governed locally by the conservation laws so transformations of coordinates are not possible. In other words, it is impossible to describe the momentum and kinetic energy of molecules in K using atomic coordinates in K' and vice versa. Molecular impulses transfer energy to the oscillator which then radiates the energy by emitting a photon.

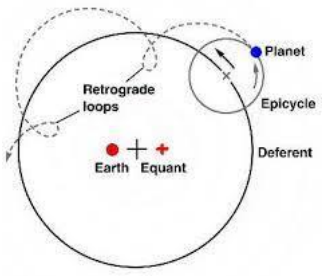
Einstein uses the conservation of momentum to justify a close causal relationship between momentum and energy exchanges in black body radiation.

> "If a radiation beam with a well-defined direction does work on a Planck resonator [quantum oscillator], the corresponding energy is taken from the beam. According to the law of conservation of momentum, this energy transfer corresponds also to a momentum transfer from the beam to the resonator."

He later explains why considerations of momentum exchange are just as important as energy exchange when deriving equations of motion at the molecular level.

> "In general one restricts oneself to a discussion of the energy exchange, without taking the momentum exchange into account. One feels easily justified in this, because the smallness of the impulses transmitted by the radiation field implies that these can almost always be neglected in practice, when compared with other effects causing the motion. For a theoretical discussion, however, such small effects should be considered on a completely equal footing with more conspicuous effects of a radiative energy transfer, since energy and momentum are linked in the closest possible way."

Energy is measurable as a magnitude and easily incorporated into the equations of motion. Momentum, on the other hand, is difficult to measure at the microscopic level so we tend to underestimate its contribution to the equations of motion. The advantage of momentum is that it can provide information about a system's time evolution. In the Ptolemaic planetary system, for example, equations of motion for retrograde loops are in clear violation of the conservation of momentum (see figure). If astronomers had understood this they would have rejected theories that suggest massive objects can reverse motion in empty space.



Ptolemaic system

Einstein is more concerned with the momentum of energy exchange than with the eigenvalues of energy states as in non-relativistic models. By doing so he places emphasis upon the continuous time evolution of states rather than upon their evaluation in absolute time. Excitation to a higher energy state is due to a continuous transformation of heat energy from K, while emission from K' is spontaneous and "corresponds to that of a radioactive reaction". The time lapse between absorption and emission is not instantaneous, but it "should be negligible compared with the times which the molecule spends in states". Thus a time dependent separation exists between absorption and emission during spontaneous emission, and the equations of motion on either side of the discontinuity must differ because of it. It is possible to obtain a picture of the time evolution of energy states by examining momentum exchange, and we find that contrasts sharply with the non-relativistic description in absolute time [3].

2.2 Heisenberg's non-relativistic matrix model

Matrix mechanics evolved from long-standing attempts to describe dispersion phenomena, the continuous change in the angle of refraction of different frequencies of light by a prism or other medium. Although light disperses continuously across the entire spectrum, at certain specific frequencies characteristic of the medium, it is completely absorbed forming lines. Heisenberg discovered complex sets of mathematical rules describing the relationships of the observed frequencies and intensities of spectral lines. This allowed him to formulate a theory of quantum mechanics reconciling the continuity of radiation fields with the discrete energy states of an atom by expressing electron transitions in the form of a matrix [4].

$$\sum_k (p_{nk} q_{km} - q_{nk} p_{km}) = \begin{matrix} i\hbar \text{ for } n=m \\ 0 \text{ for } n \neq m \end{matrix} \qquad 1)$$

Although p in the above equation refers to momentum it is not the continuous, classically described momentum in K used by Einstein, rather it is the discrete momentum of a photon designated by two indices and emitted in K'. The diagonal elements of the matrix, when $n=m$, refer to possible energy states of a quantum system. Because the theory only concerns

observables, such as emission frequencies and transition probabilities, it is formulated exclusively in coordinates relative to K'.

