

Possible Explanations for the Proposed $J/\psi + J/\psi$ and $J/\psi + \psi(2S)$ Tetraquark States

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Abstract

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The recently proposed $J/\psi + J/\psi$ and $J/\psi + \psi(2S)$ structures are investigated using a first-order tetraquark mass formula. This mass relationship is based on weakly bound $J/\psi(c\bar{c}) + J/\psi(c\bar{c})$ and $J/\psi(c\bar{c}) + \psi(2S)(c\bar{c})$ meson clusters. The first-order tetraquark mass formula provides a reasonable prediction compared to the two model fits of the ATLAS collaboration. The $J/\psi + J/\psi$ tetraquark model predicts a mass of $6.30 \text{ GeV}/c^2$ compared with the model fits of 6.41 and $6.65 \text{ GeV}/c^2$. Model fits for the $J/\psi + \psi(2S)$ tetraquark are 6.96 and $7.22 \text{ GeV}/c^2$ in comparison to the first-order model result of $6.88 \text{ GeV}/c^2$.

Using the first-order mass formula approach, both candidate tetraquarks states have primitive $\bar{J} = 1^- \times 0 \times 1^- = 0^+$, 1^+ , 2^+ configurations. The model is not sufficiently detailed to refine these assignments. No spin and parity assignments are noted by the ATLAS Collaboration.

KEYWORDS: $J/\psi + J/\psi$ and $J/\psi + \psi(2S)$ Tetraquarks; First-Order Mass Formula; Quark Model; Cluster Model.

1.0 Introduction

The ATLAS Detector observed an excess of dicharmonium events in the four-muon final state¹. A search in the 4μ final state produced through the $J/\psi + J/\psi$ and $J/\psi + \psi(2S)$ channels was performed using LHC proton-proton data collected by the ATLAS experiment at a center-of-mass energy of 13 TeV . Only the data where all detector systems are functional and recording high-quality data were used.

Ref. 1 estimated background structures based on a hybrid methodologies involving Monte Carlo simulations and data-driven methods. Statistically significant excesses with respect to backgrounds are noted in the $J/\psi + J/\psi$ channel consistent with a narrow resonance at 6.9 GeV , and a broader structure at lower mass. A statistically significant excess is also observed in the $J/\psi + \psi(2S)$ channel. The fitted masses and decay widths of the structures are reported in Ref. 1.

In this paper, the first-order tetraquark mass formulas of Refs. 2 - 17 are applied to evaluate the possible mass and \bar{J} values of the $J/\psi + J/\psi$ and $J/\psi + \psi(2S)$ tetraquarks. This mass relationship is based on weakly bound $J/\psi + J/\psi$ and J/ψ

+ $\Psi(2S)$ structures based on bound $J/\Psi(c\text{-bar}) + J/\Psi(c\text{-bar})$ and $J/\Psi(c\text{-bar}) + \Psi(2S)(c\text{-bar})$ meson clusters.

2.0 Model and Formulation

Zel'dovich and Sakharov^{18,19} proposed a semiempirical mass formula that provides a prediction of mesons and baryons in terms of effective quark masses. Within this formulation, quark wave functions are assumed to reside in their lowest 1S state. These meson mass formulas are used as the basis for deriving a first-order tetraquark mass formula. In particular, the model proposed in this paper assumes the tetraquark is partitioned into two meson clusters with the interaction between the clusters providing a minimal contribution to the tetraquark mass.

The meson mass (M_m) formula of Refs. 2 - 17 is:

$$M_m = \delta_m + m_1 + m_2 + b_m \frac{m_0^2}{m_1 m_2} \sigma_1 \cdot \sigma_2 \quad (1)$$

where m_1 (m_2) are the mass of the first (second) quark comprising the meson, m_0 is the average mass of a first generation quark^{18,19}, and the σ_i ($i = 1$ and 2) are the spin vectors for the quarks incorporated into the meson. The parameters δ_m and b_m are $40 \text{ MeV}/c^2$ and $615 \text{ MeV}/c^2$, respectively¹⁹.

The last term in Eq. 1 represents the spin-spin interaction of the quarks and $\sigma_1 \cdot \sigma_2$ is the scalar product of the quark spin vectors. $\sigma_1 \cdot \sigma_2$ has the value $-3/4$ and $+1/4$ for pseudoscalar and vector mesons, respectively¹⁹.

In formulating the tetraquark mass formula, effective quark masses provided by Griffiths²⁰ are utilized. These effective masses for d, u, s, c, b, and t quarks are 340, 336, 486, 1550, 4730, and 177000 MeV/c^2 , respectively. The effective masses are utilized in Eq. 1.

