# **Qeios**

## A pair of possible supernovae Refsdal in the Pantheon+ sample

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## Abstract

On December 1980, supernova 1980N was discovered in NGC 1316, a galaxy of the Fornax cluster. Three months later, supernova 1981D was observed in the same galaxy. The light curves of these two supernovae Ia were found to be virtually identical, suggesting that they are images of the same event, the delay between them being due to strong gravitational lensing. If so, as anticipated by Sjur Refsdal, the distance to the lens can be determined accurately, namely,  $90 \pm 1$  kpc, meaning that it belongs to the outer halo of the Milky Way.

Interestingly, there is another pair of possible images in the Pantheon+ sample, namely, supernovae 2013aa and 2017cbv, the distance to the lens being  $702 \pm 1$  kpc, that is, nearly the same as the distance to the Andromeda galaxy.

In both cases, given the relatively large angle of deviation of the supernova light by the lens, namely, 271" and 325", respectively, the lens has to be a compact object, with a mass to radius ratio over 150  $M_{\odot}R_{\odot}^{-1}$ . It is likely to be an ultra massive white dwarf.

Keywords: Supernovae; Gravitational lensing; White dwarf; Local Group.

## Introduction

In 1979, a possible pair of images of a quasar formed by a gravitational lens was described [\[1\]](#page-4-0). Comparison of the light curves of these two images later showed that there is a delay between them, of more than 400 days [\[2,](#page-4-1) [3\]](#page-4-2). As previously realized by Sjur Refsdal, when such a delay is known, the mass of the lens can be estimated while, when the mass of the lens is known, the Hubble constant can be determined [\[4\]](#page-4-3), as well as other cosmological parameters [\[5\]](#page-4-4). Based on these ideas, the H0LiCOW project has for instance provided a value of the Hubble constant at the 2.4% level [\[6\]](#page-4-5), in agreement with recent measurements of the SH0ES team [\[7\]](#page-4-6).

In 2014, four images of a supernova, coined supernova Refsdal, were observed, the lens being an early-type galaxy in the cluster MACS J1149.5+2223 [\[8\]](#page-4-7). A fifth image was observed with a delay of roughly one year, the lens being this time the whole cluster [\[9\]](#page-5-0). Of course, delays can be longer than that. For instance, in the case of supernova requiem, a fourth image is expected to be observed in 2037, that is, twenty years after the first one [\[10\]](#page-5-1).

In the later case, the three first images of the supernova were found in archival Hubble Space Telescope imaging. In the present study, taking advantage of the recent increase in size of the most complete and best characterized set of supernovae Ia, namely, the Pantheon+ sample [\[11\]](#page-5-2), other such cases were looked for.

#### Data

Like in a previous study [\[12\]](#page-5-3), Pantheon+ data were retrieved from the PantheonPlusSh0es page of the github webserver<sup>[1](#page-0-0)</sup>, equatorial coordinates being taken in the all redshifts PVs.csv file. The 798 supernovae Ia with a rather low redshift ac-

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<span id="page-0-0"></span><sup>1</sup><https://github.com/PantheonPlusSH0ES/DataRelease>



<span id="page-1-1"></span>Figure 1: The small-and-spherical lens approximation. When the lens (L) is spherical, and the light deviation angle  $(\alpha)$  by the lens large enough, the light coming from a source (S) reaches the observer following two paths, an approximately straight one and a delayed one. As a result, two images are seen by an observer, one (I) after the other (S), separated by an angle  $\theta$ .

curacy, namely, with a redshift error of more than 0.0002, were disregarded, redshifts and corresponding errors coming from the Pantheon+SH0ES.dat file. For pairs of supernovae with same heliocentric redshifts, the SALT2 light curve stretch (x1) was also picked in the Pantheon+SH0ES.dat file. When several stretch values were found for a given supernova<sup>[2](#page-1-0)</sup>, their average was considered. Photometry, maximum light curve time (SEARCH PEAKMJD) and host mass (HOST-GAL LOGMASS) of each supernova were taken in the Pantheon+ Data/1 DATA/photometry directory.

