

Research Article

Can Human Drivers and Connected Autonomous Vehicles Co-exist in Lane-Free Traffic? A Microscopic Simulation Perspective

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Recent advancements in connected autonomous vehicle (CAV) technology have sparked growing research interest in lane-free traffic (LFT). LFT envisions a scenario where all vehicles are CAVs, coordinating their movements without lanes to achieve smoother traffic flow and higher road capacity. This potentially reduces congestion without building new infrastructure. However, the transition phase will likely involve non-connected actors such as human-driven vehicles (HDVs) or independent AVs sharing the roads. This raises the question of how LFT performance is impacted when not all vehicles are CAVs, as these non-connected vehicles may prioritize their own benefits over system-wide improvements. This paper addresses this question through microscopic simulation on a ring road, where CAVs follow the potential lines (PL) controller for LFT, while HDVs adhere to a strip-based car-following model. The PL controller is also modified for safe velocities to prevent collisions. The results reveal that even a small percentage of HDVs can significantly disrupt LFT flow: 5% HDVs can reduce LFT's maximum road capacity by 16%, and a 20% HDVs nearly halves it. The study also develops an adaptive potential (APL) controller that forms APL corridors with modified PLs in the surroundings of HDVs. APL shows a peak traffic flow improvement of 23.6% over the PL controller. The study indicates that a penetration rate of approximately 60% CAVs in LFT is required before significant benefits of LFT start appearing compared to a scenario with all HDVs. These findings open a new research direction on minimizing the adverse effects of non-connected vehicles on LFT.

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Highlights

- Even a small percentage of human driver vehicles (HDVs) significantly lowers the LFT flow.
- The LFT traffic flow via Potential Lines controller halves by 20% HDV penetration rate
- 60% CAV Penetration rate is required before observing LFT's major advantages
- Introduced Adaptive Potential Line (APL) controller to improve the LFT with HDVs
- A peak improvement of 23.6% was observed via the APL strategy over PL controller

1. Introduction

With urbanization and increased usage of private vehicles (PVs), traffic congestion has been an ever-increasing problem, especially in cities. The increased traffic has led to worldwide road construction projects, yet the problem persists^[1]. The last two decades have seen significant changes in the automotive industry. There has been a steady increase in autonomous vehicle (AV) technology. Multitudes of works have suggested using shared AVs (SAVs) to sway people away from using PVs in the direction of public transport (PT) by providing first- or last-mile operation via SAVs or providing a whole mobility-on-demand (MOD) service using AV fleet^{[2][3][4]}. However, the experience with the current MOD services indicates that if not appropriately regulated, these services may contribute further to the traffic congestion with additional vehicles^[1]. Therefore, the fundamental problem remains that a vehicle occupies the same amount of road space regardless of whether a human or AV technology drives the vehicle and would require increased road infrastructure to accommodate more vehicles.

To address the above problem, there has also been research on increasing road capacities and safety by utilizing new AV technologies. In this regard, connected autonomous vehicles (CAVs) play a significant role^{[5][6]}. The real-time data sharing from vehicle-to-vehicle (V2V), vehicle-to-infrastructure (V2I), or vehicle-to-everything (V2X) allows the CAVs to constantly observe the environment from multiple perspectives, enabling technologies such as advanced traffic state estimation approaches^[7], cooperative adaptive cruise control (CACC)^[6], speed advisory systems^[8] or integrated intersection control^[9]. The most significant change that CAVs can bring is perhaps in the currently dominant driving paradigm of lane-based traffic management — the CAVs show the potential to drive even when no specific lanes are marked on the road^[10]. The concept of managed lanes was introduced in the last century to help coordinate the vehicle movements by human drivers in an era of ever-

increasing maximum possible vehicle speed^[11]. With the introduction of CAVs, the fixed lanes can be removed since the CAVs can coordinate their movements via communication channels. This has led to the concept of lane-free traffic (LFT) for CAVs^[10]. Figure 1 shows the concept of LFT. As shown in Figure 1(b), the LFT vehicles can communicate and influence the movements of upstream and downstream vehicles using wireless communication. Usually, the LFT controller algorithms achieve this by assuming artificial nudging and repulsive forces applied to vehicles in the front and the back, respectively ^{[10][12][13]}.

The LFT was inspired by the vehicles driving without strict lane discipline in some countries, referred to as lane-less traffic^{[11][10]}. This allows them to utilize the whole width of the road; however, with a significantly higher risk of accidents^[14]. Apart from automation that allows fast and accurate decisions in LFT, the main difference between the two modes is that while in lane-less traffic, communication with other drivers is limited to honking, headlight flashing, or hand gesturing, the vehicles in LFT can fully communicate their intended trajectories and other important information to significantly larger areas. This allows LFT vehicles to coordinate their movements in a way that substantially improves the system-wide maximum flow rate compared to lane-based traffic for the same road width^[13]. However, the exact flow rate improvement depends on the LFT controller used.

While the LFT strategy has a high potential to improve traffic conditions, its realization may still be decades ahead. First, significant technological developments are required to ensure flawless inter-vehicular communication and reliable autonomous driving functionality. Second, the CAVs in LFT will be driving in a completely new traffic mode where safety will be of primary concern. Consequently, LFT must undergo rigorous testing before any CAV can be driven in LFT mode. However, unlike the functionality of current AVs, which can be tested with a single vehicle, the LFT would require tests with multiple CAVs operating in an unexplored driving mode. To solve this problem, some researchers have suggested using a driving simulator to study the safety as perceived by the traveler in LFT ^[15]. Nonetheless, it is expected that even if the cities are convinced of investing in the LFT, the transition phase or even the actual operation of LFT may still involve some traffic participants who do not necessarily coordinate their movements with other vehicles. This can happen for multiple reasons, for example, the involvement of human drivers, having an older AV without a communication module, or a CAV temporarily losing its communication capability. Under all of these circumstances, the question remains: how will the efficiency of the LFT be affected by it? It is equally important to ask how much the proportion of CAVs should be to achieve the benefits of LFT.

This work attempts to answer the above questions. The paper mainly focuses on the impacts of having some traffic participants in LFT who try to improve self-interest instead of coordinating and optimizing the flow of the whole system. These participants are assumed to be human drivers trying to achieve their desired speeds while maintaining a safe distance from the vehicle in the front. The LFT traffic vehicles are assumed to be unable to exert nudging forces on these HDVs, as shown in Figure 1(c), which is expected to disrupt the functioning of the LFT controller. The paper does not explicitly model independent AV; instead, it is assumed that these HDVs can also partially represent independent AVs since AVs may already be trying to replicate human driving behavior. Nonetheless, the main focus of the paper is not on accurately modeling the human or independent AV's behavior but rather on the disruption it causes to the LFT.

To study the impact of the whole setup, microscopic simulations of a 1 km ring road are used. The human drivers are modeled using the car-following model by^[16], designed to simulate lane-less traffic. For the CAVs, the paper uses the potential line (PL) controller introduced by^[13]. Inspired by the human model of^[16], the paper also introduces the concept of safe acceleration into the PL controller as a secondary measure to avoid any potential collision. Finally, an adaptive potential lines (APL) controller is introduced that improves the flow of CAVs mixed with HDVs in LFT. The new controller forms APL corridors with modified PLs in the vicinity of HDVs.

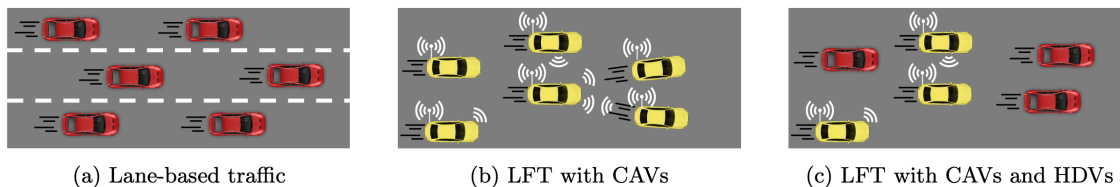


Figure 1. Traditional lane-based traffic, lane-free traffic (LFT) and LFT with a mixture of CAVs and HDVs.

1.1. Background

To address the ever-increasing traffic demand, modern roads were designed to accommodate more vehicles and alleviate congestion by rerouting traffic away from city centers. This includes the introduction of beltways or ring roads, which provide more direct and faster connections to areas around the city. However, investigations worldwide reveal the immediate benefits of beltways have been diminished due to the induced demand and relocation of jobs and housing to suburban areas^[17]

[18][19]. Consequently, innovative approaches using advanced Vehicle Automation and Communication Systems (VACS) are gaining the center stage for sustainable long-term solutions to ensure smooth traffic flow, reduce congestion, and enhance safety[20]. In the last decade, the application of CAVs comprised the vast majority of cutting-edge research towards addressing traffic congestion[6], including LFT for more exploitation of the lateral capacity of the roads[11].

Since LFT allows for lateral freedom of CAVs, compared to lane-based traffic, novel driving strategies have been proposed for vehicle navigation in this new environment. A common approach comprises defining artificial potential fields around each vehicle for collision avoidance[21], as shown in Figure 2(a), and including additional controllers for achieving other objectives such as driving close to the desired speed, staying within the road boundary, and driving energy-efficiently.[22] formulated all the objectives in an optimal control problem and solved it for each vehicle in real-time. A nonlinear model predictive control (NMPC) approach is proposed in [23] for cooperative driving among autonomous vehicles. A more structured lane-free traffic is proposed in [24]. In this approach, an artificial potential line (PL) is assigned for each vehicle as the desired lateral location, as illustrated in 2(b). This led to more laminar traffic flow, eliminating unnecessary lateral movement of vehicles. Other approaches, such as nonlinear controllers[25], have also been proposed that use a more complex dynamic model of vehicles. While the majority of LFT research focuses on freeway networks, few have considered elements of urban networks such as beltways[13], intersections[26][27], and roundabouts[28]. In addition to conventional control approaches, reinforcement learning-based approaches have also been used, showcasing comparable benefits for driving comfort and traffic efficiency[29][30][31][32]. In addition, LFT calls for novel ideas such as vehicle flocking[33][34] or snake-like platooning[35].

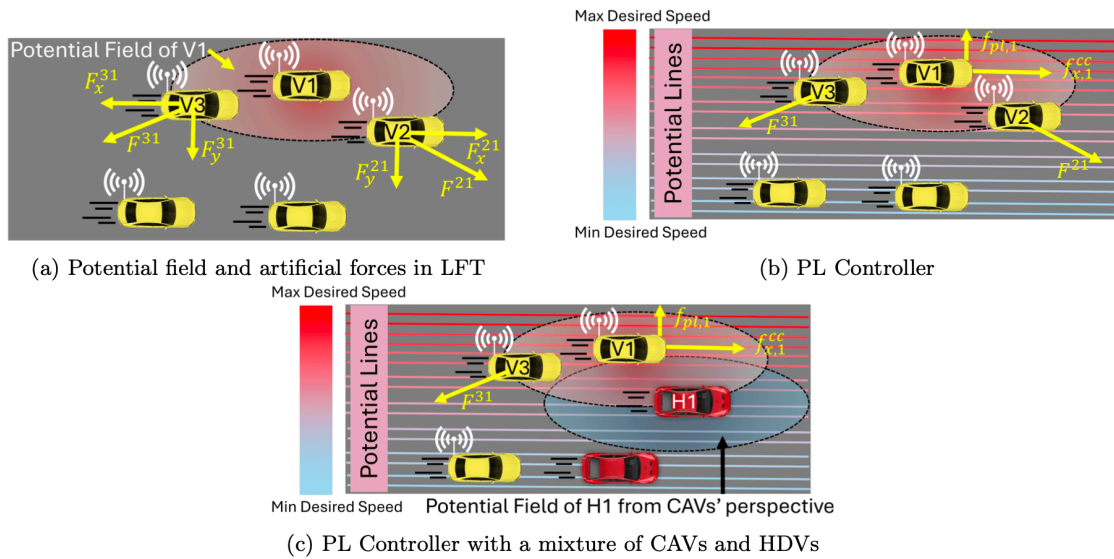


Figure 2. Potential fields, artificial forces and potential line (PL) controller for LFT.

It should be noted that almost all current LFT driving strategies rely highly on V2V communication, sharing the vehicle's current state or even planned trajectory. For example, to apply an optimal control approach, the vehicles in [22] share planned trajectories with the surrounding vehicles. Since the HDVs in the current work are assumed to make independent decisions by considering self-interests, their trajectories are not available to the LFT controller and, thus, cannot be shared with CAVs. Therefore, these latter types of LFT controllers pose a significant challenge in handling HDVs.