Off-diagonal elements, $n \neq m$, are assigned a value of zero because the energy of an absorption offsets the energy of an emission except for a difference in phase. However exchanges of momentum in K and K' originating with these same absorptions and emissions are ignored because they are unobservable. Although the collisions cannot be observed individually they are evident statistically in measurements of gas temperature. Heisenberg did not take these microscopic interactions into account because he believed that quantum mechanics should be "founded exclusively upon relationships between quantities which in principle are observable". This caused him to focus exclusively on emissions that are given by the intensity and frequency of spectral lines as measured in K'. By ignoring the thermal energy of gas molecules he effectively eliminated classical theory from consideration.

Describing energy by means of emission processes alone is a violation of energy conservation. The meaning of energy is not embodied only by observables in the form of emissions, or only in absorptions, for due to the conservation laws energy can neither be created nor destroyed. It is correctly described as a transformation from an absorption process to an emission process. Therefore the off-diagonal elements in 1) need to be assigned non-zero, classical values due to the kinetic energy.

2.3 Relativistic versus non-relativistic theory

Einstein's statistically derived quantum theory of radiation evolves continuously in time, whereas the matrix mechanical model describes the physical parameters frequency and spectral line intensity relative to absolute time. Should a non-linear gravitational field intensity be imposed the physical variables of statistical theories derived in continuous time will adapt consistently. However non-relativistic theories will not adapt because they are only valid with respect to absolute time. The question how to incorporate gravitational fields into non-relativistic theory is highly contentious and many theories are devoted to it. Einstein's statistically based theory, ignored because it is not experimentally verifiable, satisfies the conditions of relativity theory due to first principles.

Feynman's sum-over-paths approach of non-relativistic quantum mechanics describes particle dynamics without *explicit* use of a wave function thereby removing the conceptual difficulties of collapse [5]. Predictions are made by summing the probability amplitudes of all possible paths and squaring the result to obtain the probability of an event. The absorption energy is an initial condition of the experiment so the experimental apparatus is described in K and the paths of probability amplitudes (the emissions) are described in K'. All the paths contribute to the probability amplitude for an event so they are regarded as *possible* trajectories, but they are not like anything anyone has ever experienced for they are

unrestricted by the conservation laws. They may form loops, extend to infinity, go backwards in time, or exceed the speed of light; and the use of "all possible paths" is an indication that boundary conditions extend to infinity. The conservation laws are disregarded by making use of absolute time.

3. Relativistic quantum mechanics

3.1 The relativistic absorption of energy by a cconservative system

In order to describe particle motion in a way that allows for the local application of conservation laws we shall use the "the action", which is the time integral of the kinetic and potential energy extended over its entire path [6]. Each of the paths between the same two end-points will have a different action. The principle of least action asserts that the particle path actually taken is the one for which the action is a minimum. It is the basis for Newton's laws of motion. For a conservative system with constraints of fixed end positions and fixed time travel t_1 and t_2 the action minimum is equal to zero.

$$\delta S = \int_{t_1}^{t_2} (T-V)\,dt = 0 \qquad 2)$$

Although the times t_1 and t_2 are fixed the time period $t_2 - t_1$ varies continuously with respect to gravitational potential so the action integral gives a relativistically correct equation of motion.

3.2 The relativistic absorption of energy by a quantum system

An atomic oscillator absorbs energy from the environment and transforms it into a unit of electromagnetic energy, the photon. There are two possible ways for this to happen, by either induced or spontaneous emission. The way an atom emits radiation determines to a large extent how that energy was absorbed, for due to the conservation laws energy must be absorbed before it can be emitted. In the case of induced emission a photon is absorbed and a photon is emitted so there appears to be continity of exchange with the nucleus acting as a conservative force [7]. That is the assumption behind the wave function model of the emission and absorption of radiation. It is why induced emission is mistakenly believed to be due to a path independent *conservative* force. In the case of spontaneous radiation, on the other hand, the emission process occurs in a very different way. Photon emission occurs due to the decay of an electron to a lower energy level by a statistical process determined within the atom. Not only is there a discontinuity between absorption and emission, but they occur in response to physically distinct statistical laws.