## Distance to the lens

Under the assumption that light arriving first from the source is not significantly perturbed by the gravitational field of the lens, with

<span id="page-1-2"></span>
$$
\Delta_d = d_l + d_{ls} - d_s = c_0 \Delta t \tag{1}
$$

 $\Delta t$  being the delay between the observations of the source and its image,  $d_s$ , the distance to the source,  $d_l$ , the distance to the lens,  $d_{ls}$ , the distance between the source and the lens (see Figure [1\)](#page-1-1) and  $c_0$ , the speed of light,

$$
d_{ls}^2 = d_s^2 + d_l^2 - 2d_s d_l \cos \theta
$$

where  $\theta$  is the angle between the source and its image, yields:

$$
d_l = \frac{d_s \Delta_d + \frac{1}{2} \Delta_d^2}{d_s (1 - \cos \theta) + \Delta_d}
$$

and since, in the present study,  $\Delta_d \ll d_s$ , with  $\theta$ being small:

<span id="page-1-4"></span>
$$
d_l \approx \frac{2\Delta_d}{\theta^2 + \frac{2\Delta_d}{d_s}}\tag{2}
$$

## Mass of the lens

The angle of deviation of light by the lens,  $\alpha$ , is so that  $(Fig. 1)$  $(Fig. 1)$ :

$$
\alpha \approx \theta \left( 1 + \frac{d_l}{d_{ls}} \right)
$$

that is, with eqn [1,](#page-1-2) and since  $\Delta_d \ll d_l$ :

<span id="page-1-3"></span>
$$
\alpha \approx \theta \frac{d_s}{d_s - d_l} \tag{3}
$$

On the other hand:

$$
\alpha = \frac{4GM_l}{c_0^2b}
$$

where  $b$  is the distance of closest approach of the light to the lens,  $M_l$  being the mass of the lens and  $G$ , the gravitation constant. Thus, with eqn [3:](#page-1-3)

$$
\frac{M_l}{b} = \frac{c_0^2}{4G} \frac{d_s}{d_s - d_l} \theta
$$

Though b is not expected to be known, it is larger than  $R_l$ , the radius of the lens. So:

<span id="page-1-5"></span>
$$
\frac{M_l}{R_l} > \frac{c_0^2}{4G} \frac{d_s}{d_s - d_l} \theta \tag{4}
$$

#### Results

#### Selection of candidates

644 pairs of supernovae with same heliocentric redshift were found in the Pantheon+ sample, that is, pairs of supernovae with redshifts differing by no more than the sum of their respective errors.

Among these 644 pairs, only 12 have SALT2 stretches (x1) differing by less than 0.02. For seven

<span id="page-1-0"></span><sup>&</sup>lt;sup>2</sup>The magnitude of 127 supernovae of the Pantheon+ sample was measured up to four times.

Supernova Lens Host  $\vert$  z  $\vert$  first image second one  $\vert \theta$  $delay<sup>a</sup>$  distance  $\frac{M_l}{R_l}$ b (days) (kpc) NGC 1316 | 0.005871 | 1980N | 1981D | 271" | 96 | 90  $\pm$  1 | 154 NGC 5643 0.004113 | 2013aa | 2017cbv | 325" | 1490 | 702  $\pm$  1 | 193

<span id="page-2-0"></span>Table 1: Supernovae Refsdal of the Pantheon+ sample.

 $a \pm 1$  day. <sup>b</sup>Lower bound (solar units).

pairs, host masses are known for both of them. However, for five of them, their host masses differ by more than the sum of their respective errors, host masses being compared on the logarithmic scale.

Thus, only two pairs of supernovae from the Pantheon+ sample have obviously same redshift, same SALT2 stretch and same host mass, namely, 1980N and 1981D, 2013aa and 2017cbv. Actually, both pairs are siblings [\[13\]](#page-5-4), that is, they were observed in the same galaxy, namely, NGC 1316 and NGC 5643, respectively.