To simulate mixed HDVs and CAVs traffic, it is essential to have a driving model that accurately represents human drivers. Currently, there is no data on how humans would drive in an LFT environment. Therefore, the most suitable approach is to consider models developed for lane-less traffic. In the literature, models for lane-less traffic often target a traffic condition where vehicles of various types and sizes share the same road without strict lane discipline. This can include vehicles like cars, rickshaws, motorbikes, and trucks[36][37][38].

The main challenge in modeling lane-less traffic is that most car-following models assume homogeneous traffic driving in lanes. These models typically assume a pair of a leader and a follower vehicle, which suits lane-based traffic and focuses mainly on longitudinal movements. However, lane-less traffic has certain additional characteristics that are not found in lane-based traffic. For

example, smaller vehicles tend to squeeze into gaps between larger vehicles, exhibit staggered following, and show lateral shifting or even multiple leaders^{[36][39]}.

The literature shows a gradual improvement in models that can deal with lane-less traffic. For instance,^[40] incorporated non-lane-based car following into the basic model of ^[41]. Due to the off-centered vehicle positions, this model does not allow full leadership to the front vehicle, and thus making staggered following possible. Although the model is designed for homogeneous traffic, it allows weak lane discipline behavior and includes lateral discomfort.^[37] modified^[41] to include type-dependent variables for the leader-follower pair. Later, inspired by approaches with a discretized lateral axis^[42], they improved the type-dependent model using the concept of strips^[16]. A similar concept of sublane was later introduced into Simulation of Urban Mobility (SUMO)^[43] for Chinese traffic situations and became part of its standard distribution^[44]. Other methods have also been used for mixed traffic^{[45][46]}. More recently, with the possibility of collecting lane-less traffic data more efficiently, data-driven approaches have emerged in the literature^{[39][47]}. However, strip-based approaches have so far been simple and computationally effective for simulating lane-less traffic.

2. Methodology

To simulate LFT with CAVs and HDVs, the paper uses two methods simultaneously: a car-following model for HDVs and LFT controller for CAVs. After establishing the equations for vehicle dynamics, the section presents both in detail.

2.1. Vehicle Dynamics

The vehicles in the simulation move in discrete time steps using a double integrator model. The model is implemented using differential equations. Let k , ΔT and $t = k \cdot \Delta T$ represent the current time step, time step size and current simulation time, respectively. For a vehicle i , let x_i , $v_{x,i}$ and $a_{x,i}$ represent the longitudinal position, speed, and acceleration, respectively. Similarly, let y_i , $v_{y,i}$, and $a_{y,i}$ represent the same variables for the lateral axis, respectively. Then the equations for vehicle dynamics are given as:

$$x_i(k+1) = x_i(k) + \Delta T v_{x,i}(k) + \frac{1}{2} \Delta T^2 a_{x,i}(k) \quad (1a)$$

$$y_i(k+1) = y_i(k) + \Delta T v_{y,i}(k) + \frac{1}{2} \Delta T^2 a_{y,i}(k) \quad (1b)$$

$$v_{x,i}(k+1) = v_{x,i}(k) + \Delta T a_{x,i}(k) \quad (1c)$$

$$v_{y,i}(k+1) = v_{y,i}(k) + \Delta T a_{y,i}(k) \quad (1d)$$

According to the vehicle type (HDV or CAV), each time step calculates the accelerations in both directions using the human driver model or the LFT controller. Then, the above equations are used to calculate the vehicle states for the next time step. The current study also assumes that each vehicle i has a desired longitudinal speed $v_{des,i}$, which it aims to achieve during the simulation. To prevent the vehicle from leaving the road boundary, the simulation models limit the lateral acceleration such that the road boundary is not crossed.

2.2. Human Driver Model

The human driver model used is based on the strip-based model proposed by [16] for lane-less traffic. Besides being simple, the main reason for choosing the model is that it was validated in a SUMO simulation using video data from a highway in Mumbai, India. This model differs from traditional lane-based simulations in that it allows continuous movement in the lateral axis by discretizing it into strips. The current paper replicates Mathew's model without significant changes. The following summarizes its formulation for the longitudinal and lateral movements.

2.2.1. Longitudinal Movement

Mathew's model based the longitudinal movement on the Gipps model [41] originally meant for lane-based traffic. Instead of using the original Gipp's formulation, they used its simplification as presented in [48]. Nonetheless, both the original and its simplification are conceptually identical.

For a collision-free movement, the model uses a safe velocity $v_{x,i}^{safe}$ with regards to the vehicle in the front, called the leader. This causes the subject vehicle i to maintain a safe gap and stop behind the leading vehicle without collision if required. The formulation for the safe velocity is given as:

$$v_{x,i}^{safe}(k+1) = -\tau \cdot A_x^- + \sqrt{(\tau \cdot A_x^-)^2 + v_{x,leader}(k)^2 + 2 \cdot A_x^- \cdot (g_{leader}(k) - g_o)} \quad (1)$$

where τ is the reaction time in seconds (s), g_{leader} is the distance from the front of the i to the back of the leader in meters (m), A_x^- is the maximum deceleration ability of i in m/s^2 and $v_{x,leader}$ is the longitudinal velocity of the leader in m/s . g_o is the minimum distance i withholds if the leader suddenly stops.

Eq. 1 is the simplified Gipp's model. [16] extended it to account for different combinations of follower-leader vehicle types (e.g. trucks, rickshaws, motorbikes, cars). More specifically, [16] replaced the term $(g_{leader} - g_o)$ in Eq. 1 by a regression-based formulation to convert different combinations of vehicle

types into equivalent gap for car following car. However, since all vehicles in the current study are supposed to be cars, this modification is ignored in the current work. Additionally, g_0 is also set to 0 for simplification.

The next important factor in Eq. 1 is the determination of the leader vehicle. In lane-based traffic, the leader is the immediate vehicle in the front in the same lane. However, selecting a leader in traffic without lanes is more challenging since multiple vehicles could be in front.^[16] suggested discretizing the lateral axis strips of width Δs . Instead of occupying a lane, each vehicle occupies a strip. For the vehicle i , the leader is chosen by looping through all the vehicles that occupy any of the strips that i occupies and then selecting the one closest to i in the longitudinal distance. For computational efficiency, the current study only considers the vehicles within the front distance ΔD_{front} of i . If no such leading vehicle exists within ΔD_{front} , the $v_{x,i}^{safe}$ is set to the desired speed $v_{des,i}$ of the vehicle.

In the current study, after determining the $v_{x,i}^{safe}$, the difference to the current speed $v_i^{diff} = v_{safe} - v_{x,i}$ is calculated, which is then used to calculate the safe acceleration:

$$a_{x,i}^{safe}(k+1) = \begin{cases} \min\left(\frac{v_i^{diff}}{\Delta T}, A_x^-\right) & \text{if } v_i^{diff} < 0 \\ \min\left(\frac{v_i^{diff}}{\Delta T}, A_x^+\right) & \text{otherwise} \end{cases} \quad (2)$$

where A_x^+ is the vehicle's maximum acceleration ability. Eq. 2 shows that a vehicle's acceleration and deceleration are constrained by its abilities, i.e. A_x^- and A_x^+ , respectively.

2.2.2. Lateral Movement

The lateral movements in the human model of ^[16] are also based on strips rather than lanes. The decision to change a strip is dependent on the benefit of changing the strip, measured in terms of speed gain. A vehicle is allowed to change only one strip in a time step; however, the model considers multiple strips in calculating the benefits since the driver's overtaking maneuver may require changing multiple strips. Thus, the decision to change the strip involves multiple steps. First, for a vehicle i , the benefit of changing the strip from a currently occupied strip s_c to a destination strip s_d is calculated using the following formula:

$$b_{s_d,i}(k) = \frac{v_{x,i,s_d}^{safe}(k) - v_{x,i,s_c}^{safe}(k)}{v_{des,i}} \times e^{-\lambda \cdot n_s} \quad (3)$$

where v_{x,i,s_d}^{safe} and v_{x,i,s_c}^{safe} are the safe velocities in strips s_d and s_c , respectively. v_{x,i,s_d}^{safe} is calculated by imagining i to be located at s_d (instead of s_c), and calculating the leading vehicle and subsequently the

safe velocity using Eq. 1. n_s is the number of strip changes required to reach s_d from s_c and λ is a model parameter to control the impact of far-away strips. Thus, the factor $e^{-\lambda \cdot n_s}$ is used to reduce the importance of benefits as the destination strip gets farther away from the currently occupied strip. In the denominator of Eq. 3, [16] used the maximum speed possible in the current strip s_c . However, the current study replaced it with the desired speed of i . This modification is done for consistency with the PL controller for CAVs, which assumes that each CAV tries to achieve and maintain a specific desired speed. In simple words, Eq. 3 compares the potential speed in the destination strip to the speed possible in the current strip while considering the number of strip changes needed to reach the destination strip. If it is possible to gain speed in the destination strip, the benefits are positive; otherwise, they are negative.

With regards to the implementation of Eq. 3, the model assumes that the decision to change the strip is not instantaneous; rather, the driver may keep track of the benefits for multiple time steps and only move when there is a significant accumulated benefit. Therefore, for each HDV, variables are maintained that represent the driver memory for observed benefits. The current study maintains separate variables for the accumulated benefits of strip changes to the left and right sides of the currently occupied strips. In each time step, Eq. 3 is calculated for all strips on the left and the right side, and accumulated to the respective memory variable; when the accumulated benefit crosses a certain threshold L_{th} , the driver changes the strip in the corresponding direction. The side that has the largest accumulated benefit is preferred for strip change. In order to avoid continuous changing of strips, the driver's memory variable is halved if the observed benefit on the corresponding side of the strip is less than or equal to zero.

The value of L_{th} models the aggressiveness of the human driver. A low value of L_{th} would mean that the human driver makes lateral changes even for a slight gain in speed and vice versa for a high L_{th} . On the other hand, λ determines how much changes in lateral position a human considers worth considering for the acquired benefits.

2.3. Potential Lines Controller for LFT

The PL controller introduced by [13] observes that the vehicles in countries with right-hand driving rules usually overtake and move faster on the left side of the road. PL controller utilizes this observation to laterally organize the LFT based on the desired longitudinal speeds of CAVs. The CAVs are assigned virtual PLs from left to right based on their desired speeds, as shown in Figure 2b,

encouraging the CAVs to follow the designated PL. This creates a lane-independent lateral structure and avoids unnecessary lateral movements.

For this study, any LFT controller could have been chosen to study the impacts of HDVs. However, the primary reason for choosing the PL controller is its simplicity and the fact that it does not require knowledge of the paths to be taken by other traffic participants, which is usually required for some optimization-based LFT approaches [22]. This requirement may be problematic for the inclusion of HDVs since the system does not exactly know the path that humans will take. However, the current study assumes that CAVs at least know the current speed of the HDVs, which can be estimated with high accuracy.

The PL controller of [13][24] calculates the longitudinal acceleration based on two terms: (1) the goal to achieve the desired speed (the cruise control) and (2) the influential artificial forces of the other vehicles that prohibit collisions. These forces are also shown in Figure 2. However, during experiments for the current study, it was noticed that artificial forces require significant parameter tuning to avoid collisions, which may still occur under high vehicle densities. Thus, the current study introduced the concept of safe velocity into the PL controller, inspired by the human driver model. Furthermore, [13][24] used additional boundary forces using the proportional controller to force CAVs away from road boundaries. However, for consistency with the HDVs, the boundary forces are replaced by a hard constraint on the lateral acceleration that keeps the CAVs within road boundaries.

The following details the components used for the PL controller:

2.3.1. PL based Lateral Control

The PL controller uses a PL force to guide the vehicle to the assigned PL. As the first step, a lateral position $y_{pl,i}$, referred to as the PL, is assigned to the vehicle i by linearly distributing the lateral axis according to the minimum and maximum desired speeds of all vehicles:

$$y_{pl,i} = Y_r + B_{pl} + (v_{des,i} - v_{min}) \frac{Y_l - Y_r - 2B_{pl}}{v_{max} - v_{min}} \quad (4)$$

where Y_r and Y_l are the lateral positions for the right and left boundaries of the road, respectively. The parameter B_{pl} is used to leave some gap without PLs on either side of the road. This is required since the CAVs use the positions of their centers to align with the assigned PL. Therefore, B_{pl} is determined based on the width of the broadest vehicle. v_{max} and v_{min} are the maximum and minimum desired speeds of all vehicles, respectively.