To restore continuity to the equations of motion we use Hamilton's principle which expresses the meaning of the entire set of differential equations describing the paths and calls for minimizing a single physical quantity, the action, in order to obtain the path actually taken. Energy absorption consists of two simultaneously evolving processes; the excitation of an electron and the localization of fields in the creation of a photon. The transition of an electron from the ground state to an excited state is characterized in generalized coordinates with six dimensions, three to describe its position on the electron shells R_1 and R_2, and three

to describe its trajectory. The electron initiates its motion at a point on the equipotential surface R_1 of the ground state at time t_1, is excited by linear transverse fields along a path r, and upon arriving at the state R_2 the fields are localized and assume the experimentally determined orbital angular momentum for the hydrogen atom, (T-V) = 2πE.

$$S[r(t)] = \int_{R_1}^{R_2} (\mathrm{T\text{-}V}) \int_{t_1}^{t_2} dt \qquad 3)$$

The action, S[r(t)], is a functional that describes the absorption process in four dimensions. It has as its argument an infinite number of functions, the possible electron trajectories r(t). If the initial time t_1 of the excitation is arbitrarily set at zero, then the action integral for one cycle of the fields is evaluated as follows:

$$2\pi E\tau = h$$

Simplifying, we describe the localized fields of photons, $E\tau = \hbar$.

3.3 Relativistic emission of energy by an atomic system

At the relativistic or high end of the energy spectrum in quantum field theory, particles are treated as excited states of the more fundamental underlying quantum fields. As Nobel laureate Frank Wilczek noted [8], "In quantum field theory, the primary elements of reality are not individual particles, but underlying fields." Particles are described by an action functional $S[\psi(x_i)]$ where the fields $\psi(x_i)$ of particles are defined throughout space. Each of the elementary particles has a field and the action depends on all of the fields. We will follow these same practices in order to extend the field interpretation to the low energy emissions currently described by non-relativistic theory.

The region of space-time that is of interest is located in K' (see 2.1) and lies between the two states of an atomic oscillator. Within the space-time region between the excited and ground states we define a Lagrangian density of the fields and their first derivatives $£(\phi_i, \phi_{i,\mu})$ which allows for a complete accounting of the energy interactions, where ϕ_i is the current density and $\phi_{i,\mu}$ is the combined electromagnetic field strength of electron and nucleus. The action integral for a quantum oscillator with an outer electron that occupies either of two allowable energy states may now be formulated, with emission initiating from the excited state $R_2 = (x_2,y_2,z_2)$ at time t_2 and finalizing at the ground state $R_1 = (x_1,y_1,z_1)$ at time t_1. In order to apply Hamilton's principle to the field dynamics we require, following Dirac, that the integral of the Lagrangian density over the region of space-time between the excited and ground states be a minimum for all small variations of the coordinates inside the region [9].

$$S[\phi_i(t)] = \int_{R_2}^{R_1} \int_{t_2}^{t_1} L(\phi_i \phi_{i,\mu}) d^3x dt = E\tau \qquad 4)$$

Solving we obtain an expression for the localized fields of a photon $E\tau=\hbar$. The change in action yields a relativistic formulation of emission that is invariant, the same for all observers. The action $S[\phi_i(t)]$ is a functional, a function of the values of coordinates on the *discrete*

boundaries of the space-time surfaces R_2 and R_1 which are in turn functions of the *continuous* space-time variables of the fields within the surface. The field boundaries are uniquely fixed in four dimensions by the volume d^3x and the time interval t_2-t_1 causing the photon to be described as a four-dimensional localization of fields. Experimental evidence supporting the idea that two equations of motion, 3) and 4), are necessary to describe the absorption and emission of energy can be found by examining the behavior of the simplest quantum system, an electron cyclotron [3,10].