#### The 1980N–1981D pair

Supernovae Ia 1980N and 1981D were both observed in NGC 1316 [\[14\]](#page-5-5), a member of the Fornax cluster. As illustrated in Figure [2,](#page-3-0) their light curves are remarkably similar [\[14,](#page-5-5) [15\]](#page-5-6). Noteworthy, in the B band (Fig. [2,](#page-3-0) top), their SALT2 stretches are almost identical, namely,  $x1 = -1.14 \pm 0.12$  and  $x1 =$  $-1.15\pm0.36$ , respectively, while the maximum-light magnitudes of both supernovae are the same within  $\pm$  0.1 mag, in the B and V bands [\[15\]](#page-5-6) as well as in the J, H, and K ones [\[14\]](#page-5-5). Interestingly, even though the light curve of 1981D was followed during only 22 days, the color stretches [\[16\]](#page-5-7) of both supernovae look also similar (Fig. [2,](#page-3-0) bottom).

If 1980N and 1981D are two images of the same supernova, according to eqn [2,](#page-1-4) given the delay between the two images, the angle between them (see Table [1\)](#page-2-0) and the distance to NGC 1316, namely, 12.5 Mpc [\[17\]](#page-5-8), the gravitational lens is at a distance of  $90 \pm 1$  kpc, that is, at the same distance as the outer halo globular cluster Eridanus, namely, 90.1 kpc [\[18\]](#page-5-9).

Moreover, according to eqn [4,](#page-1-5) with a mass to radius ratio over 154  $M_{\odot}R_{\odot}^{-1}$ , the lens has to be a compact object, like a white dwarf, a neutron star or a black hole.

Typical white dwarfs have mass to radius ratios of  $\approx$  50  $M_{\odot}R_{\odot}^{-1}$  [\[19\]](#page-5-10). However, white dwarfs with mass to radius ratios over 300  $M_{\odot}R_{\odot}^{-1}$  have been observed [\[19,](#page-5-10) [20\]](#page-5-11). So, the 1980N–1981D gravitational lens is likely to be an ultra massive white dwarf. If so, it is expected to be located almost exactly in the direction where supernova 1981D was observed  $(\alpha = 50.659920^{\circ}, \delta = -37.232720^{\circ}).$ 

#### The 2013aa–2017cbv pair

Supernovae Ia 2013aa and 2017cbv were both observed in NGC 5643 [\[21\]](#page-5-12). As illustrated in Figure [3,](#page-3-1) their light curves are nearly identical [\[21\]](#page-5-12). Noteworthy, in the B band (Fig. [3,](#page-3-1) top), their SALT2 stretches are nearly the same, namely,  $x1 =$  $0.63 \pm 0.14$  $0.63 \pm 0.14$  $0.63 \pm 0.14$  and  $x1 = 0.62 \pm 0.05$ , respectively<sup>3</sup>. Interestingly, their color stretches seem also close (Fig. [3,](#page-3-1) bottom). Moreover, their spectra are remarkably similar [\[21\]](#page-5-12).

If 2013aa and 2017cbv are two images of the same supernova, according to eqn [2,](#page-1-4) given the delay between the two images, the angle between them (Table [1\)](#page-2-0) and the distance to NGC 5643, namely, 18.8 Mpc [\[17\]](#page-5-8), the gravitational lens is at a distance of  $702 \pm 1$  kpc, that is, nearly at the same distance as the Andromeda galaxy [\[22\]](#page-5-13).

Moreover, according to eqn [4,](#page-1-5) with a mass to radius ratio over 193  $\dot{M}_{\odot} R_{\odot}^{-1}$ , the 2013aa–2017cbv gravitational lens is also likely to be an ultra massive white dwarf. If so, it is expected to be located almost exactly in the direction where supernova 2017cbv was observed  $(\alpha = 218.143420^{\circ})$  $\delta = -44.134090^{\circ}$ ).

