On the Sagnac Effect and the Consistency of Relativity Theory

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Abstract

The Sagnac effect is an interferometric phenomenon produced by rotation. It has a rich history and presently has numerous technological applications. Despite some persistent claims to the contrary, we explain why the Sagnac effect does not prove relativity either incorrect or inconsistent. Analyzing such misunderstandings has didactic value because it allows us to review some subtle relativity concepts. It also reveals the importance of basing scientific reasoning on rigorous logical thinking to avoid confusion derived from prejudices based on our limited everyday human experience.

1 Introduction

George Sagnac (1869-1928) was a French physicist who strongly opposed Einstein's theory of relativity. In the early years following Einstein's 1905 groundbreaking article, the theory was still controversial among scientists. In 1910, Sagnac conceived a rotating interferometer that now bears his name to prove the existence of the luminiferous ether. By 1913, he published the results of the experiment confirming his ideas about the ether [1].

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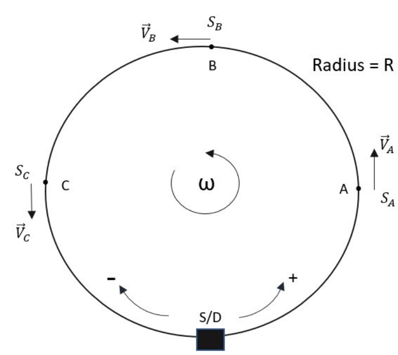
The first relativistic proofs of the effect were presented by von Laue in 1920 and with a different approach by Paul Langevin in 1921. Presumably, it took so long to find relativistic explanations because, at the time, the experiment was not widely known [2].

Today, the relativistic nature of the Sagnac effect is, of course, out of the question. There exists a vast literature with an exhaustive analysis of the phenomenon [3, 4, 5]. Its applications range from GPS, fiber optic and ring laser gyroscopes, navigation systems to geodesic and seismology [6].

However, now and then, claims contesting the correctness and consistency of relativity still appear in the scientific literature [7, 8, 9, 10, 11, 12, 13, 14, 15]. So far, such claims proved untenable. They result from an incomplete comprehension of the relativity principles, the structure of physical theories in general, and the attachment to hard-to-overcome Newtonian ideas.

Figure 1: Sagnac Interferometer



The recent resurgence of such questionings almost invariably rely on the claim that the Sagnac effect disproves relativity theory by revealing two alleged problems: light speed is anisotropic and the existence of clock synchronization inconsistencies [12, 13, 14, 15]. The argument for the supposed light-speed anisotropy is also known as Selleri's paradox [10].

Such misunderstandings give an excellent opportunity to rehearse, from a didactic viewpoint, concepts necessary for the correct relativistic description of noninertial systems. The analysis could prove helpful to undergraduate as well as graduate students specializing in relativity.

In section 2, we describe the simplest version of the Sagnac interferometer that will be enough for our purposes. We also briefly mention the two possible interpretations. Section 3 reviews the concepts and formalism necessary for a proper relativistic analysis of the alleged difficulties. Finally, section 4 analyzes and resolves the two alleged problems that presumably disprove relativity theory.

2 The Sagnac Interferometer

One configuration of the Sagnac interferometer is the ring interferometer (Fig. 1). Next, we describe its basic functioning principle and the interpretation of the result.

2.1 The Sagnac Experiment

The entire platform is rotated counterclockwise at angular speed ω (Fig. 1). A beam of light is split at the source S and the two beams follow the same path but in opposite directions. Then, the two beams are detected at D. Owed to rotation, the two beams follow asymmetric light paths producing a displacement of the interference fringes with respect to their position when the interferometer was stationary.