These six quarks are arranged in three generations: $[d(-1/3), u(+2/3)]$, $[s(-1/3), c(+2/3)]$, and $[b(-1/3), t(+2/3)]$ ²¹. The three generations are specified by the square brackets and the quark charges [in elementary charge units (e)] are given within parentheses.

3.0 First-Order Mass Formula for $J/\Psi + J/\Psi$ and $J/\Psi + \Psi(2S)$ Tetraquarks

The spin of a tetraquark within the first-order mass formula is determined by coupling the two meson clusters

$$J^\pi = J^\pi(1) \otimes L \otimes J^\pi(2)$$

where the first-order mass formula assumes a minimally interacting $L=0$ configuration²⁻¹⁷ between the meson clusters. Eq. 2 provides a primitive J^π assignment using the $J^\pi = 1^-$ spin and parity assignment for the J/Ψ and $\Psi(2S)$ mesons.

The first-order mass formula used in this paper partitions the tetraquark into two meson clusters. These clusters include the J/Ψ vector meson (v_m) and the $\Psi(2S)$ vector meson. Using these structures, two tetraquark configurations are

evaluated: $J/\psi + J/\psi$ and $J/\psi + \psi(2S)$ as follows:

$$M(J/\psi + J/\psi) = M(J/\psi) + M(J/\psi) + \Phi(3)$$

$$M(J/\psi + \psi(2S)) = M(J/\psi + J/\psi) + \Delta(\psi(2S) - J/\psi) + \Phi(4)$$

where Φ defines the interaction between the meson clusters, and Δ is the $\psi(2S)$ – $J/\psi(1S)$ mass difference ($589.2 \text{ MeV}/c^2$)²¹. Within the scope of this mass formula, the meson-meson cluster interaction is assumed to be weak and sufficiently small to be ignored. Accordingly, Eqs. 3 and 4 represent the tetraquark structures as modeled as quasimolecular four quark systems whose basic character is a weakly bound meson-meson system where the mesons reside in their ground states.

4.0 Results and Discussion

The angular momentum coupling from Eq. 2 and the first-order mass formula of Eqs. 1, 3, and 4 are used to construct the $J/\psi + J/\psi$ and $J/\psi + \psi(2S)$ tetraquark states. First-order mass formula model results are summarized in Table 1.

Configuration	Data: Model 1 ^a Mass (GeV/c^2)	Data: Model 2 ^a Mass (GeV/c^2)	First-Order Mass Formula ^b Mass (GeV/c^2)
$J/\psi + J/\psi$	6.41	6.65	6.30
$J/\psi + \psi(2S)$	7.22	6.96	6.88

^a Ref. 1. ^b This work.

The spin and parity assignments for these states are derived from Eq. 2. All first-order mass formula states have the primitive $J^\pi = 1^- \times 0 \times 1^- = 0^+, 1^+, 2^+$ configurations. The model is not sufficiently detailed to refine these assignments.

The $J/\psi + J/\psi$ configuration first-order mass formula leads to a predicted mass of $6.30 \text{ GeV}/c^2$. This result is about 2% (6%) smaller than the first (second) model result of 6.41 (6.65) GeV/c^2 extracted from the data. In a similar manner, the $J/\psi + \psi(2S)$ configuration first-order mass formula leads to a predicted mass of $6.88 \text{ GeV}/c^2$. This result is about 5% (2%) smaller than the first (second) model result of 7.22 (6.96) GeV/c^2 extracted from the data.

Although these results are encouraging, they are based on a first-order mass formula with a number of uncertainties including the assumed quark masses^{20,21} and the magnitude of the meson-meson cluster interaction. However, the model does provide an initial description of the $J/\psi + J/\psi$ and $J/\psi + \psi(2S)$ tetraquark states that are in reasonable agreement with the experimental data¹.

The reader should also note that the $J/\psi + J/\psi$ and $J/\psi + \psi(2S)$ tetraquark states summarized in Table 1, and those noted in Ref. 1 are varied in structure. This complexity is not completely represented by a first-order mass formula.

5.0 Conclusions

The recently proposed $J/\psi + J/\psi$ and $J/\psi + \psi(2S)$ structures are investigated using a first-order tetraquark mass formula. However, the tetraquark states summarized in Table 1 and Ref. 1 are varied in structure. This complexity is not completely represented by a first-order mass formula.

