# Review of: "Bell's theorem is an exercise in the statistical theory of causality"

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Bell's theorem plays a crucial role in present discussions about the foundations of quantum mechanics and the role played by measurements. The theorem lies behind the bold claims about the nature of physical reality (retrocausality, many worlds, bayesianism,...) made upon the confirmed violation of certain constraints (Bell's inequalities) observed in experiments in which two widely separated labs share pairs of entangled qubits (Bell experiments). In this paper, the author proposes a slightly re-elaborated proof of Bell's theorem in terms of graphs.

For decades, Bell's theorem attracted the attention of only a small community of physicists interested in the foundations of quantum mechanics, separated into two fiercely opposed camps by a deep rift: on one side, the "believers" - as the author of this paper usually describes himself - and, on the other side, a "vociferous opposition" - as he describes them in the Introduction of his paper.

The origin of this unfortunate rift can be found in the fact that the original formulation of Bell's theorem, as well as the decades-long discussions that it ignited, lack a clear description of the physics involved in the experiments and, moreover, quite often the arguments - both those of the "believers" and those of their "opposition" - are loose and imprecise in their maths.  This paper by Prof. R Gill, as well as the papers that it supposedly comes to refute by Prof. M. Kupczynski, are clear examples of the described situation. As a result, the discussions between the two camps long forever, without even a glimpse of little advance.

Bell's theorem may be stated in a very clear and concise way (analogous to the proof of the well-known Boole-Frechet inequalities of probabilistic logic), which is, however, useless for the interpretation of the results of actual experiments, as we shall explain below. This version of the theorem goes as follows:

Given a spreadsheet with four columns of binary entries, +1 or -1, denoted as x1, x2, y1, y2, the following constraint on their corresponding correlations averaged over the many rows holds:

|E(x1, y1) + E(x1, y2) + E(x2, y1) - E(x2, y2)| . le. 2

This constraint is known in the literature as the CHSH-Bell inequality. Its proof is straightforward, since for any row of entries,

x1 * (y1 + y2) + x2 * (y1 - y2) = -2 .or. +2,

given that they all are all binary variables. Averaging over all the rows, the inequality follows.

The heated discussion about the implications of this theorem stems from the results obtained in Bell experiments, in which the measured correlations do violate the above CHSH-Bell constraint, raising questions about the mechanism responsible for the violation.

None of these experiments, nonetheless, do actually produce the rows of four binary outcomes required by the strict proof of Bell's theorem presented above, hence, rendering the theorem irrelevant for the interpretation of the results of these experiments, as we had already advanced. Bell experiments do produce only two binary outcomes, x and y, for each pair of shared qubits, and, therefore, the four correlations needed to test the CHSH-Bell inequality must be defined differently: each term is actually measured in a different subset of realizations of the experiment. Hence, new assumptions are needed in order to make the above CHSH-Bell constraint relevant to these experiments. The question is, obviously, if these additional assumptions are fulfilled in the actual experiments. If they are not, then it should not come as a surprise that neither the inequality is fulfilled.

Which are these assumptions, and how do they appear in the proof of the theorem presented by R. Gill in the paper reviewed here?

Let's start by reviewing the notion of causal connection, represented in the reviewed paper as a directed link between nodes representing random variables, while the absence of a link indicates that the variables are causally disconnected. The notion of causality emerges in physics as a result of the Lorentz covariance of its fundamental laws: two events that lie out each other's light-cone cannot be the cause/effect of each other. Lorentz covariance, in turn, is imposed by the principle of relativity, which states that the fundamental laws of physics are invariant with respect to any two frames of reference related by a Lorentz transformation. In other words, the very notion of causality stems from the principle of relativity, according to which physical magnitudes can be properly defined only with respect to some reference frame, and transform according to well-defined rules when the frame of reference with respect to which they are described does change.

In the simplest version of Bell experiment, the two binary inputs, denoted as a and b in the reviewed paper, correspond to two possible orientations of a detector located at each one of the two labs participating in the experiment, which measure the polarization of their respective incoming qubits and produce each one a binary outcome, x and y. Since the pair of entangled qubits lacks any preferred direction, one may wonder how is it that two possible orientations for each detector play such a crucial role in the experiment: the principle of relativity mentioned above demands that only the relative orientation of the two detectors should be physically meaningful, while a rigid rotation of the two detectors should play no role (since it depends on the reference frame chosen to describe it)! In fact, in the actual experiments all pairs of entangled qubits are tested by two detectors oriented at a relative angle of 45 degrees (a measurement at a relative angle of 135 degrees is equivalent to one at 45 degrees up to a relabelling of the outcomes). The interpretation of the results, on the other hand, relies crucially on additional labels arbitrarily given by the experimenters to each one of the many repetitions of the experiment in order to assign them values to the inputs a and b.

This observation is crucial to understand when the proof of Bell's theorem provided by R. Gill holds and when it does not. The proof of the CHSH-Bell inequality requires a) an object, denoted in the paper as Lambda_H, to be shared between the two labs, and b) functions f(a,Lambda_H) and g(b,Lambda_H) defined respectively at each one of them. The definition of these two functions thus requires that the object Lambda_H can be properly defined together with the two possible orientations of each one of the two detectors, corresponding to a=-1,+1 and b=-1,+1. Otherwise, these functions could not be defined. Hence, all that is needed to bypass this proof of Bell's theorem is that the said object Lambda_H will not be properly defined at once with respect to the frames defined by the four possible orientations of the two detectors. In other words, all that is needed to overcome the constraints imposed by the theorem is a holonomy in the definition of the object Lambda_H. This subtle bypass of the proof provided by R. Gill is physically acceptable since only two such frames are actually realized for each pair of qubits, and as we noticed above only the relative orientation of the two detectors is a properly defined physical magnitude [1,2,3].

Finally, I would like to add that in spite of my above criticism of the proof of Bell's theorem provided by R. Gill, I agree with the criticism that he raises about the claims made by M. Kupczynski in his papers. This second author introduces additional random variables in the description of the responses of the detectors, and claims that they prevent the proper definition of the functions f and g mentioned above, so that Bell's theorem does not hold in such a case and the inequality can be violated up to the algebraic limit if enough new variables are available. Nonetheless, one may immediately notice that as long as these additional random variables are statistically independent from lambda_H, they can be integrated out, so that average responses f and g can still be defined, and Bell's constraint follows in a straightforward way. In response to this observation, which M. Kupczynski is aware of, he claims - using hand-waving arguments - that this straightforward derivation of Bell's constraint is not applicable to his model because the four subsets of realizations of the experiment from which the four correlations are measured must be defined on disjoint probability spaces (!?). However, since in Bell experiments all pairs of entangled qubits are tested at a relative angle of 45 degrees between the detectors, as we already remarked above, one must wonder how is it that repetitions labelled as belonging to the same subset are described by one probability space, while identical realizations labelled as belonging to different subsets need disjoint probability spaces? Last, M. Kupczynski is also wrong when he argues that the violation of Bell's inequalities is a characteristic feature of the non-relativistic Schrodinger equation, and it is absent in Lorentz-covariant quantum field theories. For example, violation of Bell's inequality has been recently reported in top-antitop quarks pair production events at LHC [4].

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