For the configuration shown in Fig. 1, the calculation of the fringe displacement is pretty straightforward. Let L be the circumference and ΔL the distance traveled by the Source/Detector along the circumference, then the time t^+ it takes the counterclockwise beam to reach the detection point verifies,

$$t^+ = \frac{L + \Delta L^+}{c} \tag{1}$$

$$\Delta L^+ = \omega R t^+ \tag{2}$$

Eliminating ΔL^+ and putting $L = 2\pi R$,

$$t^+ = \frac{2\pi R}{c - R\omega} \tag{3}$$

Analogously, for the clockwise beam we have,

$$t^- = \frac{2\pi R}{c + R\omega} \tag{4}$$

The phase difference is determined by the time difference,

$$t^+ - t^- \;=\; \frac{4\omega\pi R^2}{c^2(1-(\frac{R\omega}{c})^2)} \tag{5}$$

(5) is the "Newtonian" prediction. The relativistic result is obtained introducing in (5) the factor $1/\gamma = \sqrt{1-(R\omega)^2/c^2}$ corresponding to the time dilation effect,

$$\begin{aligned} t^+ - t^- &= \frac{1}{\gamma}\frac{4\omega\pi R^2}{c^2(1-(\frac{R\omega}{c})^2)} \qquad (6)\\ &= \frac{4\omega\pi R^2}{c^2\sqrt{1-(\frac{R\omega}{c})^2}} \qquad (7)\end{aligned}$$

Since to first-order in $R\omega/c$,

$$\frac{1}{1-(\frac{R\omega}{c})^2} \approx \frac{1}{\sqrt{1-(\frac{R\omega}{c})^2}} \approx 1 \tag{8}$$

the Newtonian (5) and relativistic (7) results coincide. Putting $A = \pi R^2$, to first-order we have,

$$\Delta t \;=\; \frac{4\omega A}{c^2} \tag{9}$$

The phase shift is given by,

$$\begin{aligned} \Delta\phi &= \frac{2\pi c\Delta t}{\lambda} \qquad (10)\\ &= \frac{8\pi\omega A}{\lambda c} \qquad (11)\end{aligned}$$

It is possible to prove the result remains valid for any closed path with a vector area $\vec{A}$, angular velocity $\vec{\omega}$, and an arbitrary center of rotation,

$$\Delta\phi \;=\; \frac{8\pi\,\vec{\omega}\cdot\vec{A}}{\lambda c} \tag{12}$$

2.2 Interpretation of the Result

According to the ether hypothesis, light is a disturbance similar to sound. It takes place in a substance called luminiferous ether. When we assume the interferometer is placed in an inertial system that is stationary with respect to the ether, then the result given by (11) is correct if the rotating platform does not drag the ether along with its motion.

On the other hand, it was already noticed by von Laue in 1911 that relativity predicts, to first-order approximation, the same result as an ether theory for a Sagnac-type experiment [16].

Since the Sagnac effect is compatible either with relativity or the ether hypothesis, the conclusion is that the phenomenon cannot discriminate between the two theories.

However, the famous 1887 Michelson-Morley experiment can be explained only if the ether is fully dragged with the Earth's motion [17]. On the other hand, in 1925, Michelson and Gale confirmed the Sagnac effect using the Earth as a rotating platform [18]. This experiment can only be explained if the ether is not dragged with the Earth's rotation.

Thus, neither the Micelson-Morley nor the Michelson-Gale experiments, taken separately, are evidence against the ether. However, when considered jointly, they are incompatible with the existence of the luminiferous ether since it cannot be consistently assumed the ether is fully dragged with the Earth translation and, at the same time, remains unperturbed by its rotation. Nonetheless, both experiments can be explained by relativity.

3 Relativity in Noninertial Frames

The invalidity of relativity theory is supposed to be revealed when the Sagnac effect is analyzed from the viewpoint of observers fixed to the platform and comoving with it. Unfortunately, the correct relativistic description of phenomena from within noninertial frames is a bit complicated, as was recognized by Einstein [19],

> All this happened in 1908. Why were another seven years required for the construction of the general theory of relativity? The main reason lies in the fact that it is not so easy to free oneself from the idea that coordinates must have a direct metric significance.