After determining $y_{pl,i}$, the PL controller tries to steer and maintain the lateral position of the vehicle on the assigned $y_{pl,i}$ using a proportional controller:

$$f_{pl,i} = K_{plf} \cdot (y_{pl,i} - y_i) \quad (5)$$

where K_{plf} is the controller gain for PL force. In the current study, it is set to 0.12.

2.3.2. Cruise Controller based Longitudinal Control

The fundamental aim of the cruise controller is to keep the longitudinal speed as close as possible to the desired speed. It achieves this using a proportional controller, given as:

$$f_{x,i}^{cc} = K_{px} [v_{x,i}^{ts} - v_{x,i}(k)] \quad (6)$$

where K_{px} is the controller gain, allowing a gradual increase in speed. $v_{x,i}^{ts}$ is the target speed for the next time step, calculated using the vehicle's acceleration ability:

$$v_{x,i}^{ts} = \min\{v_{x,i}(k) + A_x^+ \cdot \Delta T, v_{des,i}\} \quad (7)$$

where A_x^+ is the acceleration ability. [13] did not use A_x^+ in their formulation. However, this study adds it to also cater for the situation when the vehicle comes to a complete halt. The current study uses K_{px} value of 1.0, which makes the $f_{x,i}^{cc}$ to be directly determined by A_x^+ and $v_{des,i}$, in consistency with the human model (Eq. 2).

2.3.3. Potential Field based Collision Avoidance and Overtaking

In an LFT, the collision avoidance and overtaking is generally performed via the artificial potential fields and the resulting inter-vehicular forces[12]. Each surrounding vehicle is considered a moving obstacle, which the subject vehicle sees as an ellipsoid hemisphere. Figure 2(a) shows an example of the potential field of vehicle V1 (as seen by other CAVs) and corresponding artificial forces as experienced by vehicles V2 and V3.

[13] used a modified version of the original LFT forces formulation of [12]. However, during experiments for the current study, no significant difference was observed in the performance of the two formulations. With correct parameter tuning, both formulations performed quite similarly. Therefore, the current study uses the original formulation of [12] for simplicity. For a vehicle j in the surrounding of subject vehicle i , the artificial force is calculated as:

$$F^{ij} = \frac{1}{\left[\left(\frac{x_i - \delta_{ij}}{0.5s_{d1}} \right)^{p1} + \left(\frac{y_i - y_j}{0.5s_{d2}} \right)^{p2} \right]^{p3} + 1} \quad (8)$$

where $p1$, $p2$, and $p3$ are the function parameters, and s_{d1} and s_{d2} determine the longitudinal and lateral axis of the ellipsoid, respectively. δ_{ij} is a function for adjusting the longitudinal position of the ellipsoid's center for vehicle j , considering the safety gap and speeds (refer to [12] for details). The parameters $p1$, $p2$, and $p3$ were set as 2, 2, and 6, respectively[12].

The calculated F^{ij} is projected to the longitudinal and lateral axes, creating two components: F_x^{ij} and F_y^{ij} . These two components are accumulated for all the vehicles up to distance ΔD_{front} in the front and ΔD_{back} in the back, forming the nudging and repulsive forces, respectively.

2.3.4. Overall Accelerations

After calculating the cruise controller and artificial forces, the PL controller calculates the longitudinal and lateral accelerations as:

$$a_{x,i}^{pl}(k+1) = f_{x,i}^{cc} + w_n \sum_{j \in V_{front}} F_x^{ij} + w_r \sum_{j \in V_{back}} F_x^{ij} \quad (9)$$

$$a_{y,i}(k+1) = w_n \sum_{j \in V_{front}} F_y^{ij} + w_r \sum_{j \in V_{back}} F_y^{ij} + f_{pl,i} \quad (10)$$

$$(11)$$

where w_n and w_r are the weights for nudging and repulsive forces, respectively. V_{front} and V_{back} are the set of all vehicles (including HDVs) in front and back of the vehicle i up to distance ΔD_{front} and ΔD_{back} , respectively. Even though it is possible to use different values for w_n and w_r , leading to significantly different LFT behavior[13][12], for simplicity, the current work uses a value 1.5 for both parameters.

2.3.5. Extension of Collision Avoidance via Safe Acceleration

So far, the PL components described above have been taken from the literature without significant modification. In contrast, the following describes an essential modification introduced by the current study.

Even though the PL controller already considers artificial forces to avoid collisions, it is observed that these forces may cancel each other out under certain conditions. This causes the vehicle to not decelerate on time, leading to collisions, especially at high vehicle densities. This can be avoided by adjusting the individual weights given to different forces; however, tuning these parameters is a time-consuming process and may not necessarily produce a general parameter set suitable for all situations.

In view of the above, the current study introduces the constraint of safe acceleration into LFT controller. The first step in this process is selecting a leader vehicle. Similar to the human model, the vehicle with the shortest longitudinal distance and an overlap in the lateral axis is chosen as the leader for vehicle i . However, unlike the human model, the overlap is directly calculated using vehicle widths and lateral positions without discretizing the lateral axis into strips. After determining the leader, the safe velocity $v_{x,i}^{safe}$ and the safe acceleration $a_{x,i}^{safe}$ are calculated using Eq. 1 and Eq. 2, respectively. $a_{x,i}^{safe}$ is then used to bound the longitudinal acceleration calculated by the PL controller; thus, the overall longitudinal acceleration is given as:

$$a_{x,i}(k+1) = \min\{a_{x,i}^{pl}(k+1), a_{x,i}^{safe}(k+1)\} \quad (12)$$

2.4. Adaptive Potential Lines

The presence of HDVs can degrade the performance of a LFT controller. This degradation primarily occurs because CAVs in LFT cannot exert artificial forces on HDVs, which restricts the free movement of CAVs. This will eventually affect the performance of each LFT controller differently. However, for the PL controller, a significant restriction in CAV movement can occur when HDVs drive on PLs assigned to CAVs. In such cases, if a CAV is following an HDV and is already on the assigned PL, there will be no PL force ($f_{pl,i}$) to steer the CAV away from the PL and overtake the slow-moving HDV ahead, causing a cumulative effect on upstream vehicles.

According to the functioning of the LFT controller, each controller must devise a strategy to cater to the HDVs. Since this study uses a PL controller to study the effects of HDVs on LFT, it also modifies the PL controller to lower the impacts of HDVs on LFT. Accordingly, this study introduces the concept of APL controller, in which the PLs are modified in the vicinity of HDVs. This section outlines the APL strategy for LFT, beginning with a general description of how the vicinity of modified PLs around HDVs is calculated. It then discusses the various methods tested in the study for activating APL around HDVs. These two steps are calculated and applied in each simulation time step.

2.4.1. Adaptive Potential Lines (APL) Corridors

The APL controller primarily compresses the PLs into the lateral spaces between HDVs, as shown in Figure 3. This adjustment enables CAVs to overtake slow-moving HDVs and enhances the overall flow of CAVs. Since CAVs already move in a coordinated manner due to the PL controller, they can also overtake HDVs in a coordinated way by simply modifying the assigned PL position ($y_{pl,i}$) around HDVs.

Additionally, these areas can create long road regions where CAVs are longitudinally dominated, allowing them higher chances to exert artificial forces on the downstream CAVs and accelerate without being blocked by HDVs. This functions as a sort of corridor for CAVs with comparatively higher freedom and, thus, is referred to as the APL corridor in the study.

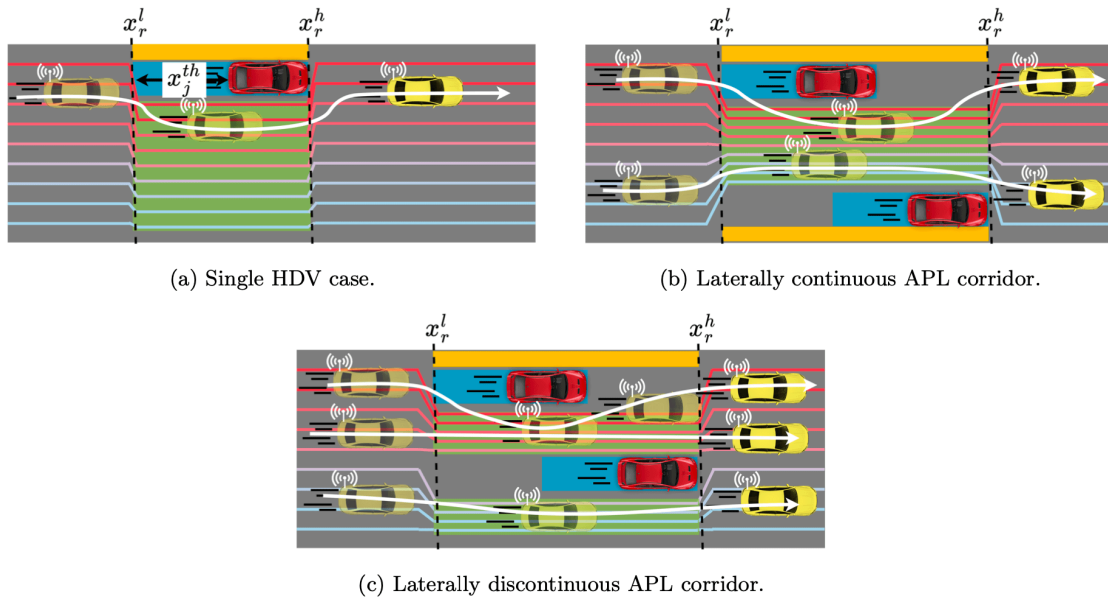


Figure 3. Adaptive potential lines (APL) controller. It adapts the PL areas near HDVs, represented by the green color and referred to as APL corridor in the study. The yellow color shows the areas excluded from PLs due to insufficient space to fit a vehicle. The blue color shows the areas marked as occupied by HDVs.

To determine the areas where APL corridors can be formed, the method first divides the road into a set of regions R in the longitudinal axis, with x_r^l and x_r^h denoting the longitudinal positions of the start and end of the region $r \in R$, respectively. A region starts at a distance x_j^{th} behind the HDV j and extends to its front in the longitudinal axis. x_j^{th} provides additional safety distance for the CAVs to orient themselves on the modified PLs without excessive deceleration near the HDVs. This region does not extend beyond the front of the HDV to avoid influencing CAVs in front, which is problematic since any HDV approaching from behind would force the CAV to move away from its front, giving unnecessary priority to HDVs. If multiple regions overlap in the longitudinal direction, they are merged to form a single region — an extended APL corridor. Figures 3(b) and 3(c) show examples of such longitudinal overlap and extended APL corridor.

After determining the longitudinal regions, the method divides the lateral axis of each region. For this purpose, combinations of lateral positions are computed for the stretch of the road from x_r^l to x_r^h . These combinations of lateral positions remain fixed for the entire region. They are calculated by leaving a lateral gap of B_{apl} on either side of HDVs. Similar to the PL formulation in Eq. 4, this gap is necessary since the CAVs use center positions to align themselves to the assigned PLs. B_{apl} is determined by the width of the broadest vehicle. Without B_{apl} , the overtaking maneuver of the CAVs may be restricted due to lateral overlap with HDVs and the constraint of safe acceleration. Furthermore, if the lateral gap between two HDVs is less than $2B_{apl}$ (meaning a CAV may not fit into the gap), the corresponding lateral gap is not considered for APL corridor. Thus, within a single longitudinal stretch (X_l , X_h), the APL corridor can be laterally continuous (Figure 3(b)) or discontinuous (Figure 3(c)), depending on the situation. If there is not enough lateral gap in the region to fit even a single CAV, then the region is not considered for an APL corridor.

2.4.2. Activation Condition for APL Corridors

The previous section focused on the potential areas for APL corridors. The two critical factors for applying APL corridors are (1) determining x_j^{th} and (2) determining the HDVs whose surroundings should be included for APL corridors. The paper evaluates four approaches that mainly differ in these two aspects, as described below:

1. **Constant Margin (CM):** This is the simplest method. It keeps x_j^{th} constant (denoted as X_{CM}) and continuously uses all HDVs' vicinities to define APL corridors.
2. **Neighbouring Speed based Constant Margin (NSCM):** Similar to CM method, this method uses the same X_{CM} for all HDVs; however, the vicinity (i.e. longitudinal distance X_{CM} behind an HDV) of an HDV is only included into APL corridor if its speed is less than the average speed of the surrounding vehicles. For an HDV j , the average surrounding speed, denoted by v_j^s , is computed by taking the mean longitudinal speed of all vehicles (both CAVs and HDVs) whose positions do not laterally overlap with the boundaries of j and are within a longitudinal distance of 20 m behind j . The idea here is that the APL should only be applied in the surroundings of an HDV if it is blocking the vehicles behind it, which can be detected if the vehicles with different lateral positions than j are moving faster than j .
3. **Follower-based Adaptive Margin (FAM):** The PL controller in this study uses the leader-follower relation along with safe acceleration as a hard constraint to avoid collisions (Eq. 12). The HDV can

potentially hinder any CAV that has an HDV leader. FAM uses this characteristic to apply the APL. It first checks if the HDV j is the leader of any CAV within a longitudinal distance of X_{AM} behind the HDV. Then, it checks if the speed of j is lower than the average speed of the surrounding vehicles (the same as in the NSCM strategy). If both conditions are fulfilled, x_j^{th} is set as the distance from the back of HDV to the back of the follower CAV. The purpose here is to only apply APL behind an HDV up to a point that allows the overtaking of the follower CAV. Thus, x_j^{th} in this strategy is not constant; rather, it depends on the distance of the follower CAV, as reflected in its name.

