Review of: "Quantum mechanics and symplectic topology"

Inge Svein Helland[1]

1 University of Oslo

Potential competing interests: No potential competing interests to declare.

Brief review of ‘Quantum mechanics and
symplectic topology’ by Andreas Henrikson.

Inge S. Helland, Professor emeritus, Dept. of Math., Univ. of Oslo

To understand the essence of this article, one needs to have some insight i symplectic geometry/topology. There is some literature on symplectic topology in connection to quantum mechanics, but as said by Ian Stewart [1] in 1987 “...we are witnessing just the tip of the symplectic iceberg.” This remark can probably also be said to be valid today. De
Gosson [2], who also has written a book in the field, said in 2009: “Unfortunately, this iceberg seems not to have received the attention it deserves in the physical literature.”

Andersson starts with saying the the uncertainty principle do not play a fundamental role in the various formulation of quantum mechanics. One subjective remark here: In a recent paper [3] by this author, essential elements of quantum mechanics are derived from the notion of two related maximal accessible variables, a notion which is very close to the uncertainty principle.

Andersson attempt to formulate the theory of quantum mechanics within the language of symplectic topology, taking as his foundation 4 postulates (; equations are written in Latex code here):

I. The state of a system is represented by its set of symplectic capacities on the complex-valued phase space.
II. The symplective capacity of a state is constructed from below by the Gromov width $c_G = h/2$.
III. The probability F that the identity of a state ξ is mistaken for a given member of its M-dimensional quantum ensemble $\{\eta_1,...,\eta_j,..., \eta_M$ is given by

$F=\sum_{i01}^M |a_j|^2 |\Omega(\xi, \eta_j]|^2 + \sum_{j=1}^M \sum_{i\ne j}^M a_j^*a_i \Omega^*(\xi,\eta_j)\Omega(\xi,\eta_i),$ (1)

where $\Omega(\xi, \eta_j$ is the overlap between the sympletic capacities of the pair of states ξ and η_j, and where a given state's quantum ensemble is supposed to be of the form

$\omega(\xi |\eta_1,...,\eta_M)=\sum_{j=1}^M a_j \Omega(\xi,\eta_j).$ (2)

IV. For a closed Hamiltonian system, the probability is conserved in time.

He remarks that the first condition seems ad hoc, but "signifies the fundamental mystery associated with the appearance of complex-valuedness in quantum mechanics which is present in all contemporary formulations". Further: "We have not been able to bring any clarity to this issue, which is thus a significant drawback to the article."

The main contribution of the article seems to be a discussion of state overlap, which leads to the probability III. The conservation of probability is related to the Schrödinger equation. The superposition principle, as given by (2), is fundamental.

The English language of the article could have been improved.

References

[1] Stewart, L.: The symplectic camel. Nature 329 (6134), 17-18 (1987).
[2] de Gosson, M.A.: The symplectic camel and the uncertainty principle: The tip of an iceberg? Foundations of Physics 39, 194-214 (2009).
[3] Helland, I.S.: On reconstructing parts of quantum theory from two related maximal conceptual variables. International Journal of Theoretical Physics 61, 69 (2022). Correction to appear.