The number of free electrons per atom in a metallic conductor

Peter F. Lang[1]

1 Birkbeck College, University of London

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Abstract

This work examines some of the important principles of classical physics on metallic conduction and free electrons. This work shows that the speed of nerve conduction is much higher than calculated the drift velocity of electrons in a metal. Inadequacies te of the Drude model of metallic structure and conduction are described. Calculations in this work show that the drift velocity of electrons in metals is much higher than expected with the Drude assumptions and there are fewer free electrons per atom in a metal than conventionally accepted.

Peter F. Lang

Birkbeck College, University of London.

Email: p.f.lang@gmail.com

ORCID: 0000-0002-8530-5422

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Introduction

Many university and high school physics text books show that there is one or at the most two free/conduction electrons per atom in a metal. In most standard text books[1] the drift velocity of conduction electrons is calculated by the following equation:

$$v = I/nAq,$$

I is the current in Amps, n is the number of carriers which are usually electrons. In the case of a copper wire there are assumed to be 8.5 x 10^{28} electrons per cubic meter and equivalent to one conduction electron per atom. Here q is the charge of each carrier which is 1.6 x 10^{-19} C and A is the cross sectional area of the conductor. It is assumed for a copper wire there is one electron per atom participating in current flow. If the current is 10 Amp and the cross sectional area A is

1mm^2, the calculated drift velocity is less than 10^3 ms^{-1}. This contrasts with the speed of an electron between two electrodes 1 mm apart in a cathode ray tube, starting from rest at the cathode with the PD between the electrodes 10V, that reaches 1.9 x 10^6 ms^{-1}. It is also known that in a lightning flash[2] that lasts about 10^{-6} seconds the speed of the lightning approaches 1 x 10^8 ms^{-1}.

Nerve impulses travel through a neuron by means of a change in the electrical charge across the cell membrane. When the nerve impulse reaches the end of a axon, the nerve membrane becomes permeable to calcium ions in the synapse and releases neurotransmitters which transmit the impulse to the dendrites of the next neuron. In humans and vertebrates some axons are insulated by a layer of myelin allowing nerves impulse [3] to travel to up to 120 ms^{1}. In this case the nerve impulse is carried by ions which collide with numerous giant molecules in the nerve. Compared to the calculated drift velocity of conduction electrons in copper which is less than 10^{-3} ms^{-1} it seems quite incredible.

This work aims to show that there are fewer conduction electrons and the electron drift velocity is much higher than usually assumed and that it is not physically possible to have an equivalent of one or than one conduction electrons per atom participating in current flow. The supposition of one conduction electron per atom is part of assumptions of the Sommerfeld/Drude theory sometimes called the free electron theory. There is no theoretical justification or experimental evidence to support the assumption that there is at least one electron per atom contributing to current flow. When physics students discover the flaw in the assumptions in electron drift speed which they have been taught as physical fact their confidence in physics as a essential science will waver. Should there be10^{28} conduction electrons per cubic metre the tremendous amount of heat generated during electrical conduction can definitely detected should be detected.

The Sommerfeld/Drude theory

At the first half of the nineteenth century not much is known about electrical conduction. At the beginning of the twentieth century, Paul Drude [4] devised a model that laid the modern atomic foundation of Ohm’s law of resistivity. He suggested that in a solid free electrons form a gas of independent particles surrounding positive metal ions. When there is a potential difference between the ends of a metallic solid a current is produced and the free electrons are accelerated by the applied electric field. However, when the free electrons are travelling through the solid they collide with atoms and other conducting electrons. The collisions slow down the electrons. The acceleration drops to zero when the speed reaches the drift speed. Other physicists had contributed to the free electron theory and the most noticeable contributor to the free electron model was Sommerfeld[5]. Other theoretical models of metals have been developed and some models such as the “ions in a sea of electrons” model[6] of metals which cannot describe any metallic properties have been shown to be a very poor representation of metals[7][8].

Assumptions of the Drude free electron theory and their consequences

In the free electron theory, conduction electrons are assumed to act as an “electron gas”. In a unit volume of the electron gas the number n of conduction electrons[9] is calculated by:

$$n = 6.022x10^{23} Z\rho/A.$$

Here 6.022x 10^{23} is the Avogadro number, Z is the number of conduction electrons per atom, ρ is the density of the metal and A is the atomic mass. Since the number of electrons n is equal to N/V then the volume of a conduction electron is:

$$V/N = 1/n = 4/3\pi r_s^3.$$

The radius, r_s, of the volume occupied by a conduction electron is then:

$$r_s = (¾/(n\pi).$$

The radius r_s can range from approximately 90pm to 300 pm or 0.9 Å to 3 Å where one Å is equal to 1x 10^{10}m.

The second important assumption of the theory is that Z, the number of conduction electrons per atom, is equal to the valency of the metal. The values r_s and n for each metal can be easily calculated by equations stated above. For example, for the metals lithium and silver r_s are respectively 172 pm and 160 pm and n for the two metals[10][11] are respectively 4.7 x 10^{28} m^{-3} and 5.86 x 10^{28} m^{-3}. However, recent work has shown that it is physically impossible[12] to have 10^{28} m^{-3} conduction electrons and that the calculated r_s values are unrealistic.