4. **Safe Velocity based Adaptive Margin (SVAM):** This approach extends the FAM method. In addition to checking if HDV is the leader of a CAV, it also checks if the speed of the follower CAV is constrained by the safe velocity, i.e., $v_{x,i} \leq (1 + \epsilon)v_{x,i}^{safe}$, where i is the follower CAV and ϵ is a small number, set to 0.05 in the study. The main idea here is to activate APL only if the follower CAV's speed is limited due to HDV. The rest of the conditions of SVAM are the same as FAM.

3. Experimental Setup

To study the effects of HDVs, a ring road of 1 km is simulated in a microscopic simulation. A custom extension of SUMO^[43] for LFT called TrafficFluid-Sim^[49] is used for this purpose. The HDVs and PL controller are implemented via a C++ interface, which allows getting the necessary information on individual vehicles and setting up accelerations for the next time step. For consistency with other LFT works, the road width is set to 10.2 m. The road emulates a continuous beltway, with the vehicles leaving from one end and entering back into the road from the other with the same lateral positions and speeds.

Regarding the simulation scenarios, five types of vehicles are simulated with (length, width) in meters given as (3.2, 1.6), (3.4, 1.7), (3.9, 1.7), (4.55, 1.82), and (5.2, 1.88). The experiments simulate vehicle densities ranging from 50 veh/km to 400 veh/km with a step size of 50 veh/km, with an equal proportion of vehicles out of the five above-mentioned categories. Different penetration rates of HDVs are simulated. If the penetration rate leads to a fractional outcome for the number of HDVs, it is rounded to the closest integer. Using uniform distribution, the vehicles are initialized at random positions with zero speed without vehicle overlap. The desired speeds are also assigned using uniform distribution ranging from 25 m/s to 35 m/s, which remain constant throughout the simulation. Each

scenario is run for one hour of simulation time with a step size ΔT of 0.25 s. Each scenario is also run with five random seeds to ensure the results are statistically reliable.

The following values are used for the model parameters. For the human model, Δs , λ and L_{th} are set to 0.1 m, 0.1, and 10, respectively. The values of Δs and λ are based on the values used in [16], while L_{th} is not explicitly mentioned by the authors, and thus, the value is chosen by experiments to allow sufficient strip changing maneuvers. For calculating $v_{x,i}^{safe}$, the maximum deceleration A_x^- is set as -4.5 m/s^2 . τ is fixed as 0.5 s for all CAVs. However, to bring variety to the individual driving style of each human driver, for HDVs, τ is drawn from a normal distribution with a mean and standard deviation of 1.5 and 0.5, respectively. The acceleration ability A_x^+ of all vehicles is set to 2.6 m/s^2 . The values of both A_x^- and A_x^+ are taken from the standard SUMO configuration. ΔD_{front} and ΔD_{back} are set to 50 m. For the APL controller, X_{CM} and X_{AM} are set to 40 m by default unless explicitly stated otherwise.

4. Results and Discussion

The results are divided into two main sections. The first section discusses the effects of increasing HDVs in LFT. The second section then analyses how this performance drop can be improved by using the APL controller.

4.1. Impact of HDVs on the LFT performance

The study first analyses the impact of different penetration rates of HDVs on the overall LFT performance, especially in terms of traffic flow and average speed, as shown in Figure 4. The first thing to observe is the significant difference in the road capacities (the highest traffic flow in Figure 4) with all-HDVs and all-CAVs scenarios. With 100% HDVs, the road capacity is limited to only 8,233 *veh/h* (speed: 23.2 *m/s*) at a density of only 100 *veh/km*, which is roughly equivalent to the flow of lane-based traffic with four lanes (approximately 2000 *veh/h/lane* [$\text{m}/48/1$]). Notably, the road width used in the simulation (10.2 meters) is typically divided into three lanes in lane-based traffic management. However, since the lanes are usually wider than the vehicles, removing the lanes allowed four vehicles to fit within the same road width, resulting in a flow roughly equivalent to four lanes. Nevertheless, this flow value also depends on the tolerance of individual drivers for changing lateral positions. For instance, in the experiments conducted for this study, the maximum road capacity with all-HDVs decreased from 8,233 *veh/h* to 6,973 *veh/h* when the threshold for benefits of changing strip (L_{th}) was reduced from 10 to 0.1.

In contrast, the scenario with all-CAVs achieves a significantly higher road capacity of 20,800 *veh/h* (speed: 23.1 *m/s*) at a density of 250 *veh/km* — an increase of almost 153% in road capacity over all-HDVs scenarios. The key difference between HDVs and the LFT is the existence of artificial forces, indicating that the coordinated movement of CAVs through artificial forces and virtual PLs plays a crucial role in achieving higher flow. Additionally, despite using the same desired speeds for individual vehicles in both scenarios, the average speed observed in the all-CAVs scenario is significantly higher across all vehicle densities.

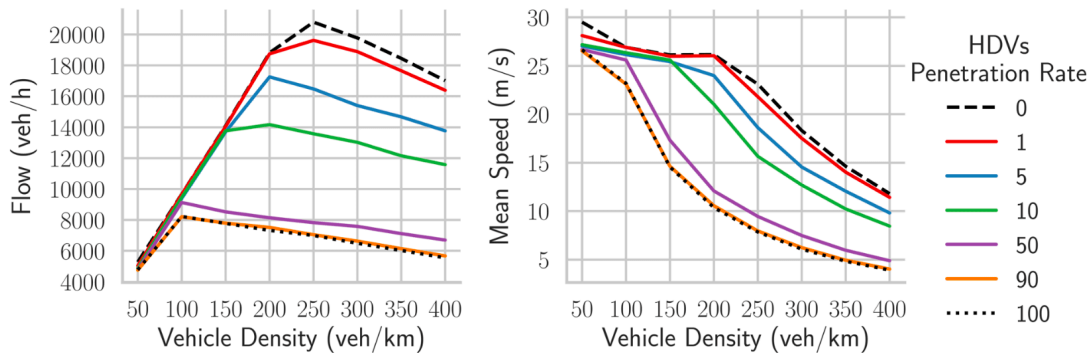


Figure 4. Traffic flow rate and mean speed of PL controller with different HDV penetration rates. The 0% and 100% penetration rates mark all-CAVs and all-HDVs scenarios, respectively.

Figure 4 also shows that as the penetration rate of HDVs increases, the flow of LFT is severely affected. Even a small percentage of HDVs can significantly lower LFT performance: with just a 5% penetration rate, road capacity drops to 17,252 *veh/h* (a drop of nearly 16%), and with a 10% penetration rate, it drops to 14,162 *veh/h* (a drop of nearly 32%). This phenomenon is further analyzed in Figure 5, illustrating traffic flow and lateral speeds against increasing HDVs penetration rates. It is observed that the performance drop is significantly high for the initial 0–20% HDVs penetration. For individual vehicle densities above 200 *veh/km*, the traffic flow is almost halved at 20% penetration, which continues to drop further at a lower rate until 40% penetration. Beyond this point, the curves are almost flat without much performance gain over all-HDVs scenarios. The performance decline is more pronounced at higher vehicle densities.

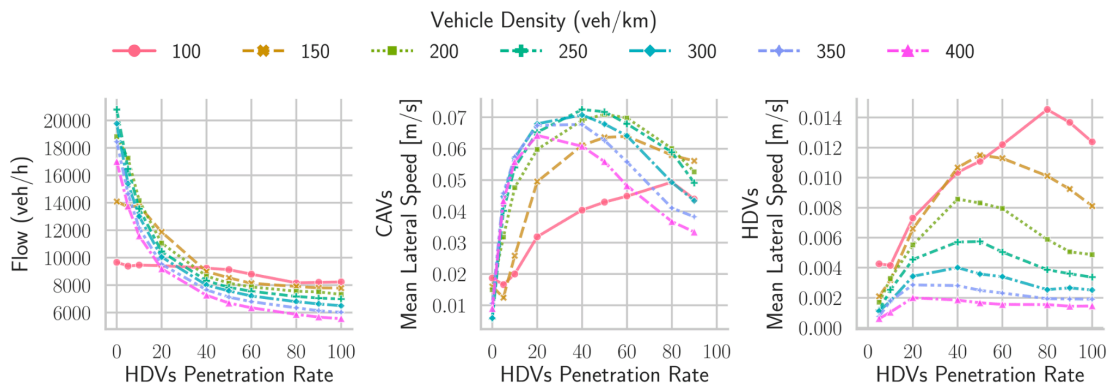


Figure 5. Traffic flow rate and mean lateral speeds for increasing penetration of HDV penetration into LFT.

The above phenomena can be further understood by looking at the mean lateral speed in Figure 5. First, with all-CAVs scenario, the vehicles can better coordinate their motion and stay on their respective PLs, leading to lower lateral movements. For all-CAVs scenarios, it is also observed that the vehicles generally show higher lateral movements at lower vehicle densities compared to higher vehicle densities. This can be explained by the higher available space at lower densities, causing the overtaking vehicles to fully utilize their artificial forces to nudge the vehicles in the front. As the vehicle densities increase, the nudging possibility decreases due to the artificial forces from the surrounding vehicles countering each other. Nonetheless, the vehicles can still coordinate their movements at higher densities, and flow remains significantly higher than all-HDV scenarios.

As the proportion of HDVs increases, lateral movements rise significantly. CAVs find it increasingly difficult to nudge other vehicles, as nudging does not affect HDVs, resulting in continuous disruptions to the LFT flow. These disruptions propagate upstream, where the PL controller attempts to adjust the CAVs in the available gaps as they encounter hindrances caused by HDVs, leading to increased lateral movements. At vehicle densities lower than 100 veh/km (as shown in Figure 5), these hindrances only cause an increase in lateral movements without significantly reducing LFT flow, as there is sufficient space available for the PL controller to maneuver the CAVs through the HDVs. However, at higher vehicle densities and HDV penetrations, many CAVs are hindered by the HDVs, with CAVs adjusting their speeds and acceleration according to the HDVs. This leads to the formation of traffic waves, as shown in Figure 6, causing a significant drop in performance.

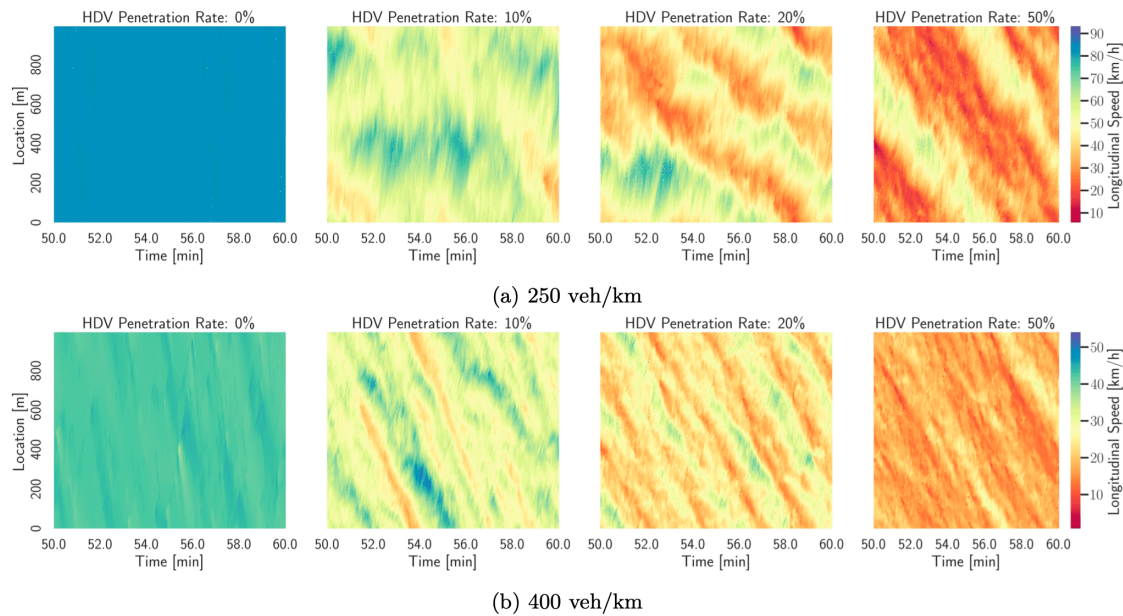


Figure 6. Spatio-temporal speed plot for PL controller. The formation of traffic waves is visible with the increasing penetration of HDVs.