This brings us to another conceptual issue. Do we need general relativity (GR) for the relativistic description of accelerated reference frames?

If by GR we mean Einstein's theory of gravitation, then we do not need GR to describe noninertial frames. But if by GR we mean the extension of special relativity to accelerated systems, the answer is yes. We accomplish such an extension by locally applying special relativity through the use of "comoving frames" or "rest frames" irrespective of gravitation [20].

So, the belief that we need GR to describe accelerated frames is incorrect when by GR we mean Einstein's gravitation theory. As recently pointed out by Pepino and Mabile [21], the last point is a widespread misconception. The occasional claim that we need gravitation to analyze rotating systems has raised justified doubts [15, 22, 23]. In particular, Ref. [24] emphasizes the fact that *the Sagnac effect is of a purely special-relativistic nature.*

Next, we introduce the basic formalism we need to correctly analyze the physics of the Sagnac effect from within the rotating platform. Note that we need neither gravitation nor the equivalence principle.

There is a caveat though because it is also true that the basic formalism can be considered a particular case of gravitation theory. When the Riemann or curvature tensor vanishes, we are in flat spacetime. In this case, when the metric tensor $g_{\mu\nu}$ differs from the Minkowski metric $\eta_{\mu\nu}$, it may be the case that we are in a noninertial frame without gravitation. So, in a sense, we need GR to describe noninertial frames because we are borrowing the general formalism necessary to describe gravitation only this time without gravitation!

3.1 The metric tensor

Let x^μ, $\mu \in \{0,1,2,3\}$ be the spacetime coordinates with $x^0 = ct$ the time coordinate and x^i, $i \in \{1,2,3\}$ the spatial coordinates. We shall use Greek indices for spacetime coordinates and Latin indices for spatial coordinates. As usual, Einstein's summation convention is used throughout. In inertial frames we have that the interval is,

$$\begin{aligned} ds^2 &= (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 \qquad (13) \\ &= \eta_{\mu\nu} dx^\mu dx^\nu \qquad (14) \end{aligned}$$

The metric tensor $\eta_{\mu\nu} = diag\{1, -1, -1, -1\}$ remains invariant after a coordinate (Lorentz) transformation between inertial frames,

$$y^{\mu} = \Lambda^{\mu}_{\cdot\,\nu}\, x^{\nu} \tag{15}$$

$$\eta_{\mu\nu} dy^{\mu} dy^{\nu} = \eta_{\mu\nu} dx^{\mu} dx^{\nu} \tag{16}$$

If we allow noninertial systems, we must replace the linear transformation (15) by a general one, $y^{\mu} = y^{\mu}(x^0, x^1, x^2, x^3)$. The invariant interval now is,

$$ds^2 = g_{\mu\nu} dy^{\mu} dy^{\nu} \tag{17}$$

$$g_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial y^{\mu}} \frac{\partial x^{\beta}}{\partial y^{\nu}}\, \eta_{\alpha\beta} \tag{18}$$

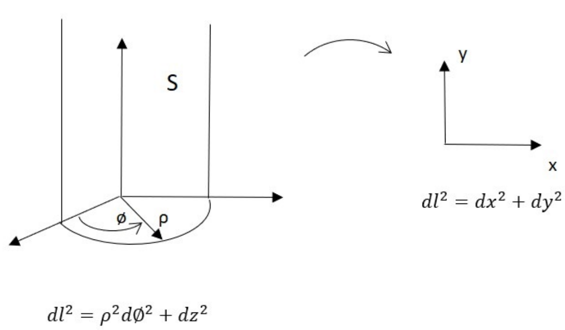
The metric tensor components become functions of the spacetime coordinates and $g_{\mu\nu}(y^0, y^1, y^2, y^3) \neq \eta_{\mu\nu}$.