If the free electron theory is correct, in a cubic meter of lithium and silver there are 4.7 x 10^{28} and 5.86 x 10^{28} conduction electrons for each metal and each conduction electron occupies a volume of $4/3\pi(1.72x10^{-10})^3$ $m^3$and $4/3\pi(1.6x10^{-10})^3$ m^3 respectively. Therefore, in a cubic meter of space, the total calculated volume of conduction electrons equals $4/3\pi(1.72x10^{-10})^3$ x 4.7 x 10^{28} $m^3$and $4/3\pi(1.6 x10^{-10})^3$ x5.86 x 10^{28} m^3 which for lithium and silver come to 1.002 m^3 and 1.005 m^3 respectively and physically impossible. Even if the total volume of the conduction electrons occupy slightly less than one cubic meter this still violates the rules of closest packing of spheres [13] which states at with hexagonal or cubic closest packing only 74.05% of the space can be occupied.

A second problem with the assumptions is the calculation of r_s. Covalent radii[14], which in calculations were able to accurately replicate experimental determined inter-nuclear distances of molecules in the gas phase, are much smaller the calculated r_s calculated by the equation $r_s = (¾/(n\pi)$ for all members of Group I metals and some other metals. In the crystalline state, the nearest neighbour distances [15], which equal atomic diameters, when divided by two are smaller than calculated r_s values for Groups I and II metals.

The equation $n = 6.022x\ 10^{23}\ Z\rho/A$ can only be used to calculate the value of n when the metal possesses a single valency[16]. However, with the exception of Group I, most metals have multiple valencies. For example, vanadium has valencies ranging from two to five. Manganese has a range of valencies[17] from one to seven.

According to the theory, in a cm^3 of copper there are $8.49x10^{22}$ conduction electrons and with each conduction electron travelling at a velocity of about 1.57 x 10^6 ms^{-1}. The total amount kinetic energy of the conduction electrons amounts to approximately 4 x 10^6 J. The electrons are slowed down by collisions and the mean time between collisions is of the order of 10^{-14} seconds[18]. That means there are nearly 10^{36} collisions together in a cm^3 of copper. Since the calculated electron drift velocity in a copper wire carrying a current of 10 amperes is less than 10^{-3} ms^{-1} most of the 10^6 J of kinetic energy resulting from the numerous collisions has to be dissipated as heat and the copper wire should heat up very quickly likely to melt. However there is little evidence that occurs so we can assume that there are much fewer conduction electrons with fewer collisions and more details are available in the original work [12].

The number of free electrons for conduction

Previous work has shown that about 0.01% of valence electrons participate in current flow. At high speeds, the kinetic energy [19] of an electron is not $\frac{1}{2}mv^2$ but:

$$= \frac{1}{2}mv^2 + \frac{3}{8} mv^4/c^2 + (5/16)\, mv^6/c^4 + \text{higher terms.}$$

The term showing the relativistic gain in energy is $\frac{3}{8}mv^4/c^2$ and the other higher terms are too small can be ignored. This term can be expressed as $(\frac{1}{2} mv^2)(\frac{3}{4}\, v^2/c^2)$. The velocity of light is $3\text{x}10^8 ms^{-2}$ and the speed of a valence electron is usually over 10^6 ms^{-1}. Depending on the element, it is of the order of about 2 to 3 $\text{x}10^6 ms^{-2}$. According to quantum mechanical band theory [20] electrons with sufficient energy can be promoted into the unfilled orbitals in the conduction band and be able to conduct electricity. Only a proportion of electrons with a high velocity have enough relativistic gain in energy can be promoted to the conduction band. $(\frac{3}{4}\, v^2/c^2)$ can be approximated to about 0.0001 (or 0.01%). An electron can only be a conduction electron when the value of $(\frac{3}{4}\, v^2/c^2)$ approaches or exceed 0.0001 otherwise there is no enough gain in energy to promote the electron into the conduction band. Further evidence that shows 0.01% of the electrons participate in current flow are fully described in the original work [12] and not repeated here.

Discussion

Total kinetic energy of the conducting electrons in one cm^3 is reduced to 0.01% of 10^6 J which is 10^2J. When the number n of conduction electrons is decreased by a factor of 10^4 the drift velocity or terminal velocity increases by a factor of 10^4. Therefore the total momentum loss is much less through collisions is considerably less and the amount of momentum loss due to each collision is minute and kinetic energy loss is so small that the heat generated is not usually detectable. Again, when there is 0.01% rather than 1 *free electron* per atom the drift velocity is much higher and comparable to electron velocities under other circumstances such as in a cathode ray tube.

Conclusion

There is no experimental evidence for the assumption of one conduction electron per metal atom. This assumption is shown to be incorrect. If there is one conduction electron per atom of copper the number of collisions between the electrons themselves and with the ions will cause a copper wire to heat up as soon as a current passes. Evidence from recent work shows 0.01% of electrons per atom are responsible for current flow and the electron drift velocity is much higher.

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