Figure 5 also shows that the lateral movements continue to increase with higher penetration rates of HDVs till a saturation point is reached, after which they begin to decline again. As the vehicle densities increase, the saturation point occurs at a lower penetration rate of HDVs. This phenomenon can be explained as follows: at lower penetration rates, the upstream CAVs can better adjust their positions in front of the hindrances caused by HDVs since the CAVs can influence other CAVs, leading to better utilization of space and higher lateral movement; however, as the proportion of HDVs increases, CAVs are unable to do that leading to lower lateral movement and higher gaps between vehicles. This is visible by comparing Figure 7(c), Figure 7(d), where the portions of road with tightly packed CAVs are larger than scenarios with 5% HDVs scenario. This happens till the saturation point, after which the proportion of HDVs is so high that the CAVs find limited opportunity to exert artificial forces, and consequently, the lateral movement and the efficient road space utilization decreases, as illustrated in Figure 7(e). This saturation point happens earlier for higher densities because the number of HDVs needed to cause this lateral movement decline is achieved earlier for higher vehicle densities.

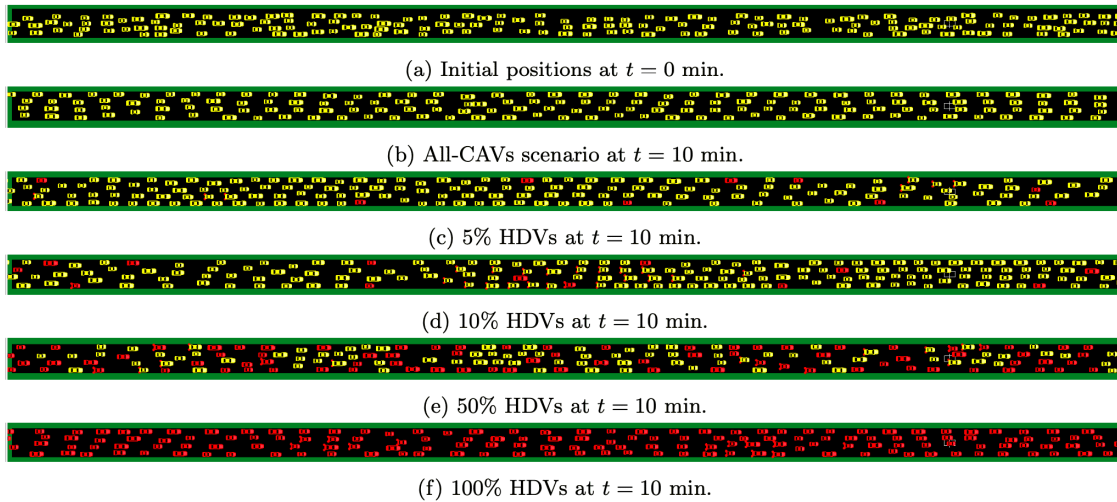


Figure 7. Examples of traffic situations with vehicle density of $400 \text{ veh}/\text{km}$ and PL controller. The starting 400 m of the ring road is shown. The yellow and red vehicles represent CAVs and HDVs, respectively. The initial positions for scenarios with HDVs remain the same as with all-CAVs scenario; only the appropriate proportion of vehicles are set as HDVs.

The above phenomena can also be observed from the development of lateral positions of CAVs over time, as shown in Figure 8. The behavior of the PL controller with all-CAVs scenario is discussed first. The PL controller tries to minimize the lateral movement by assigning PL according to desired speeds. At the beginning of the simulation, there are usually higher lateral movements since all CAVs try to reach their assigned PLs. Over time, the CAVs settle down at a lateral position and the lateral movements are minimized subsequently. It is possible that these settling points are not exactly on the assigned PL and are slightly shifted, according to the location of the PL and the artificial forces from surrounding vehicles. It is observed that this deviation of the settling point is larger near the road boundaries, as shown by all-CAVs scenarios in Figure 8. The CAVs with assigned PLs near the boundaries experience artificial forces from the surrounding vehicles from one side only, and in the absence of artificial boundary forces to counter them (due to the modifications of PL controller in the current study), these CAVs are pushed further towards the boundaries. In contrast, the CAVs with assigned PLs near the center, experience artificial forces from both sides, making it easier for them to stay on the assigned PL. However, at higher vehicle densities, even the CAVs at the center may have significant deviation due to the CAVs not finding enough space and opportunity to remain on the assigned PL, as shown by the all-CAVs scenario in Figure 8(b).

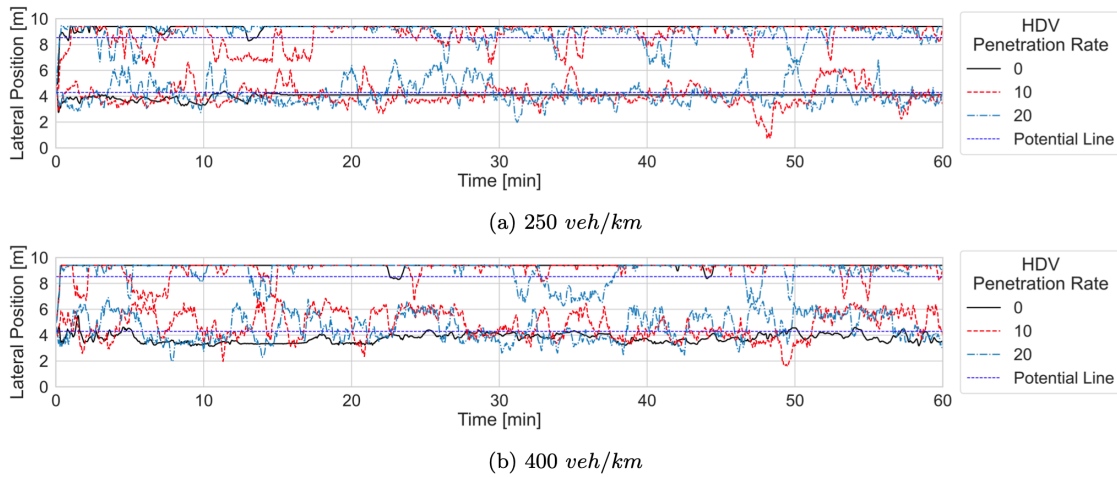


Figure 8. The lateral positions of two CAVs. The inclusion of HDVs causes significant changes in the lateral movement of CAVs.

As the penetration of HDVs increases, Figure 8 shows that the lateral movement of CAVs is significantly impacted. The main reason for this is that the HDVs do not target aligning themselves with specific PLs. This causes them to travel without coordination with CAVs, resulting in uncoordinated LFT forces on CAVs and significantly higher lateral movements. This also results in CAVs experiencing long episodes of going far away from the assigned PL. This phenomenon is observed to be more severe for higher vehicle densities; however, it is not limited to higher vehicle densities and can be observed even for small vehicle densities. Nonetheless, unlike at higher vehicle densities, at lower vehicle densities, this does not significantly affect traffic flow due to having sufficient space. Overall, this shows that even a small proportion of HDVs has the potential to not only degrade LFT flow but also significantly affect the characteristic features of certain LFT controllers, for example, the feature of reduced lateral movements in the case of PL controller.

4.2. Improvements using Adaptive Potential Lines Controller

The results in the previous section highlighted the significant performance drop observed with the penetration of HDVs into LFT. This section presents how much the APL controller introduced can improve this behavior. While the current study focused on the overtaking maneuver of the follower CAVs to develop APL activation procedures, the interaction of multiple CAVs in LFT can have significant impacts. To better understand these interactions in LFT, Figure 9 compares the initial

temporal development of PL, CM, and FAM strategies as a CAV (marked with a white rectangle) overtakes an HDV (marked with a blue rectangle). The marked CAV has a high desired speed; thus, the assigned PL is typically on the left side of the road. The CAV starts ($t = 0$) in the middle but ends up behind the marked HDV by aligning itself to the assigned PL ($t = 19s$). With a simple PL controller, the CAV remains behind the HDV. The gap between the two increases ($t = 65s$) and decreases ($t = 92s$) with the acceleration and deceleration of the HDV, contributing to the formation of traffic waves. The marked CAV overtakes the HDV when the following CAV exerts lateral force, allowing the marked CAV to change its lateral position and overtake the HDV ($t = 156s$). Nonetheless, such an overtake is purely dependent on the developing situation, and the PL controller is not taking active measures to avoid the adverse effects of HDVs.

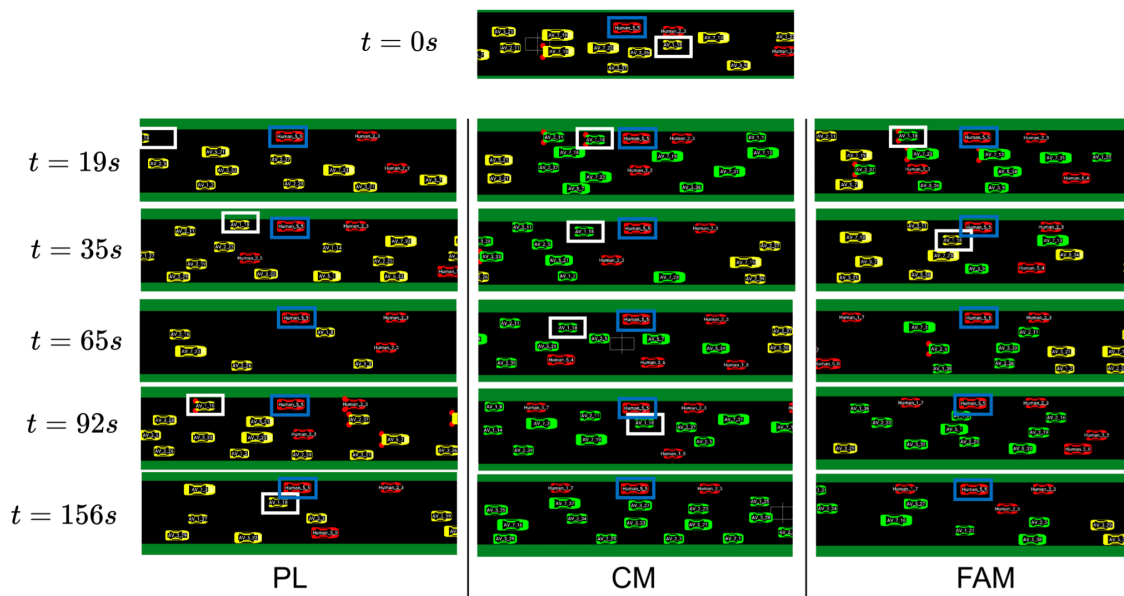


Figure 9. Comparison of the temporal development of PL and two APL controllers for 250 *veh/km* and 20% HDVs. The red and yellow vehicles represent HDVs and CAVs, respectively. The green vehicles represent the CAVs under the influence of APL corridors. The blue and white rectangles show the HDV and the CAV focused on in the discussion, respectively. The X_{CM} and X_{AM} are set to 20 m in this example.

In contrast, the APL controllers allow the marked CAV to overtake the HDV significantly earlier. However, the specific details depend on the APL strategy used. For example, since CM applies APL within a fixed distance X_{CM} behind each HDV, the APL corridor extends to multiple CAVs (including the ones behind the marked CAV), as shown at $t = 19s$. Consequently, many CAVs try to reach the

newly assigned PLs. The new PLs would be more accessible for CAVs laterally close to modified PLs ($t = 35s$). Thus, these CAVs can accelerate more quickly than the marked CAV since HDVs do not block them in the APL corridor. In turn, they nudge the marked CAV away as it tries to overtake the HDV, as shown at $t = 65s$ for CM. However, as soon as sufficient space is available, the marked CAV and the one behind it change their lateral position and overtake the HDV at $t = 92s$. Figure 9 presents FAM's temporal development as the other example of APL controller. Compared to the CM, the FAM strategy limits the APL corridor up to the marked CAV ($t = 19s$) till it changes its lateral position and the corridor is extended to the newer follower CAV; thus, fewer CAVs are competing with the marked CAV, or they compete with some delay when the APL corridor is formed due to the newer follower CAV. This provides enough time and space for the marked CAV to change its lateral position and overtake the HDV earlier than the CM strategy. It is also worth noting that even when the modified PL is deactivated in FAM for some time ($t = 35s$) due to the condition of the average surrounding speed, the marked CAV does not go back behind the HDV; instead, it continues to overtake due to the artificial forces exerted by the new follower CAV of HDV. This shows that the problem of developing LFT strategies with a mixture of HDVs is not merely a matter of how a blocked CAV overtakes HDVs; instead, it has multiple facets regarding how multiple CAVs would interact to accommodate HDVs while improving the overall situation.