A concrete example may help to clarify the nature of the spacetime metric of a noninertial frame. In 3.3 we shall see that the spacetime metric of a rotating platform is,

$$\{g_{\mu\nu}\} = \begin{bmatrix} 1-\beta^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{19}$$

Compare (19) with the metric λ_{ik} of a cylindrical surface S of radius ρ in cylindrical coordinates (ρ, ϕ) (Fig. 2),

Figure 2: Cylindrical Surface



$$\{\lambda_{ik}\} = \begin{bmatrix} \rho^2 & 0 \\ 0 & 1 \end{bmatrix}, \qquad i, k \in \{1, 2\} \tag{20}$$

Both metrics, $g_{\mu\nu}$ and λ_{ik} are flat so their Riemann tensor components vanish, $R^{(4)}_{\mu\nu\sigma\xi} = 0$ and $R^{(2)}_{ikjn} = 0$. This means that it is possible to find a global coordinate transformation that reduces their components $g_{\mu\nu}$ and λ_{ik} to the their Lorentzian $\eta_{\mu\nu} = diag\{1, -1, -1, -1\}$ and Euclidean $\lambda_{ik} = diag\{1, 1\}$ forms respectively.

Note, however, that to the flat spacetime metric (19) corresponds a non-flat three-space metric (34) with one independent nonvanishing Riemann tensor component [24],

$$R^{(3)}_{r\phi r\phi} = \frac{3\omega^2 r^2}{c^2(1-\beta^2)} \tag{21}$$

That means the spatial geometry corresponding to the rotating platform, and generally to a noninertial frame, is non-Euclidean.

3.2 Interpretation of the spacetime coordinates

This is the delicate part that Einstein complained about, as mentioned at the beginning of section 3. Fortunately, by now, this is well understood. We do not have to figure it out ourselves like he had to.

Without a correct interpretation of the spacetime coordinates with respect to noninertial frames, we will not be able to analyze and understand the Sagnac effect from the relativistic perspective of observers moving with the platform.

Here we merely describe the necessary formalism for our purposes and do not present any derivations. We refer the interested reader to the standard literature on GR, for instance, [25, 26].

3.2.1 Time intervals and clock synchronization

In an inertial frame, the time coordinate $x^0 = ct$ is directly related to the time t of the system of reference, i.e., t is the time read in a net of synchronized clocks stationary in that reference frame. Similarly, the spatial coordinates have an immediate metrical meaning, the spatial distance d between points $x^i = a^i$ and $x^i = b^i$ is

$$d = \sqrt{(b^1 - a^1)^2 + (b^2 - a^2)^2 + (b^3 - a^3)^2} \tag{22}$$

Luckily, the Earth can be considered very close to an inertial system for all practical purposes of our daily endeavors. Our clocks remain synchronized for most practical uses,[1] and we can safely employ the Pythagorean theorem for calculating finite distances.

In a noninertial frame, the measuring properties of space and time are encoded in the metric tensor $g_{\mu\nu}$. The coordinates are mere labels that univocally determine spacetime events. While x^0 is associated with time, its value is not equal to the reading of an actual clock. Analogously for the spatial coordinates x^i.

Global synchronization of clocks, like in inertial frames, is generally impossible. We cannot arbitrarily assume we have a variable that can be identified with the readings of a net of synchronized clocks inside the noninertial frame. The time interval marked by a clock fixed at $P \equiv (x^1, x^2, x^3)$ in our reference frame is given by

$$d\tau = \frac{\sqrt{g_{00}}}{c} dx^0 \tag{23}$$

$$\tau_B - \tau_A = \int_{x_A^0}^{x_B^0} \frac{\sqrt{g_{00}}}{c} dx^0 \tag{24}$$

In a noninertial frame, even clocks in the same reference frame have different rates since g_{00} can be a function of the coordinates.

