Possible Hexaquark Explanation for the State X(2600) in the $\pi^+ + \pi^- + \eta'$ System Observed in the Process $J/\Psi \rightarrow \gamma \pi^+ \pi^- \eta'$

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Abstract

The recently proposed X(2600) structure is investigated using a first-order hexaquark mass formula. This mass relationship is based on weakly coupled $\pi^+ + \pi^- + \eta'$ and $\pi^+ + \pi^- + \eta'^*$ meson clusters. The first-order hexaquark mass formula provides a reasonable prediction (within about 2.8%) of the measured mass, and predicts a $J^\pi = 2^-$ state in agreement with data that suggests possible assignments of 0^+, 0^-, 2^+, and 2^- for the X(2600) resonance.

1.0 Introduction

Based on about 10^4 J/ψ events collected with the BESIII detector, the process $J/\Psi \rightarrow \gamma \pi^+ \pi^- \eta'$ suggests the existence of a new resonance designated as X(2600) with a mass of 2618.3 ± 2.0 (statistical) $^{+16.3}_{-1.4}$ (systematic) MeV[1]. The proposed resonance is observed with a statistical significance larger than 20σ in the $\pi^+ \pi^- \eta'$ invariant mass spectrum. Reference 1 suggests possible J^π assignments of 0^+, 0^-, 2^+, and 2^- for the X(2600) resonance.

The BESIII analysis did not specify the structure of the X(2600). In order to understand the nature of the X(2600) (e.g, an η radial excitation or an exotic hadron), it is necessary to determine its spin and parity and to study its production and decay properties in other J/ψ decay channels. Herein, the X(2600) is investigated in terms of a weakly coupled meson-meson-meson hexaquark.

In this paper, the first-order hexaquark mass formulas of Refs. 2 and 3 are applied to evaluate the possible mass and J^π values of the candidate X(2600) hexaquark. This mass relationship is based on weakly coupled $\pi^+ + \pi^- + \eta'$ and $\pi^+ + \pi^- + \eta'^*$ meson clusters.

The approach of this paper differs from previous hexaquark models[2,3] that utilized two weakly coupled baryon clusters. This paper weakly couples three meson clusters

2.0 Model and Formulation

Zel'dovich and Sakharov[4,5] proposed a semiempirical mass formula that provides a prediction of meson and baryon masses in terms of effective quark masses. Within this formulation, quark wave functions are assumed to reside in their lowest 1S state. These meson mass formulas are used as the basis for deriving a first-order hexaquark mass formula. In particular, the model proposed in this paper assumes the hexaquark is partitioned into three meson clusters with the interaction between the clusters providing a minimal contribution to its mass.

The meson mass (M_m) formula of Refs. 4 and 5 is:

$$\mathbf{M_m = \delta_m + m_1 + m_2 + b_m\,[m_o^2 / (m_1\, m_2)]\,\sigma_1 \cdot \sigma_2 \quad (1)}$$

where m_1 (m_2) are the mass of the first (second) quark comprising the meson, m_0 is the average mass of a first generation quark[6,7], and the σ_i (i = 1 and 2) are the spin vectors for the quarks incorporated into the meson. The parameters δ_m and b_m are 40 MeV/c^2 and 615 MeV/c^2, respectively[5].

The last term in Eq. 1 represents the spin-spin interaction of the quarks and $\boldsymbol{\sigma_1 \cdot \sigma_2}$ is the scalar product of the quark spin vectors. $\boldsymbol{\sigma_1 \cdot \sigma_2}$ has the value -3/4 and +1/4 for pseudoscalar and vector mesons, respectively[5].

In formulating the hexaquark mass formula, effective quark masses provided by Griffiths[6] are utilized. These effective masses for d, u, s, c, b, and t quarks are 340, 336, 486, 1550, 4730, and 177000 MeV/c^2, respectively. The effective masses are utilized in Eq. 1.

These six quarks are arranged in three generations: [d(-1/3), u(+2/3)], [s(-1/3), c(+2/3)], and [b(-1/3), t(+2/3)][7]. The three generations are specified by the square brackets and the quark charges [in elementary charge units (e)] are given within parentheses.

The hexaquark mass (M) is modeled in terms of three weakly interacting meson clusters M_m(i)

$$\mathbf{M = M_m(1) + M_m(2) + M_m(3) + \Phi_{12} + \Phi_{23} \quad (2)}$$

where Φ represents the interaction between the clusters.(e.g., Φ_{12} is the interaction between meson cluster 1 and cluster 2). Other interaction structures (e.g., $\Phi_{12} + \Phi_{13} + \Phi_{23}$) are possible, but the form of Eq. 2 is representative given the assumed zero angular momentum value between the weakly coupled clusters.