For the APL strategies, $t = 92s$ and $t = 156s$ show another phenomenon. Since the longitudinally overlapping regions are combined into single APL corridors, it is observed that many HDVs start to follow each other, forming longitudinal groups of HDVs. This is also visible in Figure 10 for an extended road portion, where compared to PL case, HDVs are less dispersed in CM and FAM case. Figure 10 also shows that since CM (and NSCM) use a fixed distance (X_{CM}) behind each HDV to form APL corridors, it can create long stretches of APL corridors. In contrast, FAM (and SVAM) create shorter APL corridors. Consequently, it can be said that while the adaptive margin-based strategies favor the overtaking maneuvers of the follower CAVs, the constant margin-based strategies favor the movement of a larger group of CAVs using longer APL corridors.

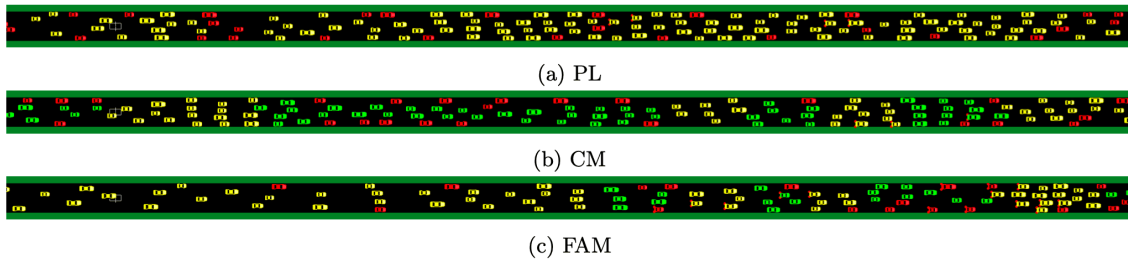


Figure 10. The formation of APL corridors between HDVs for CM and FAM strategies. The CAVs following APL corridors are represented with green color. The figure represents the situation at $t = 10$ min with 250 *veh/km* and 20% HDVs. The X_{CM} and X_{AM} are set to 20 m in this example.

Figure 11 compares the overall flow and the average lateral speeds of APL and PL controllers for a density of 250 *veh/km*. The first thing to note is the general trend of a significant decrease in LFT traffic flow with HDV penetration, even with the APL controller. Nonetheless, the APL controller significantly improves over the PL controller, especially in the beginning 0–50% HDVs penetration. To compare this further, Figure 12 shows the difference in traffic flow values for APL and PL controllers. For each APL strategy, the improvement generally peaks at a certain HDV penetration and then declines. The CM and NSCM provide the best improvements over PL for an HDV penetration lower than 40% with peak improvement of 2205 *veh/h* (23.6%) achieved with NSCM for a density of 200 *veh/km* and 30% HDV.

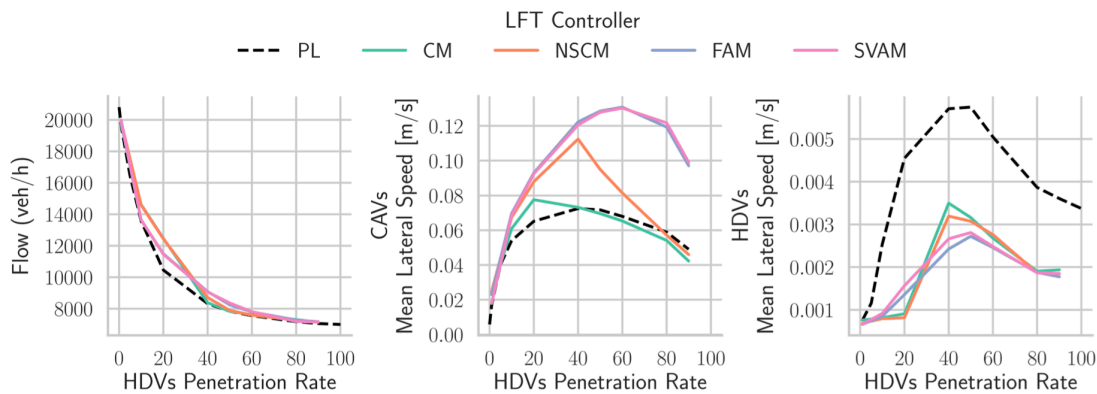


Figure 11. The traffic flow rates and mean lateral speeds for different APL strategies and HDV penetrations. The vehicle density is set to 250 *veh/km*.

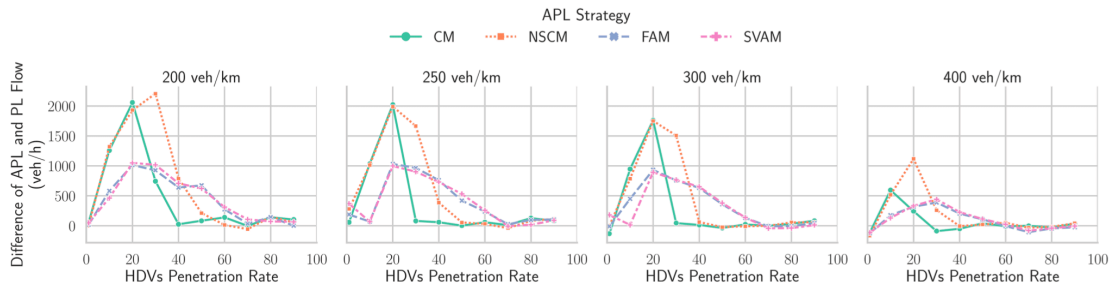


Figure 12. The difference of traffic flow values of APL and PL controllers under the same settings.

Additionally, the improvements of NSCM are found to be more general regarding HDV penetration than CM. This means that NSCM continues to provide performance improvement for almost 10% more HDVs than CM. This is due to the way APL corridors are formed, where both longitudinally overlapping regions are first combined into a single region. Then, the lateral space available between HDVs in each region is used for APL corridors. With increasing HDV penetration, there are longitudinally more overlapping regions and less empty lateral space available for each region to form APL corridor. Consequently, since CM uses all HDVs in this process, it finds it challenging to form APL corridors and shows degraded performance earlier. This reason also applies when higher values of X_{CM} are used with CM, as demonstrated in Figure 13, or when a higher vehicle density is simulated. Consequently, it is worth noting in Figure 13 that CM's performance is improved for 400 *veh* when lower X_{CM} is used. In contrast, NSCM only considers an HDV for APL corridors when the HDV is moving slower than the surrounding vehicles. This reduces the overlapping longitudinal regions and allows NSCM to form more APL corridors than CM. This is also visible through the lateral trajectory plots in Figure 14(b), where the vehicle experiences APL corridors in NSCM much more than CM.

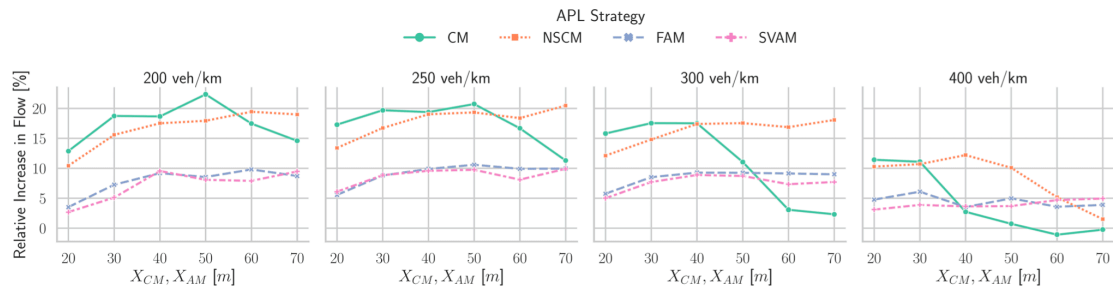


Figure 13. Impact of changing the parameter values for various APL strategies and 20% HDVs. The y-axis shows the relative increase in the traffic flow compared to the PL controller's flow under the same setting.

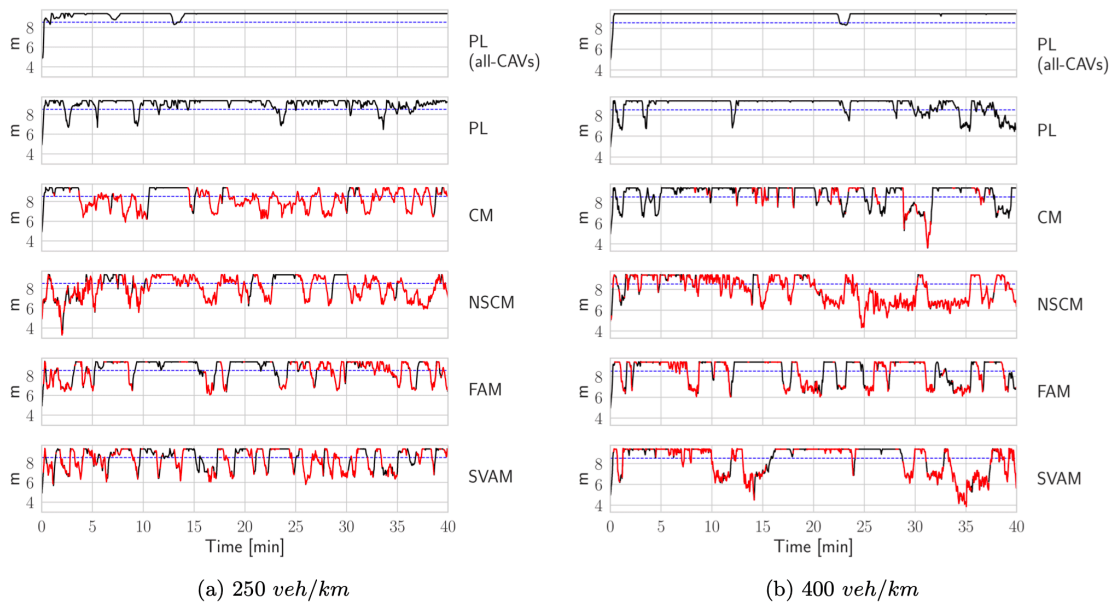


Figure 14. The lateral positions of a CAV under APL controllers with 20% HDVs (except all-CAVs case). The red lines indicate the trajectory under the influence of APL corridors.

Figure 12 also shows that the improvements of FAM and SVAM are smaller than CM and NSCM, almost half for 0-30% HDV penetration. However, FAM and SVAM are observed to be even more general than NSCM strategy in terms of HDV penetration, producing at least some improvement over PL controller upto 70% HDV penetration. This can be explained by their adaptive nature for forming APL corridors. Since they only consider the distance up to the follower CAV for the APL corridor, both strategies have fewer overlapping regions, resulting in a short but higher number of APL corridors than CM and

NSCM. For smaller HDV penetration (0–30%), such short corridors would be less efficient than long corridors of CM and NSCM since many CAVs would have to laterally adjust their movements whenever they enter or exit such short APL corridors. This is visible from the higher lateral speeds of CAVs in Figure 11 for FAM and SVAM as well as from the significantly larger lateral movements in the trajectory plots of Figure 14.

On the other hand, these short APL corridors benefit from high HDV penetration and vehicle density. Under these conditions, the FAM and SVAM can still form APL corridors, unlike CM and NSCM strategies. This allows CAVs to overtake the blocking HDV and accelerate, making these strategies applicable to even high HDV penetration. However, as the HDV penetration goes above 70%, forming APL corridors becomes difficult for almost all APL strategies, and the performance becomes similar to the PL controller.

Figure 11 also shows that the lateral movement of HDVs in the APL controller is significantly lower than in the PL controller. As described above, the APL corridors also push HDVs to form longitudinal groups via CAV movements, unlike PL where HDVs are more dispersed in between CAVs. These HDVs in APL either do not have enough speed incentives to change lateral position or do not find enough space for lateral movements among fast-moving CAVs in the APL corridors. Thus, for the 0–30% HDV penetration where the CM and NSCM form large APL corridors, the lateral movement of HDVs is minimal. However, as the number of long APL corridors decreases with increasing HDV penetration, the HDVs find more chances for lateral movements.