<span id="page-2-1"></span><sup>3</sup>Data coming from the Carnegie Supernova Project, during which both supernovae were studied.



<span id="page-3-0"></span>Figure 2: Light curves of supernovae Ia 1980N and 1981D, as a function of time since their respective maximum in the B band. Stars: supernova 1980N. Open symbols: supernova 1981D. Top: B magnitudes. Bottom: Difference between magnitudes in the B and V bands.

## Discussion

#### Where are both lenses ?

Lens 1980N–1981D is roughly in the same direction as the Fornax dwarf spheroidal galaxy  $(\alpha = 39.997200^{\circ}, \delta = -34.449187^{\circ}).$  However the later is significantly farer, namely, at a distance of  $143 \pm 3$  kpc [\[23\]](#page-5-14). On the other hand, it is at the same distance as the outer halo globular cluster Eridanus ( $\alpha = 66.185417^{\circ}, \delta = -21.186943^{\circ}$ ) [\[18\]](#page-5-9). However, it is more than 30 kpc away, far from and not in line with its tidal tails [\[24\]](#page-5-15).

Lens 2013aa–2017cbv seems even farer from any known galaxy of the Local Group but, given its large distance, it could for instance belong to some yet unknown cluster or stream of stars.



<span id="page-3-1"></span>Figure 3: Light curves of supernovae Ia 2013aa and 2017cbv, as a function of time since their respective maximum in the B band. Open symbols: supernova 2017cbv. For supernova 2013aa, data come from SWIFT (crosses) or from the Carnegie Supernova Project (stars). Top: B magnitudes (for the sake of clarity, error bars have been omitted). Bottom: Difference between magnitudes in the B and V bands.

#### Are both lenses observable ?

Both lenses are quite far, namely, at a distance of  $\approx 90$  and 700 kpc, respectively (Table [1\)](#page-2-0), while most studies of remote white dwarfs have focused on cases located in rather close globular or open star clusters [\[25,](#page-5-16) [26,](#page-6-0) [27\]](#page-6-1). However, an accreting white dwarf has already been detected in the Andromeda galaxy, thanks to its recurrent X-ray emission [\[28\]](#page-6-2). Note also that LISA has the potential to detect detached double white dwarf binaries in neighboring galaxies, up to the border of the Local Group [\[29\]](#page-6-3).

#### How many of them ?

In the present study, pairs of images of the same supernova have been looked for using rather strict criteria, meaning that some pairs may have been overlooked. For instance, comparison of host masses was considered as a selection criterion, but host masses have not been recorded for all supernovae of the Pantheon+ sample. Moreover, it has been assumed that errors on host mass estimates can be safely trusted. In this respect, note that it would be more rigorous to compare the total surface brightnesses of both hosts, since this is the quantity that is expected to be conserved upon light deflection [\[30\]](#page-6-4).

## Conclusion

Two likely pairs of images have been found in the Pantheon+ sample of supernovae Ia, that is, two pairs of supernovae with strikingly similar light curves [\[15,](#page-5-6) [21\]](#page-5-12) (Fig. [2](#page-3-0) and [3\)](#page-3-1). In both cases, the gravitational lens has to be a compact object with a mass to radius ratio over 150  $M_{\odot}R_{\odot}^{-1}$  (Table [1\)](#page-2-0), being likely an ultra massive white dwarf. While lens 1980N–1981D is located in the outer halo of the Milky Way, lens 2013aa–2017cbv is at the same distance as the Andromeda galaxy.

Masses of nearby white dwarfs were previously measured using related effects, like astrometric microlensing [\[31\]](#page-6-5) or self-lensing [\[32\]](#page-6-6). However, to my knowledge, this is the first time strong gravitational lensing is used for measuring the distance and estimating the mass to radius ratio of a star.