Although clock synchronization is generally impossible, we can calculate the time coordinates of infinitesimally close clocks that correspond to the simultaneous reading of our clock. This process is sometimes also called "synchronization". However, it does not mean that clock readings are simultaneously equal. Let x_1^0 be the coordinate time at $P_1 \equiv x_1^i$. Then, the value of the coordinate time $(x_1^0)_2$ at $P_2 \equiv x_2^i = x_1^i + dx^i$ that is simultaneous with x_1^0 is,

$$(x_1^0)_2 = x_1^0 + g_i dx^i \tag{25}$$

$$g_i = -\frac{g_{0i}}{g_{00}} \tag{26}$$

Thus, in general, simultaneous events at P_1 and P_2 do not have the same value of the time coordinate $(x_1^0)_2 - x_1^0 = g_i dx^i \neq 0$. Although we can integrate $g_i dx^i$ along a given worldline, in GR, the simultaneity of distant events, hence synchronization of distant clocks, does not make sense in general. We shall further elaborate on this in section 4.2.

[1] A notable exception is the GPS, which most of us use daily and is notoriously sensitive to relativistic effects.

3.2.2 Spatial distances

The spatial distance between P_1 and P_2 is defined along a given path $x^\mu(\alpha)$

$$dl = \sqrt{\gamma_{ik} dx^i dx^k}\,, \qquad l_{12} = \int_{\alpha_1}^{\alpha_2} \sqrt{\gamma_{ik} dx^i dx^k} \tag{27}$$

where $P_1 \equiv x^\mu(\alpha_1)$, $P_2 \equiv x^\mu(\alpha_2)$, and γ_{ik} is the metric tensor of space given by

$$\gamma_{ik} = -g_{ik} + \frac{g_{0i} g_{0k}}{g_{00}} \tag{28}$$

As we mentioned before, the geometry that corresponds to this metric is non-Euclidean (see, (21)).

3.3 Spacetime in a rotating reference frame

Let us refer the rotating platform to an inertial frame with origin in its center and with the platform lying in the $x' - y'$ plane (Fig. 2). The spacetime coordinates for this inertial frame in cylindrical coordinates are (ct', r', ϕ', z'). We now perform a rotational transformation passing to a frame rotating with the platform

$$t = t', \qquad r = r', \qquad \phi = \phi' - \omega t, \qquad z = z' \tag{29}$$

The interval expressed in the primed inertial system is,

$$ds^2 = c^2 dt'^2 - dr'^2 - r'^2 d\phi'^2 - dz'^2 \tag{30}$$

After the transformation, according to (29) and putting $\beta = \omega r/c$

$$ds^2 = \left(1 - \beta^2\right) c^2 dt^2 - dr^2 - r^2 d\phi^2 - dz^2 - 2\beta rc\, dt\, d\phi \tag{31}$$

In this case the metric tensor does not depend on time and is called stationary. According to (26), (28), and (31), we have

$$\{g_i\} \equiv \left(0, \tfrac{\beta r}{1-\beta^2}, 0\right) \tag{32}$$

$$\{g_{ik}\} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -r^2 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \left\{\frac{g_{0i} g_{0k}}{g_{00}}\right\} = \frac{1}{1-\beta^2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \beta^2 r^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{33}$$

$$\{\gamma_{ik}\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{r^2}{1-\beta^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{34}$$

We also need to find the path of light rays in our rotating platform, i.e., null geodesics. Again, to avoid misunderstandings, we remark that this is a flat spacetime concept, so we are not using Einstein's theory of gravitation. We shall be concerned only with light rays traveling on the rim of the disc, so our initial conditions are r=const., z=0. From (31), setting $dr = dz = 0$, $ds^2 = 0$, and discarding the solution $dt < 0$

$$dt^+ = \frac{r d\phi^+}{c(1-\beta)}, \qquad d\phi^+ > 0 \tag{35}$$

$$dt^- = -\frac{r d\phi^-}{c(1+\beta)}, \qquad d\phi^- < 0 \tag{36}$$

4 Presumed Disproves of Relativity

The formalism presented in the former section allows us to perform the correct analysis of the following two recurrent claims [10, 11, 12, 14, 15]:

- The Sagnac effect proves that light speed is anisotropic and different from c (Selleri's paradox).
- The Sagnac effect proves that relativity leads to contradictory results regarding clocks synchronization and the existence of unphysical "time gaps."