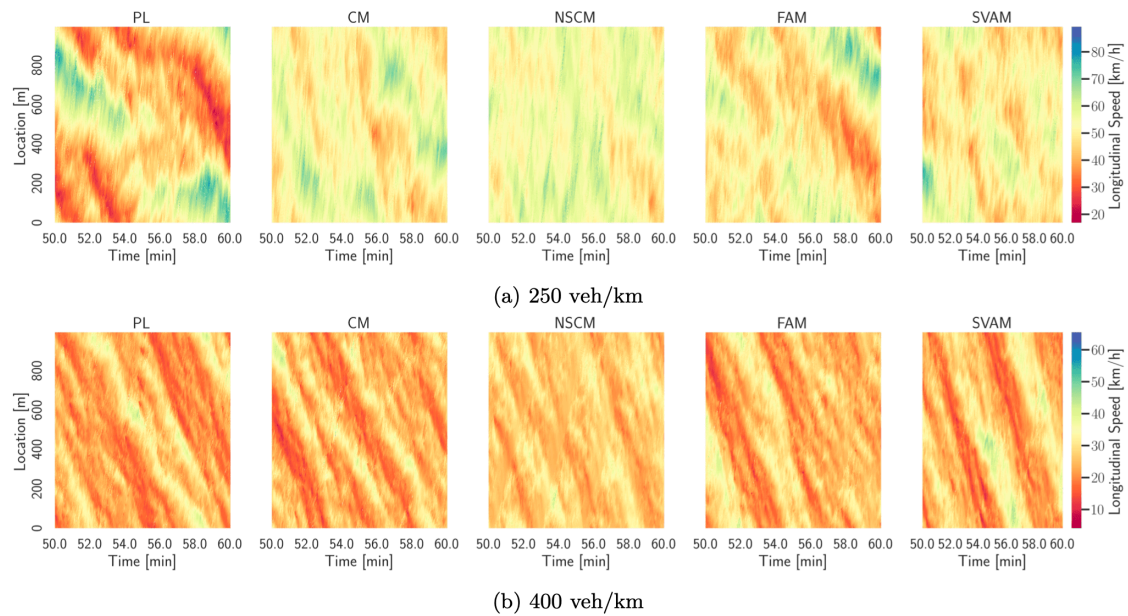


Figure 15. Spatio-temporal speed plot for APL controllers with penetration of 20% HDVs. The APL controllers reduce the intensity of traffic waves.

The previous section showed that the PL controller forms traffic waves with increasing HDV penetration. Therefore, it is also important to analyze the impact of the APL controller on this phenomenon. As shown in Figure 15, the APL strategy does not completely remove the traffic waves but reduces their intensity. The long APL corridors in CM and NSCM help to reduce the traffic waves the most; however, the short APL corridors in FAM and SVAM are found to be less effective against traffic waves. At a very high vehicle density (400 veh/km), all APL strategies struggle to avoid forming traffic waves.

Overall, the results in this section show that for a relatively low number of HDVs (almost 30% HDVs), an APL strategy that favors unhindered movements of groups of CAVs like CM or NSCM produces better results. However, an APL strategy that favors the overtaking maneuvers of follower CAVs is more general and produces better results with higher HDV penetration. Nevertheless, even with the APL controller, the results indicate that penetration of at least 60% CAVs into LFT is necessary before any major benefits of LFT to the overall traffic flow start to appear.

5. Summary

The ever-increasing problem of traffic congestion in cities, coupled with advancements in AV technologies, has spurred the search for innovative solutions. Rather than investing in expensive and time-consuming new road infrastructures like beltways, there is a recent push to enhance the capacity and safety of existing infrastructure. One promising approach involves using CAVs. These vehicles can communicate with each other and the infrastructure to better coordinate their movements and improve traffic flow.

One significant way CAV technology can transform traffic management is through LFT. In LFT, vehicles coordinate their movements without relying on traditional fixed lanes. Recent studies have shown that LFT has the potential to significantly increase road capacity through the coordinated movements of CAVs. However, the transition to LFT may not be straightforward and could require the coexistence of independent traffic vehicles, either HDVs or simple AVs, during the transition phase.

This study examines the impact of HDVs on LFT using a microscopic simulation of a 1 *km* ring road with a width of 10.2 meters. While HDVs make driving decisions based on individual benefits, CAVs use the PL controller for coordinated movements. The results indicate that a pure LFT scenario significantly improves road capacity compared to scenarios with only HDVs without lanes; a maximum flow of 20,400 *veh/h* is observed for LFT compared to only 8,044 *veh/h* for the latter. However, the flow improvement of LFT significantly drops as soon as HDVs are introduced into the system. Even a small penetration of HDVs, such as 5%, reduces the maximum flow by 16% (to 17,252 *veh/h*). With 10% HDVs, it drops by 32% (to 14,162 *veh/h*), and almost halves with 20% HDVs. In literature, the LFT is also marked by its characteristic feature of avoiding the formation of traffic waves. However, this study found that with the penetration of HDVs, traffic waves start to appear in LFT and worsen with higher HDV penetrations.

The study also introduced an APL controller approach to reduce the above performance drop. Unlike the simple PL controller, where the assigned PLs remain fixed, the APL controller adapts the PLs in the vicinity of HDVs. These areas with modified PLs are referred to as APL corridors. The study developed four APL variants that mainly differ on (1) the distance behind each HDV included in the APL corridor and (2) the conditions required for individual HDV whose vicinity is included in the corridor. The study found that the APL variants that favor the group movement of CAVs by forming long APL corridors perform better for a lower range of HDV penetration (up to almost 30%). In comparison, the APL

strategies that favor the overtaking maneuvers of individual CAVs perform better for higher HDV penetration. Overall, the NSCM strategy for APL controller provides the best performance.

Even with the improved performance of the APL controller, the results indicate that the proportion of CAVs in LFT needs to be significantly high (almost 60%) before the practical benefits of LFT start to appear in terms of traffic flow.

6. Limitations and Future Work

Even though this study aimed to be as comprehensive as possible in its methodology, some limitations must be considered when interpreting the outcomes. First, the study used a specific model for the HDVs, which may not accurately represent real driving behavior without lanes, especially in LFT scenarios. A major limitation of the HDVs used is their complete inability to communicate with one another. In reality, HDVs may use signals such as honking or flashing headlights to indicate their intention to overtake the leading vehicle. Similarly, an HDV can also partially observe the movements of CAVs and adjust the driving behavior accordingly. However, the HDV model in this study does not consider such HDV-HDV or CAV-HDV interactions, which may not represent real-world behavior.

Additionally, the driving behavior of individual humans varies significantly, while the study model used similar parameters for all HDVs. An important parameter for the HDVs is the reaction time and deceleration capability for calculating safe velocities. This can have a significant impact on outcomes. The same applies to CAVs, i.e., the study used a specific LFT controller and set of parameter values for the experiments. The outcomes may differ significantly if these parameters are changed. It is also possible that the impacts of HDVs are amplified due to the use of a ring road; even with a small proportion of HDVs, the CAVs repeatedly encounter the HDVs in each rotation, leading to an amplified effect on upstream vehicles.

The HDV model can be enhanced in the future to better represent human behavior in LFT scenarios. Driving simulator studies of LFT may be crucial for this purpose^[15], especially for estimating the reaction time and deceleration for HDVs in an LFT environment. The impact of HDVs should also be studied under different simulation scenarios to provide further insights into their effects. Similarly, the impact of HDVs on other LFT controllers should be investigated. It is important to note that the previously concluded penetration rate of 60% CAVs to start seeing the benefits of LFT indicates that LFT controller design should also consider boundary cases, such as handling situations when not all

vehicles are CAVs. Even though APL controller reduces the impact of HDVs, there is still potential to further improve the LFT controller for these cases.

Statements and Declarations

CRedit authorship contribution statement

Arslan Ali Syed: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology. Majid Rostami-Shahrbabaki: Writing – review & editing, Supervision, Conceptualization. Klaus Bogenberger: Writing – review, Supervision, Conceptualization, Funding acquisition.

Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the authors used Grammarly and Microsoft Copilot to improve the language of the draft they prepared. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

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References

1. ^{a, b}Rahman MM, Najaf P, Fields MG, Thill J-C (2022). "Traffic Congestion and Its Urban Scale Factors: Empirical Evidence from American Urban Areas". *International Journal of Sustainable Transportation*. 16 (5): 406–421. doi:[10.1080/15568318.2021.1885085](https://doi.org/10.1080/15568318.2021.1885085).
2. ^ΔNarayanan S, Chaniotakis E, Antoniou C (2020). "Shared autonomous vehicle services: A comprehensive review". *Transportation Research Part C: Emerging Technologies*. 111: 255–293. doi:[10.1016/j.trc.2019.12.008](https://doi.org/10.1016/j.trc.2019.12.008).

3. [△]Syed AA, Dandl F, Bogenberger K. User-assignment strategy considering future imbalance impacts for ride hailing. In: 2021 IEEE International Intelligent Transportation Systems Conference (ITSC). Indiana polis, IN, USA: IEEE; 2021. p. 2441-2446. doi:[10.1109/ITSC48978.2021.9564559](https://doi.org/10.1109/ITSC48978.2021.9564559).
4. [△]Syed AA, Akhnouk K, Kaltenhaeuser B, Bogenberger K. Neural network based large neighborhood search algorithm for ride hailing services. In: Moura Oliveira P, Novais P, Reis LP, editors. Progress in Artificial Intelligence. Cham: Springer International Publishing; 2019. p. 584-595. doi:[10.1007/978-3-030-30241-2_49](https://doi.org/10.1007/978-3-030-30241-2_49).
5. [△]Narayanan S, Chaniotakis E, Antoniou C. Factors affecting traffic flow efficiency implications of connected and autonomous vehicles: A review and policy recommendations. In: Advances in Transport Policy and Planning. Elsevier; 2020. p. 1-50. doi:[10.1016/bs.atpp.2020.02.004](https://doi.org/10.1016/bs.atpp.2020.02.004). ISBN 978-0-12-820191-6.
6. [△], [▷], [◁]Ahmed HU, Huang Y, Lu P, Bridgelall R. "Technology Developments and Impacts of Connected and Autonomous Vehicles: an Overview." Smart Cities. 5(1):382-404, Mar. 2022. doi:[10.3390/smartcities5010022](https://doi.org/10.3390/smartcities5010022).
7. [△]Rostami-Shahrbabaki M, Safavi AA, Papageorgiou M, Setoodeh P, Papamichail I (2020). "State estimation in urban traffic networks: A two-layer approach". Transportation Research Part C: Emerging Technologies. 115: 102616. doi:[10.1016/j.trc.2020.102616](https://doi.org/10.1016/j.trc.2020.102616). [Link](#).
8. [△]Rostami-Shahrbabaki M, Niels T, Hamzehi S, Bogenberger K (2020). "An Independent Trajectory Advisory System in a Mixed-Traffic Condition: A Reinforcement Learning-Based Approach". IFAC-PapersOnline. 53 (2): 15667-15673. doi:[10.1016/j.ifacol.2020.12.2550](https://doi.org/10.1016/j.ifacol.2020.12.2550). [Link](#). 21st IFAC World Congress.
9. [△]Niels T, Mitrovic N, Dobrota N, Bogenberger K, Stevanovic A, Bertini R (2020). "Simulation-Based Evaluation of a New Integrated Intersection Control Scheme for Connected Automated Vehicles and Pedestrians". Transportation Research Record: Journal of the Transportation Research Board. 2674 (11): 779-793. doi:[10.1177/0361198120949531](https://doi.org/10.1177/0361198120949531).
10. [△], [▷], [◁], [◻]Papageorgiou M, Mountakis KS, Karafyllis I, Papamichail I, Wang Y (2021). "Lane-free artificial-fluid concept for vehicular traffic". Proceedings of the IEEE. 109 (2): 114-121.
11. [△], [▷], [◁]Sekeran M, Rostami-Shahrbabaki M, Syed AA, Margreiter M, Bogenberger K (2022). "Lane-Free Traffic: History and State of the Art". In: 2022 IEEE 25th International Conference on Intelligent Transportation Systems (ITSC), pages 1037-1042. doi:[10.1109/ITSC55140.2022.9922282](https://doi.org/10.1109/ITSC55140.2022.9922282).
12. [△], [▷], [◁], [◻], [◼], [◽]Yanumula VK, Typaldos P, Troullinos D, Malekzadeh M, Papamichail I, Papageorgiou M. "Optimal Path Planning for Connected and Automated Vehicles in Lane-free Traffic." In: 2021 IEEE Inte