Using this approach, ongoing surveys of supernovae Ia, like the Zwicky Transient Facility [\[33\]](#page-6-7) or the Dark Energy Survey [\[34\]](#page-6-8), are expected to provide more cases. Of course, compact lenses in the Local Group could also be found by studying other kinds of well-characterized transients.

With more statistics, the study of the population of compact objects in the halo of the Milky Way or within the Local Group may prove interesting, for instance for gaining better insights about the substructures of the former or the history of the later. Understanding the origin of apparently isolated objects like those discovered in the course of the present study could also reveal key features of the dynamics of the Local Group.

## References

- <span id="page-4-0"></span>[1] Walsh, D., Carswell, R.F. & Weymann, R.J. (1979). 0957+561 A, B: twin quasistellar objects or gravitational lens ? Nature 279(5712), 381–384.
- <span id="page-4-1"></span>[2] Pelt, J., Kayser, R., Refsdal, S. & Schramm, T. (1996). The light curve and the time delay of QSO 0957+561. Astron. Astrophys. 305, 97.
- <span id="page-4-2"></span>[3] Oscoz, A., Alcalde, D., Serra-Ricart, M., Mediavilla, E., Abajas, C., Barrena, R., Licandro, J., Motta, V. & Munoz, J. (2001). Time delay in QSO 0957+561 from 1984–1999 optical data.  $Ap. J. 552(1), 81.$
- <span id="page-4-3"></span>[4] Refsdal, S. (1964). On the possibility of determining Hubble's parameter and the masses of galaxies from the gravitational lens effect. Mon. Not. R. Astron. Soc. 128(4), 307–310.
- <span id="page-4-4"></span>[5] Dobke, B.M., King, L.J., Fassnacht, C.D. & Auger, M.W. (2009). Estimating cosmological parameters from future gravitational lens surveys. Mon. Not. R. Astron. Soc.  $397(1)$ , 311–319.
- <span id="page-4-5"></span>[6] Wong, K.C., Suyu, S.H., Chen, G.C., Rusu, C.E., Millon, M., Sluse, D., Bonvin, V., Fassnacht, C.D., Taubenberger, S., Auger, M.W. et al. (2020). H0LiCOW XIII. A 2.4% measurement of H0 from lensed quasars:  $5.3\sigma$  tension between early and late-Universe probes. Mon. Not. R. Astron. Soc. 498, 1420–1439.
- <span id="page-4-6"></span>[7] Riess, A.G., Yuan, W., Macri, L.M., Scolnic, D., Brout, D., Casertano, S., Jones, D.O., Murakami, Y., Anand, G.S., Breuval, L. et al. (2022). A comprehensive measurement of the local value of the Hubble constant with 1 km s<sup>-1</sup> Mpc<sup>-1</sup> uncertainty from the Hubble Space Telescope and the SH0ES team. Ap. J. letters 934(1), L7.