4.1 Anisotropy of the speed of light

Setting r=R and $\omega R = v$, according to (35) and (36),

$$R\frac{d\phi^+}{dt^+} = c - v \tag{37}$$

$$R\frac{d\phi^-}{dt^-} = -(c + v) \tag{38}$$

(37) and (38) determine the propagation of the counterclockwise and clockwise beams, respectively. They seem to have different speeds relative to the

rotating platform. By integration we again find the classical formulas (3) and (4) for t^+ and t^-.

One could think the apparent speeds distinct from c happen because we disregarded length contraction and proper time dilation. However, if ds is the differential distance as seen by the inertial observer and ds_p the one corresponding to the local observer fixed to the platform, we have $ds_p = \gamma ds$ and $dt_p^{\pm} = dt^{\pm}/\gamma$. The observer fixed to platform at p would find

$$\frac{ds_p}{dt_p^+} = \gamma^2 \frac{ds}{dt^+} = \gamma^2(c - v) \tag{39}$$

$$\frac{ds_p}{dt_p^-} = \gamma^2 \frac{ds}{dt^-} = \gamma^2(c + v) \tag{40}$$

So, the argument goes, even considering the known relativistic effects, we still have light speed $\neq c$ inside the platform. Next, we turn to the correct relativistic approach.

4.1.1 The correct approach

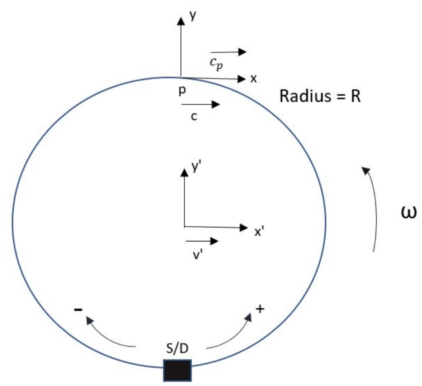
The first hint at the incorrectness of the previous argument is that it contradicts the relativist addition formula of velocities (41).

Indeed, if the inertial observer in the primed system (Fig. 2) measures a light ray velocity c, since the rotating observer at p sees the primed systems moving at v', according to the relativistic addition of velocities,

$$c_p = \frac{v' + c}{1 + \frac{v'c}{c^2}} = c \tag{41}$$

But what can be wrong with (39) and (40)? Although the usual argument is straightforward, it hides a subtle mistake. Inside the rotating platform, the time coordinate $x^0 = ct$ is a mere label for the "physical time" that a given observer fixed to the platform experiences.

Figure 3: Rotating Platform



The relation between x^0 and actual time in the platform is given by (23) with g_{00} given by (31),

$$d\tau^{\pm} = \sqrt{1-\beta^2}\, dt = \frac{dt^{\pm}}{\gamma} \tag{42}$$

The differential distance dl traveled by light for an observer inside the platform is given by (27), (31), (37), and (38)

$$dl^{+} = \frac{R}{\sqrt{1-\beta^2}} \frac{c-v}{R}\, dt^{+} = \gamma(c-v)dt^{+} \tag{43}$$

$$dl^{-} = \frac{R}{\sqrt{1-\beta^2}} \frac{c+v}{R}\, dt^{-} = \gamma(c+v)dt^{-} \tag{44}$$

But then again $dl^{\pm}/d\tau^{\pm}$ give us (39) and (40)! The usual argument of dividing (43) and (44) by (42) to obtain the speed is indeed incorrect (see, for instance, [25]). To see why, let us recall how speed should be correctly calculated.