This mass relationship is based on weakly bound $J/\psi + J/\psi$ and $J/\psi + \psi(2S)$ meson clusters. The first-order tetraquark mass formula provides a reasonable prediction compared to the two model fits of the ATLAS collaboration. The $J/\psi + J/\psi$ tetraquark model predicts a mass of $6.30 \text{ GeV}/c^2$ compared with the model fits of 6.41 and $6.65 \text{ GeV}/c^2$. Model fits for the $J/\psi + \psi(2S)$ tetraquark are 6.96 and $7.22 \text{ GeV}/c^2$ in comparison to the first-order model result of $6.88 \text{ GeV}/c^2$.

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References

- 1) ATLAS Collaboration, Observation of an Excess of Dicharmonium Events in the Four-Muon Final State with the ATLAS Detector, *Phys. Rev. Lett.* **131**, 151902 (2023)
- 2) J. J. Bevelacqua, First-Order Tetraquark Mass Formula, *Physics Essays***29**, 198 (2016).
- 3) J. J. Bevelacqua, Description of the X(5568) and Proposed $750 \text{ GeV}/c^2$ State in Terms of a First-Order Tetraquark Mass Formula, *Physics Essays* **29**, 367 (2016).
- 4) J. J. Bevelacqua, Fusion of Doubly Heavy Mesons into a Tetraquark, *Physics Essays***31**, 167 (2018).
- 5) J. J. Bevelacqua, Possible Tetraquark Explanation for the Proposed X(3872), *Physics Essays***32**, 469 (2019).
- 6) J. J. Bevelacqua, Description of the X(6900) as a Four Charmed Quark State in Terms of a First-Order Tetraquark Mass Formula, *Qeios* **KLXLKJ**, 1 (2020). <https://doi.org/10.32388/KLXLKJ>.
- 7) J. J. Bevelacqua, Description of the X(2900) as an Open Flavor Tetraquark in Terms of a First-Order Mass Formula, *Qeios*, **OVLMEB**, 1 (2020). <https://doi.org/10.32388/OVLMEB>.
- 8) J. J. Bevelacqua, Possible Tetraquark Explanation for the Proposed $Z_{cs}(3985)^-$, *Qeios* **GLTEU2**, 1 (2021). <https://doi.org/10.32388/GLTEU2>.
- 9) J. J. Bevelacqua, Possible Tetraquark Explanation for the X(6200), *Qeios***J6AFYW**, 1 (2021). <https://doi.org/10.32388/J6AFYW>.
- 10) J. J. Bevelacqua, Possible Tetraquark Explanation for the T_{cc}^+ , *Qeios* **OMDGAQ**, 1 (2021). <https://doi.org/10.32388/OMDGAQ>.
- 11) J. J. Bevelacqua, Possible Tetraquark Explanation for the Proposed $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$, *Qeios* **PPLMWV**, 1

(2021). <https://doi.org/10.32388/PPLMWV>.

12) J. J. Bevelacqua, Possible Tetraquark Explanation for the Proposed X(3960), Qeios **O1L0YM**, 1 (2022).
<https://doi.org/10.32388/O1L0YM>.

13) J. J. Bevelacqua, Possible Tetraquark Explanation for the Proposed T(2900)⁺ and T(2900)⁰ Structures, Qeios **V6WLTS**, 1 (2022). <https://doi.org/10.32388/V6WLTS>.

14) J. J. Bevelacqua, Possible K K bar Tetraquark Explanation for the $\chi(1370)$, Qeios **HBDQXV**, 1 (2023).
<https://doi.org/10.32388/HBDQXV>.

15) J. J. Bevelacqua, Possible f Quark Model of Tetraquarks and Pentaquarks, Qeios **8T3IVE**, 1 (2023).
<https://doi.org/10.32388/8T3IVE>.

16) J. J. Bevelacqua, Possible Tetraquark Explanation for the Y(10753), Qeios **NZRGH3**, 1 (2023).
<https://doi.org/10.32388/NZRGH3>.

17) J. J. Bevelacqua, Possible Tetraquark Explanation for the $\psi(4230)$, $\psi(4360)$, and $\psi(4415)$, Qeios **D5HKO0**, 1 (2024).
<https://doi.org/10.32388/D5HKO0>.

18) Ya. B. Zel'dovich and A. D. Sakharov, *Kvarkovaya struktura i massy sil'novzaimodeistvuyushchikh chastits*, Yad. Fiz. **4**, 395 (1966).

19) A. D. Sakharov, Mass formula for mesons and baryons, Sov. Phys. JETP **51**, 1059 (1980).

20) D. Griffiths, *Introduction to Elementary Particles*, 2nd ed., (Wiley-VCH, Weinheim, 2008).

21) Particle Data Group, Review of Particle Physics, Prog. Theor. Exp. Phys **2022**, 083C01 (2022).