- <span id="page-4-7"></span>[8] Kelly, P.L., Rodney, S.A., Treu, T., Foley, R.J., Brammer, G., Schmidt, K.B., Zitrin, A., Sonnenfeld, A., Strolger, L.G., Graur, O. et al. (2015). Multiple images of a highly magnified supernova formed by an early-type cluster galaxy lens. Science 347(6226), 1123–1126.
- <span id="page-5-0"></span>[9] Kelly, P.L., Rodney, S., Treu, T., Strolger, L.G., Foley, R., Jha, S., Selsing, J., Brammer, G., Bradač, M., Cenko, S.B. *et al.* (2016). Deja vu all over again: the reappearance of supernova refsdal. Ap. J. letters 819(1), L8.
- <span id="page-5-1"></span>[10] Rodney, S.A., Brammer, G.B., Pierel, J.D., Richard, J., Toft, S., O'Connor, K.F., Akhshik, M. & Whitaker, K.E. (2021). A gravitationally lensed supernova with an observable two-decade time delay. Nature Astronomy 5(11), 1118–1125.
- <span id="page-5-2"></span>[11] Scolnic, D., Brout, D., Carr, A., Riess, A.G., Davis, T.M., Dwomoh, A., Jones, D.O., Ali, N., Charvu, P., Chen, R. et al. (2022). The Pantheon+ analysis: the full data set and light-curve release.  $Ap. J. 938(2), 113.$
- <span id="page-5-3"></span>[12] Sanejouand, Y.H. (2023). A robust assessment of the local anisotropy of the Hubble constant. arXiv 2312, 05896.
- <span id="page-5-4"></span>[13] Scolnic, D., Smith, M., Massiah, A., Wiseman, P., Brout, D., Kessler, R., Davis, T., Foley, R., Galbany, L., Hinton, S.R. et al. (2020). Supernova siblings: assessing the consistency of properties of type Ia supernovae that share the same parent galaxies. Ap. J. letters  $896(1)$ , L13.
- <span id="page-5-5"></span>[14] Elias, J.H., Frogel, J.A., Hackwell, J.A. & Persson, S.E. (1981). Infrared light curves of Type I supernovae. Astrophys. J. 251, L13– L16.
- <span id="page-5-6"></span>[15] Hamuy, M., Phillips, M., Maza, J., Wischnjewsky, M., Uomoto, A., Landolt, A.U. & Khatwani, R. (1991). The optical light curves of SN 1980N and SN 1981D in NGC 1316 (Fornax A). A. J. 102, 208–217.
- <span id="page-5-7"></span>[16] Burns, C.R., Stritzinger, M., Phillips, M., Hsiao, E., Contreras, C., Persson, S., Folatelli, G., Boldt, L., Campillay, A., Castellón, S. et al. (2014). The Carnegie supernova project: intrinsic colors of type Ia supernovae. Ap. J. 789(1), 32.
- <span id="page-5-8"></span>[17] Hoyt, T.J., Beaton, R.L., Freedman, W.L., Jang, I.S., Lee, M.G., Madore, B.F., Monson, A.J., Neeley, J.R., Rich, J.A. & Seibert, M. (2021). The carnegie chicago hubble program