The speed of a moving particle at P_2 is obtained dividing the distance dl between to successive points $P_1 \equiv x^i$ and $P_2 \equiv x^i + dx^i$ along its trajectory by the time dt it took to traverse dl. If the reading of the clock at P_1 is $t = t_1$ and the reading of the clock at P_2 is $t = t_2$, the time it took the particle to travel the distance dl is obtained by the difference of the readings of two clocks placed at different positions in space. In an inertial frame there is no problem with taking $dt = t_2 - t_1$ because we can synchronize all the clocks within a given reference frame.

The subtle mistake with the usual argument is that it overlooks the fact that clocks at P_1 and P_2 are not synchronized. The correct evaluation of the time it took the particle to go from P_1 to P_2 cannot be obtained by the mere difference of the readings of two clocks that are not synchronized.

The necessary correction is introduced with use of (25) as we explain in section 3.2.1. We cannot use the formula $d\tau = \sqrt{g_{00}}\, dx^0$, as we did in (42), when $dx^0 = x_2^0 - x_1^0$ is the time coordinate difference corresponding to different points in space. The corrected time coordinate difference is,

$$
\begin{aligned}
x_2^0 - (x_1^0)_2 &= \delta x^0 && (45)\\
x_2^0 - (x_1^0 + g_i\, dx^i) &= \delta x^0 && (46)\\
x_2^0 - x_1^0 - g_i\, dx^i) &= \delta x^0 && (47)\\
dx^0 - g_i\, dx^i &= \delta x^0 && (48)
\end{aligned}
$$

Now that we have corrected for the lack of synchronization, we evaluate the correct elapsed time with the clock at P_2 using (23), and (48),

$$\delta\tau = \frac{\sqrt{g_{00}}}{c}\, \delta x^0 \tag{49}$$

$$\delta\tau = \frac{\sqrt{g_{00}}}{c}(dx^0 - g_i\, dx^i) \tag{50}$$

As an example, let us calculate the speed of the counterclockwise light ray with respect to the platform. According to (31),(32), (37), (50), considering that in our case $x^0 = ct$,

$$
\begin{aligned}
\frac{dl^+}{\delta\tau^+} &= \frac{\gamma(c-v)dt^+}{\sqrt{1-\beta^2}(dt^+ - \frac{\beta R}{c(1-\beta^2)} d\phi^+)} && (51)\\
&= \frac{\gamma^2(c-v)}{1 - \frac{\beta}{c(1-\beta^2)} R \frac{d\phi^+}{dt^+}} && (52)\\
&= \frac{\gamma^2 c(1-\beta)}{1 - \frac{\beta}{1+\beta}} && (53)\\
&= \gamma^2 c(1-\beta^2) && (54)\\
&= c && (55)
\end{aligned}
$$

Thus, a local observer inside the platform actually measures with his clocks and rulers a speed of light equal to "c" in agreement with the result obtained through the relativistic addition of velocities (41). Similarly, we can prove that $dl^-/\delta\tau^- = c$, confirming the consistency of the relativity principles.

4.2 Inconsistent self-synchronization

Another recurrent claim of relativity's inconsistency purportedly revealed by the Sagnac effect is an hypothetical failure of synchronizing a rotating clock with itself. This effect is usually referred to as the "time gap" or "synchronization gap" [12, 14, 15, 22].

Unfortunately, the meaning of the word synchronization in this context is ambiguous. This contributes to the existing confusion. In an inertial reference frame, when we say that clocks are synchronized, we mean that those clocks show the same readings "simultaneously". As we noted before, in a noninertial frame, that is generally not possible. Simultaneity only makes sense between infinitesimally closed points in space.

Even when we say that we synchronize infinitesimally closed clocks, we do not mean that we make them show the same readings simultaneously. As we have seen in sections 3.2.1 and 4.1.1, synchronizing infinitesimally closed clocks signifies finding the value of the time coordinate $(x_1^0)_2$ at P_2 that corresponds to the simultaneous instant given by x_1^0 at P_1. In other words, the instant x_1^0 at P_1 is simultaneous with x_2^0 at P_2, not when $x_2^0 = x_1^0$, but when $x_2^0 = x_1^0 + g_i\,dx^i$, where $x_2^i = x_1^i + dx^i$.