3.0 First-Order Mass Formula for the X(2600)

The spin and parity of a hexaquark within the first-order mass formula, summarized in Section 2.0, is determined by coupling the three meson clusters

$$\mathbf{J^\pi = J^\pi(1) \times L(12) \times J^\pi(2) \times L(23) \times J^\pi(3) \quad (3)}$$

where the first-order mass formula assumes a minimally interacting L(12) = L(23) = 0 configuration[2,3] between the clusters. Eq. 3 provides a primitive J^π assignment using the possible meson clusters. The π^+, π^-, and η' have a 0^- assignment. The excited η'^* states have both 0^- and 2^- assignments. These states and their impact on the X(2600) first-order mass prediction are summarized in Table 1. Considering that L(12) = L(23) = 0, Eq. 3 simplifies the weakly coupled $\pi^+ + \pi^- + \eta'$ system to the form

$$J^\pi = J^\pi(1) \times J^\pi(2) \times J^\pi(3) \quad (4)$$

For the $\pi^+ + \pi^- + \eta'$ and $\pi^+ + \pi^- + \eta'^*$ system states built upon 0^- η' and 0^- η'* configurations, Eq. 4 leads to a predicted 0^- hexaquark system ($0^- \times 0^- \times 0^- = 0^-$). A 2^- hexaquark state results from Eq. 4 for other $\eta'^* = 2^-$ ($0^- \times 0^- \times 2^- = 2^-$) states.

The first-order mass formula used in this paper partitions the hexaquark into three meson clusters. These clusters include the $\pi^+ + \pi^- + \eta'$ pseudoscalar mesons (sm). Using this structure, the hexaquark first-order mass formula of Eq. 2 involving ground state $\pi^+ + \pi^- + \eta'$ meson clusters is

$$M(\pi^+ + \pi^- + \eta') = M_{sm}(\pi^+) + M_{sm}(\pi^-) + M_{sm}(\eta') \quad (5)$$

Within the scope of this mass formula, the meson-meson cluster interaction is assumed to be weak and sufficiently small to be ignored in Eq. 5. Accordingly, Eq. 5 represents the X(2600) structure as a quasimolecular six quark systems whose basic character is a weakly bound meson-meson-meson systems where the mesons reside in their ground states.

The π^+ and π^- mesons are modeled as suggested in Refs. 4 and 5. The η' is a superposition of u ubar, d dbar, and s sbar quarks[7]. This configuration type is not addressed in Refs. 4 and 5. As a matter of specificity, this paper calculates the η' mass as a sum of its constituent parts with the methodology of Refs 4 and 5 applied to each constituent meson pair.

4.0 Results and Discussion

The angular momentum coupling from Eq. 4 and the first-order hexaquark mass formula of Eqs. 2 and 5 are used to construct the $\pi^+ + \pi^- + \eta'$ X(2600) state. As noted previously, the spin and parity assignment for the X(2600) state is derived from Eq. 4. The resulting $J^\pi = 0^-$ assignment is in agreement with one of the possible assignments of Ref. 1. As noted in Refs 2 and 3, the first-order mass formula only provides a primitive spin and parity assignment for the meson-meson-meson cluster configuration.

Eqs. 2 and 5 lead to a predicted X(2600) mass of 1807.8 MeV/c^2 for the $\pi^+ + \pi^- + \eta'$ configuration. This result is about 31% smaller than the experimental value[1]. Although these results are encouraging, they are based on a first-order mass formula with a number of uncertainties including the assumed quark masses[6] and the magnitude of the meson-meson cluster interaction. However, the model does provide an initial description of the $\pi^+ + \pi^- + \eta'$ X(2600) state that is reasonable agreement with the experimental data[1].

As noted in Ref. 1, other states are possible including the $\pi^+ + \pi^- + \eta'^*$ configuration. Table 1 summarizes calculations incorporating the η'^*. When incorporating excited meson states, the hexaquark mass is calculated following the format of Eq. 6[2,3]

$$M(\pi^+ + \pi^- + \eta'^*) = M(\pi^+ + \pi^- + \eta') + \Delta \quad (6)$$

where

$\Delta = M(\eta'^*) - M(\eta')$ (7)

Experimental mass values[7] are utilized in Eq. 7.

Table 1 X(2600) First-Order Hexaquark Mass Formula Results

Configuration	M(Mev/c^2)	J^π
$\pi^+ \pi^- \eta'$	1807.8	0^-
$\pi^+ \pi^- \eta(1295)$	2144.0	0^-
$\pi^+ \pi^- \eta(1405)$	2258.8	0^-
$\pi^+ \pi^- \eta(1475)$	2325.0	0^-
$\pi^+ \pi^- \eta_2(1645)$	2467.0	2^-
$\pi^+ \pi^- \eta_2(1870)$	2692.0	2^-
Experiment[a]	2618.30±2.0 (+16.3 – 1.4)	$0^\pm$, $2^\pm$

[a] Ref. 1.

Using Eq. 6, the mass results improve using excited η'^* states. Table 1 suggests the best mass results are obtained for the $\pi^+ \pi^- \eta_2(1870)$ configuration with a predicted mass of 2692.0 MeV/c^2 that is about 2.8% larger than the experimental value. A $J^\pi = 2^-$ value is obtained that is agreement with a portion of the set of possible experimental values.

5.0 Conclusions

The recently proposed X(2600) structure[1] is investigated using a first-order hexaquark mass formula. This mass relationship is based on weakly coupled $\pi^+ + \pi^- + \eta'$ and $\pi^+ + \pi^- + \eta^*$ meson clusters.

The assumed $\pi^+ + \pi^- + \eta'_2(1870)$ configuration leads to a mass result that is within about 2.8% of the measured X(2600) value[1]. The predicted first-order J^π value of 2^- is also in agreement with the possible values suggested by the data. These results suggest the X(2600) structure[1] could be a weakly bound $\pi^+ + \pi^- + \eta'$ or $\pi^+ + \pi^- + \eta^*$ meson cluster system.

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