X: Tip of the red giant branch distances to NGC 5643 and NGC 1404. Ap. J. 915(1), 34.

- <span id="page-5-9"></span>[18] Baumgardt, H., Hilker, M., Sollima, A. & Bellini, A. (2019). Mean proper motions, space orbits, and velocity dispersion profiles of Galactic globular clusters derived from Gaia DR2 data. Mon. Not. R. Astron. Soc. 482(4), 5138–5155.
- <span id="page-5-10"></span>[19] Bédard, A., Bergeron, P. & Fontaine, G. (2017). Measurements of physical parameters of white dwarfs: A test of the mass–radius relation.  $Ap. J. 848(1), 11.$
- <span id="page-5-11"></span>[20] Pshirkov, M.S., Dodin, A.V., Belinski, A.A., Zheltoukhov, S.G., Fedoteva, A.A., Voziakova, O.V., Potanin, S.A., Blinnikov, S.I. & Postnov, K.A. (2020). Discovery of a hot ultramassive rapidly rotating DBA white dwarf. Mon. Not. R. Astron. Soc. lett. 499(1), L21–L25.
- <span id="page-5-12"></span>[21] Burns, C.R., Ashall, C., Contreras, C., Brown, P., Stritzinger, M., Phillips, M., Flores, R., Suntzeff, N.B., Hsiao, E.Y., Uddin, S. et al. (2020). SN 2013aa and SN 2017cbv: Two Sibling Type Ia Supernovae in the spiral galaxy NGC 5643. Ap. J. 895(2), 118.
- <span id="page-5-13"></span>[22] Vilardell, F., Ribas, I., Jordi, C., Fitzpatrick, E.L. & Guinan, E.F. (2010). The distance to the Andromeda galaxy from eclipsing binaries. Astronomy & Astrophysics 509, A70.
- <span id="page-5-14"></span>[23] Oakes, E.K., Hoyt, T.J., Freedman, W.L., Madore, B.F., Tran, Q.H., Cerny, W., Beaton, R.L. & Seibert, M. (2022). Distances to Local Group Galaxies via Population II, Stellar Distance Indicators. II. The Fornax Dwarf Spheroidal. Ap. J. 929(2), 116.
- <span id="page-5-15"></span>[24] Myeong, G., Jerjen, H., Mackey, D. & Da Costa, G.S. (2017). Tidal Tails around the Outer Halo Globular Clusters Eridanus and Palomar 15. Ap. J. letters 840(2), L25.
- <span id="page-5-16"></span>[25] Campos, F., Bergeron, P., Romero, A.D., Kepler, S.O., Ourique, G., Costa, J.E.d.S., Bonatto, C.J., Winget, D.E., Montgomery, M.H., Pacheco, T.A. et al. (2016). A comparative analysis of the observed white dwarf cooling sequence from globular clusters. Mon. Not. R. Astron. Soc. 456(4), 3729–3742.
- <span id="page-6-0"></span>[26] Williams, K.A., Canton, P.A., Bellini, A., Bolte, M., Rubin, K.H., Gianninas, A. & Kilic, M. (2018). Ensemble properties of the white dwarf population of the old, solar metallicity open star cluster Messier 67. Ap. J.  $867(1)$ , 62.
- <span id="page-6-1"></span>[27] Richer, H.B., Caiazzo, I., Du, H., Grondin, S., Hegarty, J., Heyl, J., Kerr, R., Miller, D.R. & Thiele, S. (2021). Massive white dwarfs in young star clusters.  $Ap. J. 912(2), 165.$
- <span id="page-6-2"></span>[28] Tang, S., Bildsten, L., Wolf, W.M., Li, K., Kong, A.K., Cao, Y., Cenko, S.B., De Cia, A., Kasliwal, M.M., Kulkarni, S.R. et al. (2014). An accreting white dwarf near the Chandrasekhar limit in the Andromeda galaxy. Ap.  $J. 786(1), 61.$
- <span id="page-6-3"></span>[29] Korol, V., Koop, O. & Rossi, E.M. (2018). Detectability of double white dwarfs in the local group with LISA.  $Ap. J. letters 866(2), L20.$
- <span id="page-6-4"></span>[30] Virbhadra, K. (2022). Distortions of images of Schwarzschild lensing. Phys. Rev. D 106(6), 064038.
- <span id="page-6-5"></span>[31] Sahu, K.C., Anderson, J., Casertano, S., Bond, H.E., Bergeron, P., Nelan, E.P., Pueyo, L., Brown, T.M., Bellini, A., Levay, Z.G. et al. (2017). Relativistic deflection of background starlight measures the mass of a nearby white dwarf star. Science 356(6342), 1046–1050.
- <span id="page-6-6"></span>[32] Masuda, K., Kawahara, H., Latham, D.W., Bieryla, A., Kunitomo, M., MacLeod, M. & Aoki, W. (2019). Self-lensing discovery of a  $0.2 \text{ M}$  white dwarf in an unusually wide orbit around a Sun-like star.  $Ap. J. 881(1), L3.$
- <span id="page-6-7"></span>[33] Dhawan, S., Goobar, A., Smith, M., Johansson, J., Rigault, M., Nordin, J., Biswas, R., Goldstein, D., Nugent, P., Kim, Y. et al. (2022). The Zwicky Transient Facility Type Ia supernova survey: first data release and results. Mon. Not. R. Astron. Soc.  $510(2)$ , 2228–2241.
- <span id="page-6-8"></span>[34] Abbott, T.M.C., Acevedo, M., Aguena, M., Alarcon, A., Allam, S., Alves, O., Amon, A., Andrade-Oliveira, F., Annis, J., Armstrong, P. et al. (2024). The Dark Energy Survey:

Cosmology Results With  $\approx 1500$  New Highredshift Type Ia Supernovae Using The Full 5-year Dataset. arXiv 2401, 02929.