Up to this point everything should be uncontroversial. The confusion appears when we extent this concept of synchronization to finite distances by integration of (25),

$$x^0(P_2) = x^0(P_1) + \int_{P_1}^{P_2} g_i\,dx^i \tag{56}$$

There are three reasons revealing that (56) is inconsistent as a definition of distant simultaneity:

1. The coordinate time $x^0(P_2)$ simultaneous with $x^0(P_1)$ can depend on the path.
2. In noninertial frames, simultaneity, therefore synchronization of clocks, is not a transitive property [27].
3. If we follow a closed path, upon returning to the same point, we would find a different value of $x^0(P_1)$, usually referred to as the "time gap".

The "time gap" interpreted as the failure to synchronize a clock with itself only arises if we accept (56) as a valid definition of distant simultaneity. To avoid this problem, some authors define simultaneity only over open paths. However, this is misleading and, in fact, contradictory.

The main reason (56) cannot, in general, be a consistent definition of simultaneity of distant events is the lack of transitivity mentioned in point 2 above. Indeed, to reach P_2 from P_1 we have to successively pass through an infinite sequence of intermediate points $P_1, P_{1'} \ldots, P_{n'}, P_2$ conserving, in each step, the simultaneity with the previous one ($n' \to \infty$).

Transitivity of simultaneity holds in inertial frames, as explicitly stated by Einstein in 1905 [28]. However, it is easy to qualitatively understand why transitivity does not hold in general and in particular in our rotating platform. Let us take three points A, B, and C fixed to the rim of our platform (Fig. 1). Consider the three comoving inertial systems S_A, S_B, and S_C. Let us assume $E_A = (x_A^0, 0, 0, 0)$ in S_A simultaneous (in S_A) with $E_B = (x_B^0, 0, 0, 0)$ in S_B and E_B simultaneous (in S_B) with $E_C = (x_C^0, 0, 0, 0)$ in S_C. If $\vec{V}_{BA}$ represents the relative velocity of S_B with respect to S_A, since $\vec{V}_{BA} \neq 0$ the simultaneous events E_B and E_C in S_B are not simultaneous in S_A. Therefore, although we follow a path of successive simultaneity,

$$E_A \xrightarrow{simult.} E_B \xrightarrow{simult.} E_C \xrightarrow{not\ simult.} E_A \tag{57}$$

upon returning to the initial point, we have lost simultaneity. So, the "time gap" is a natural consequence of the relative character of simultaneity between systems in relative motion, notwithstanding that A, B, and C are fixed in the platform frame.

5 Conclusions

The correct application of relativity to noninertial frames proves light speed is invariant, so the Sagnac effect neither constitutes empirical evidence against relativity nor discloses any internal inconsistencies. The proof also resolves the so-called Selleri's paradox [10].

We explained that the presumed "unphysical" time discontinuity or time gap is a natural consequence of the nontransitivity of the synchronization process and has nothing to do with synchronizing a clock with itself or with time being discontinuous.

Unlike later claims based on his interferometric experiment, it is timely to recognize that Sagnac held a coherent position because he intended to prove the existence of the luminiferous ether, which had to be discarded only on empirical evidence.

So far, the alleged inconsistencies, paradoxes, and incorrectness presumably affecting relativity theory ultimately reduce to the persistent illusion claiming the simultaneity of distant events ought to be absolute and the difficulties of correctly assessing the logical consequences of its relative character.

Regarding the last point, the great French physicist, mathematician and philosopher Henri Poincaré said [29]:

> We have not a direct intuition of simultaneity, nor of the equality of two durations. If we think we have this intuition, this is an illusion. We replace it by the aid of certain rules which we apply almost always without taking count of them.

Soon afterward Einstein took count of them revolutionizing our concepts about space and time.

Data availability statement

No new data were created or analysed in this study.

Author declaration

The author has no conflicts to disclose.

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