- national Intelligent Transportation Systems Conference (ITSC), Indianapolis, IN, USA. IEEE; 2021. p. 3545–3552. doi:[10.1109/ITSC48978.2021.9564698](https://doi.org/10.1109/ITSC48978.2021.9564698).
13. ^{a, b, c, d, e, f, g, h, i, j}Rostami-Shahrbabaki M, Zhang H, Sekeran M, Bogenberger K (2023). Increasing the Capacity of a Lane-Free Beltway for Connected and Automated Vehicles Using Potential Lines. Technical report.
 14. [^]Mukherjee D, Mitra S (2022). "Development of a Systematic Methodology to Enhance the Safety of Vulnerable Road Users in Developing Countries". *Transportation in Developing Economies*. 8 (2): 28. doi:[10.1007/s40890-022-00165-4](https://doi.org/10.1007/s40890-022-00165-4).
 15. ^{a, b}Sekeran M, Syed AA, Lindner J, Margreiter M, Bogenberger K. "Investigating lane-free traffic with a dynamic driving simulator." In: 2023 IEEE 26th International Conference on Intelligent Transportation Systems (ITSC), Bilbao, Spain. IEEE; 2023. p. 3417–3422. doi:[10.1109/ITSC57777.2023.10421947](https://doi.org/10.1109/ITSC57777.2023.10421947).
 16. ^{a, b, c, d, e, f, g, h, i, j}Mathew TV, Munigety CR, Bajpai A (2015). "Strip-Based Approach for the Simulation of Mixed Traffic Conditions". *Journal of Computing in Civil Engineering*. 29 (5): 04014069. doi:[10.1061/\(ASCE\)CP.1943-5487.0000378](https://doi.org/10.1061/(ASCE)CP.1943-5487.0000378).
 17. [^]Ishikura T, Yokoyama F (2022). "Regional economic effects of the Ring Road project in the Greater Tokyo Area: A spatial CGE approach". *Papers in Regional Science*.
 18. [^]Nugmanova, Arndt, Hossain, Kim. "Effectiveness of ring roads in reducing traffic congestion in cities for long run: Big Almaty ring road case study." *Sustainability*. 11 (18): 4973, 2019. doi:[10.3390/su11184973](https://doi.org/10.3390/su11184973).
 19. [^]Zhiyuan G, Fucheng Z. Study on the Traffic Impact Analysis of the Second Ring Road in Beijing. In: 2009 Second International Symposium on Information Science and Engineering. 2009 Dec. p. 505–508. doi:[10.1109/ISISE.2009.71](https://doi.org/10.1109/ISISE.2009.71).
 20. [^]Diakaki C, Papageorgiou M, Papamichail I, Nikolos I (2015). "Overview and analysis of vehicle automation and communication systems from a motorway traffic management perspective". *Transportation Research Part A: Policy and Practice*. 75: 147–165.
 21. [^]Malekzadeh M, Manolis D, Papamichail I, Papageorgiou M. "Empirical investigation of properties of lane-free automated vehicle traffic." In: 2022 IEEE 25th international conference on intelligent transportation systems (ITSC). IEEE; 2022. p. 2393–2400.
 22. ^{a, b, c}Yanumula VK, Typaldos P, Troullinos D, Malekzadeh M, Papamichail I, Papageorgiou M (2023). "Optimal Trajectory Planning for Connected and Automated Vehicles in Lane-Free Traffic With Vehicle Nudging". *IEEE Transactions on Intelligent Vehicles*. 8 (3): 2385–2399. doi:[10.1109/TIV.2023.3241200](https://doi.org/10.1109/TIV.2023.3241200).

23. [△]Levy R, Haddad J (2022). "Cooperative path and trajectory planning for autonomous vehicles on roads without lanes: A laboratory experimental demonstration". *Transportation Research Part C: Emerging Technologies*. 144: 103813. doi:[10.1016/j.trc.2022.103813](https://doi.org/10.1016/j.trc.2022.103813). [Link](#).
24. ^{a, b, c}Zhang H, Rostami-Shahrbabaki M, Troullinos D, Bogenberger K. "A Novel Potential Line Strategy for Autonomous Vehicle Control in Lane-Free Traffic." In: 2023 IEEE 26th International Conference on Intelligent Transportation Systems (ITSC). IEEE; 2023. p. 4173–4180.
25. [△]Theodosis D, Karafyllis I, Papageorgiou M (2023). "Cruise controllers for lane-free ring-roads based on control Lyapunov functions". *Journal of the Franklin Institute*. 360 (9): 6131–6161. doi:[10.1016/j.jfranklin.2023.02.003](https://doi.org/10.1016/j.jfranklin.2023.02.003). [Link](#).
26. [△]Stüger P, Malcolm P, Niels T, Bogenberger K (2023). "Integrated Intersection Control for Lane-Free Connected and Automated Driving". *Proceedings*. pages 1–7. doi:[10.1109/MT-ITS56129.2023.10241608](https://doi.org/10.1109/MT-ITS56129.2023.10241608).
27. [△]Malcolm P, Bogenberger K (2025). "Lane-free intersection control for connected automated vehicles prioritizing vulnerable road users". *Transportation Research Part C: Emerging Technologies*. 170: 104918. doi:[10.1016/j.trc.2024.104918](https://doi.org/10.1016/j.trc.2024.104918). [Link](#).
28. [△]Naderi M, Papageorgiou M, Troullinos D, Karafyllis I, Papamichail I (2024). "Controlling Automated Vehicles on Large Lane-Free Roundabouts". *IEEE Transactions on Intelligent Vehicles*. 9 (1): 3061–3074. doi:[10.1109/TIV.2023.3338261](https://doi.org/10.1109/TIV.2023.3338261).
29. [△]Berahman M, Rostami-Shahrbabaki M, Bogenberger K (2024). "Multi-Task Lane-Free Driving Strategy for Connected and Automated Vehicles: A Multi-Agent Deep Reinforcement Learning Approach". *arXiv*. Available from: <https://arxiv.org/abs/2406.14766>.
30. [△]Berahman M, Rostami-Shahrbabaki M, Bogenberger K (2022). "Driving strategy for vehicles in lane-free traffic environment based on deep deterministic policy gradient and artificial forces". *IFAC-Papers on Line*. 55 (14): 14–21. doi:[10.1016/j.ifacol.2022.07.576](https://doi.org/10.1016/j.ifacol.2022.07.576). <https://www.sciencedirect.com/science/article/pii/S2405896322009879>. 11th IFAC Symposium on Intelligent Autonomous Vehicles IAV 2022.
31. [△]Karakalou A, Troullinos D, Chalkiadakis G, Papageorgiou M. Deep RL reward function design for lane-free autonomous driving. In: *Advances in Practical Applications of Agents, Multi-Agent Systems, and Complex Systems Simulation. The PAAMS Collection*. Cham: Springer International Publishing; 2022. p. 254–266. ISBN 978-3-031-18192-4.
32. [△]Karakalou A, Troullinos D, Chalkiadakis G, Papageorgiou M (2023). "Deep reinforcement learning reward function design for autonomous driving in lane-free traffic". *Systems*. 11 (3): 134. doi:[10.3390/systems11030134](https://doi.org/10.3390/systems11030134). <https://www.mdpi.com/2079-8954/11/3/134>.

33. [△]Rostami-Shahrbabaki M, Weigl S, Niels T, Bogenberger K (2023). "Modeling Vehicle Flocking in Lane-Free Automated Traffic". *Transportation Research Record*. 2677 (9): 499–512. doi:[10.1177/03611981231159405](https://doi.org/10.1177/03611981231159405).
34. [△]Rostami-Shahrbabaki M, Weigl S, Akbarzadeh M, Bogenberger K. A two-layer approach for vehicular flocking in lane-free environment. In: *11th Triennial Symposium on Transportation Analysis (TRISTAN)*; 2022.
35. [△]Dabestani N, Typaldos P, Papamichail I, Papageorgiou M (2024). "Joint Vehicle Path Planning for Interruptible 1-D Snake-like Platoons and 2D Flocks in Lane-free Traffic". *IFAC-PapersOnLine*. 58 (10): 61–68. doi:[10.1016/j.ifacol.2024.07.319](https://doi.org/10.1016/j.ifacol.2024.07.319). [Link](#). 17th IFAC Symposium on Control of Transportation Systems CTS 2024.
36. [△], [▷]Munigety CR, Mathew TV (2016). "Towards Behavioral Modeling of Drivers in Mixed Traffic Conditions". *Transportation in Developing Economies*. 2 (1): 6. doi:[10.1007/s40890-016-0012-y](https://doi.org/10.1007/s40890-016-0012-y).
37. [△], [▷]Ravishankar KVR, Mathew TV. Vehicle-type dependent car-following model for heterogeneous traffic conditions. *Journal of Transportation Engineering*. 137(11):775–781, Nov. 2011. doi:[10.1061/\(ASCE\)TE.1943-5436.0000273](https://doi.org/10.1061/(ASCE)TE.1943-5436.0000273).
38. [△]Kanagaraj V, Treiber M (2018). "Self-driven particle model for mixed traffic and other disordered flows". *Physica A: Statistical Mechanics and its Applications*. 509: 1–11. doi:[10.1016/j.physa.2018.05.086](https://doi.org/10.1016/j.physa.2018.05.086). [Link](#).
39. [△], [▷]Kashyap NR, Chilukuri BR, Srinivasan KK, Asaithambi G (2020). "Analysis of Vehicle-Following Behavior in Mixed Traffic Conditions Using Vehicle Trajectory Data". *Transportation Research Record: Journal of the Transportation Research Board*. 2674 (11): 842–855. doi:[10.1177/0361198120949874](https://doi.org/10.1177/0361198120949874).
40. [△]Gunay B. Car following theory with lateral discomfort. *Transportation Research Part B: Methodological*. 41 (7): 722–735, Aug. 2007. doi:[10.1016/j.trb.2007.02.002](https://doi.org/10.1016/j.trb.2007.02.002).
41. [△], [▷], [◁]Gipps PG (1981). "A Behavioural Car-Following Model for Computer Simulation". *Transportation Research Part B: Methodological*. 15 (2): 105–111. doi:[10.1016/0191-2615\(81\)90037-0](https://doi.org/10.1016/0191-2615(81)90037-0).
42. [△]George Oketch T. New modeling approach for mixed-traffic streams with nonmotorized vehicles. *Transportation Research Record: Journal of the Transportation Research Board*. 2000 Jan; 1705(1):61–69. doi:[10.3141/1705-10](https://doi.org/10.3141/1705-10).
43. [△], [▷]Behrisch M, Bieker L, Erdmann J, Krajzewicz D. "SUMO--simulation of urban mobility: an overview." In: *Proceedings of SIMUL 2011, The Third International Conference on Advances in System Simulation*. ThinkMind; 2011.

44. [△]Semrau M, Erdmann J (2016). "Simulation framework for testing ADAS in Chinese traffic situations". *S UMO 2016--Traffic, Mobility, and Logistics*. 30: 103–115.
45. [△]Hossain M, McDonald P. Modelling of traffic operations in urban networks of developing countries: A computer aided simulation approach. *Computers, Environment and Urban Systems*. 22(5):465–483, 1998. doi:[10.1016/S0198-9715\(98\)00040-4](https://doi.org/10.1016/S0198-9715(98)00040-4).
46. [△]Arasan VT, Koshy RZ (2005). "Methodology for Modeling Highly Heterogeneous Traffic Flow". *Journal of Transportation Engineering*. 131 (7): 544–551. doi:[10.1061/\(ASCE\)0733-947X\(2005\)131:7\(544\)](https://doi.org/10.1061/(ASCE)0733-947X(2005)131:7(544)).
47. [△]Papathanasopoulou V, Antoniou C (2018). "Flexible car--following models for mixed traffic and weak lane--discipline conditions". *European transport research review*. 10: 1--14.
48. [△]Kesting A, Treiber M. *Traffic flow dynamics: data, models and simulation*. no. **Book, Whole**(Springer Berlin Heidelberg, Berlin, Heidelberg, 2013); 2013.
49. [△]Troullinos D, Chalkiadakis G, Manolis D, Papamichail I, Papageorgiou M. "Lane- Free Microscopic Simulation for Connected and Automated Vehicles." In: *2021 IEEE International Intelligent Transportation Systems Conference (ITSC)*, Indianapolis, IN, USA. IEEE; 2021. p. 3292–3299. doi:[10.1109/ITSC48978.2021.9564637](https://doi.org/10.1109/ITSC48978.2021.9564637).

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