3.4 Comparison of the relativistic and non-relativistic models

The principal difference between relativistic and non-relativistic models is in their underlying physical assumptions. In relativistic theory the emission of energy is described with a Lagrangian (T-V) as a four-dimensional localization of fields $£(\phi_i, \phi_{i,\mu})$, while in non-relativistic theory it is described with a Hamiltonian (T+V) as the potential of a point electron. To demonstrate their equivalence we will show how the two methods relate physically. In non-relativistic theory a particular type of wave function or "spinor", is used to describe energy emission. It defies explanation in ordinary space-time for it acts like an ordinary vector for infinitesimal rotations, but transforms to its negative for complete rotations and requires two complete rotations to return to its original state. Thus attempts to visualize spinor behavior make use of imaginary geometries such as the Möbius strip. Normally rotations and transformations of vector components and the coordinates describing them are carried out continuously. The same cannot be said for spinors. Spinors require two complete rotations to return to the original state, but they are constructed in such a way that they are sensitive to how the gradual rotation of coordinates was carried out to arrive there. In other words, they exhibit path dependence. More specifically, for any final configuration of the coordinates there are two topologically inequivalent continuous rotations of the coordinate system that result in this same configuration. It is impossible to carry out transformations between the two inequivalent rotations of coordinates even though they arrive at the same configuration. Because the spinor follows two independent paths it represents a *non-conservative force*.

A simpler explanation is possible by using physical arguments from the relativistic model. Rather than treat quantum mechanical energy emission as a single event carried out by a single particle described by a spinor, we use equations 3) and 4) to describe it with two field sources, an electron of field ϕ_i and a photon of field $\phi_{i,\mu}$, during two distinct physical processes, absorption and emission. The field ϕ_i of an electron has spin described by Pauli matrices oriented in three-dimensional space and the field $\phi_{i,\mu}$ of a photon is oriented due to polarization in four-dimensional space-time by Maxwell's equations. We interpret the spinor therefore as a superposition of two field geometries that undergo continuous change as they cycle through the physical processes of excitation, localization, and emission. There are two paths possible, spin-up or spin-down, and two rotations are necessary to complete an entire photon emission, where rotations represent changes in phase of the photon's electromagnetic

field. Thus non-relativistic spinors are rotations in abstract "space-time", which we interpret relativistically as the time evolution of *real* particle field geometries.

4. Conclusion

The wave function describes all possible paths of an electron, and the one that is detected is singled out as the only true path. The true path is not determinable in advance through experimental means, rather it is predicted probabilistically and determined after the fact by measurement. Thus the path with the greatest probability does not necessarily minimize the action. In contrast, Hamilton's principle singles out the path that minimizes the action before particles are detected. In quantum mechanics the paths that are realized are the ones with action minimum ħ. Electron excitation is one path and decay is another. The fact that two transitions, or paths, cause one result, a photon emission; and that they proceed according to distinct laws of motion is to a large extent what makes quantum theory so different from classical theory and difficult to understand.

The significance of Einstein's 1917 paper, "On the quantum theory of radiation", is now clear. He begins by noting that when atoms absorb energy from a radiation field in K, it constitutes a causal process, "To this transfer of energy there also corresponds a momentum transfer from radiation bundle to resonator, by momentum conservation." And when the atoms decay, an equal amount of energy is emitted, relative to K', causing recoil momentum in a direction opposite to that of propagation. Einstein concludes that two reference frames and two *independently derived statistical equations of motion* are required to describe black body radiation (see 2.1). The experimentally determined spinor from non-relativistic theory requires two independent paths for a complete rotation (see 3.4). Due to these closely related supporting arguments we hypothesize that two equations of motion are required to describe individual events; the time integral of a Lagrangian (Eqn 3) in K and the time integral of a Lagrangian density (Eqn 4) in K'. When applied to a quantum oscillator consecutively in time the equations provide a complete description of energy absorption, transforrmation, and photon emission.

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The non-relativistic models do not include the photon so they treat the atom incorrectly as a *conservative system*.