Does energy always have mass?

Germano D'Abramo

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Germano D'Abramo
Ministero dell'Istruzione, dell'Università e della Ricerca,
00041, Albano Laziale, RM, Italy
E–mail: germano.dabramo@gmail.com
ORCID: 0000-0003-1277-7418

Abstract

One of the most widespread interpretations of the mass-energy equivalence establishes that not only can mass be transformed into energy but that every type of energy also has mass (via the mass-energy equivalence formula $m = \mathcal{E}/c^2$). Here, we show that this is not always the case. With a simple thought experiment, we show that, for instance, the electric potential energy of a charged capacitor should not contribute to the capacitor's rest mass (while still contributing to its linear momentum).

Keywords: special relativity; general relativity; mass-energy equivalence; gravitational frequency shift; conservation of energy; conservation of linear momentum; thought experiments

1 Introduction

In a recently published paper [1], we reexamined Einstein's 1905 derivation of mass-energy equivalence [2]. Einstein's original approach consisted in studying, in different reference frames, the energy balance of a body emitting electromagnetic radiation. In our paper, we showed that an unsupported assumption stands behind the validity of Einstein's celebrated result, namely that the motion of the body, in the form of its kinetic energy K relative to a stationary observer O, does contribute to the increase in the 'internal reservoir' of energy from which the electromagnetic emission originates with

Figure 1: Does a gasoline tank in motion have more internal (chemical) energy than a stationary one? That appears to be a necessary consequence of the crucial assumption made by Einstein in his 1905 derivation of mass-energy equivalence [1].

respect to O. We pointed out that with electromagnetic emissions or with any non-mechanical process, the consequences implied by that assumption are not unproblematic. As a matter of fact, in cases like those, it is much like taking for granted that, for instance, the kinetic energy of an electric battery in motion relative to us can contribute, for us, to the increase in the electrical energy content of that battery. Or that the kinetic energy of a car in motion relative to us can contribute, for us, to the increase in the energy content of the gasoline and, ultimately, to the increase in the gasoline mass (see Fig. 1).

Moreover, in the same paper, we gave strong evidence that the mentioned Einstein's assumption is logically equivalent, although not in a trivial way, to assuming mass-energy equivalence from the outset. We concluded that Einstein's original result was not *proving* that mass and energy are equivalent but, more correctly, that *if* mass transforms into energy, it does it according to the relation $\mathcal{E} = mc^2$.

Furthermore, inspired by the abovementioned results, we ended up asking whether energy always has mass. To be precise, if and when mass transforms into energy, like, for instance, in nuclear reactions (fission, fusion, annihilation, etc.), mass and energy are indeed related according to the equation $\mathcal{E} = mc^2$. However, the question is whether every form of energy (heat, electrical or gravitational potential energy, etc.) always has an inertial/gravitational mass.

At the end of [1], we questioned that indiscriminate energy-to-mass conversion belief by analyzing and revising the following thought experiment by Misner, Thorne, and Wheeler [3] on the gravitational frequency shift derived from the conservation of energy:

> That a photon must be affected by a gravitational field Einstein (1911) showed from the law of conservation of energy, applied in the context of Newtonian gravitation theory. Let a particle of rest mass m start from rest in a gravitational field g at point $\mathcal{A}$ and fall freely for a distance h to point $\mathcal{B}$. It gains kinetic energy mgh. Its total energy, including rest mass, becomes
>
> $$m + mgh.$$
>
> Now, let the particle undergo an annihilation at $\mathcal{B}$, converting its total rest mass plus kinetic energy into a photon of the same energy. Let this photon travel upward in the gravitational field to $\mathcal{A}$. If it does not interact with gravity, it will have its original energy on arrival at $\mathcal{A}$. At this point it could be converted by a suitable apparatus into another particle of rest mass m (which could then repeat the whole process) plus an excess energy mgh that costs nothing to produce. To avoid this contradiction of the principal [*sic*] of conservation of energy, which can also be stated in purely classical terms, Einstein saw that the photon must suffer a red shift. [*The speed of light is set as* $c = 1$]

Unfortunately, Misner, Thorne, and Wheeler's argument appears to be problematic. If a particle of rest mass m starts from rest in a gravitational field g at point $\mathcal{A}$ and falls freely for a distance h to point $\mathcal{B}$, that particle possesses also an energy equal to mgh already at point $\mathcal{A}$. It is called gravitational potential energy. Therefore, *owing to the complete mass-energy equivalence*, at point $\mathcal{A}$, that particle already has a total mass/energy equal[1] to $m + mgh$. Now, if the energy of the photon generated in the particle annihilation and traveling upward does not have its original value on arrival at $\mathcal{A}$ (i.e., $m + mgh$), the mass of the particle created by the suitable apparatus at the end of the process would not have the same mass as the original particle (again, $m + mgh$), and the total energy/mass would not be

[1] It can be shown that, in a uniform gravitational field g, the mass m_h of a particle at height h is $m_h = me^{\frac{gh}{c^2}}$, where m is the proper mass at height taken as zero. The total energy E_{tot}, proper mass plus gravitational potential energy, at height h is given by $E_{tot} = mc^2 e^{\frac{gh}{c^2}}$. For small distances h, we have $m_h \simeq m + \frac{mgh}{c^2}$ and $E_{tot} \simeq mc^2 + mgh$. By assuming $c = 1$, like in [3], we have that the mass and the total energy of the particle at the height h (point $\mathcal{A}$ in [3]) are $m + mgh$.

conserved. When Misner, Thorne, and Wheeler say that the particle "gains kinetic energy mgh" on arrival at point $\mathcal{B}$, and "its total energy, including rest mass, becomes $m + mgh$", they seem to forget that the particle already has a gravitational potential energy mgh, and total energy $m + mgh$, just before starting to fall. That is demanded by the principle of conservation of energy.

Therefore, the widely-held assumption that every energy always has mass is at odds with the conservation of energy and the existence of the gravitational frequency shift taken together. The thought experiment by Misner, Thorne, and Wheeler pits the above three assumptions one against the other. They cannot be simultaneously true. However, we concluded our paper [1] by saying that it is still not clear which one, among the three, is actually at fault. The only exception we felt like making was for the conservation of energy.

The present paper aims to clarify that issue. First, by applying energy and linear momentum conservation, we prove that, in the case of Misner, Thorne, and Wheeler's derivation, the gravitational potential energy of a body does, in fact, have mass and does contribute to the total mass of the body (Section 2). Within that proof, we also show that the gravitational frequency shift is incompatible with the conservation of linear momentum. Therefore, returning to the conclusion of the paper [1], the culprit seems to be the soundness of the gravitational frequency shift phenomenon.

In Section 3, we provide a different proof showing that the gravitational frequency shift, taken alone, is incompatible with energy conservation. That proof does not require the assumption of complete mass-energy equivalence. In particular, we do not even need to assume that the gravitational potential energy of a body contributes to the total mass of the body, as we have done in our revision of Misner, Thorne, and Wheeler's derivation.

Finally, in Section 4, by using the same type of thought experiment given in Section 3, we prove that energy does not always have mass. Specifically, we analyze the case of the energy stored in a charged capacitor. We show that the electric potential energy of a charged capacitor does not contribute to the capacitor's rest mass while still contributing to its momentum.

In the concluding section, we summarize the results achieved in this paper.

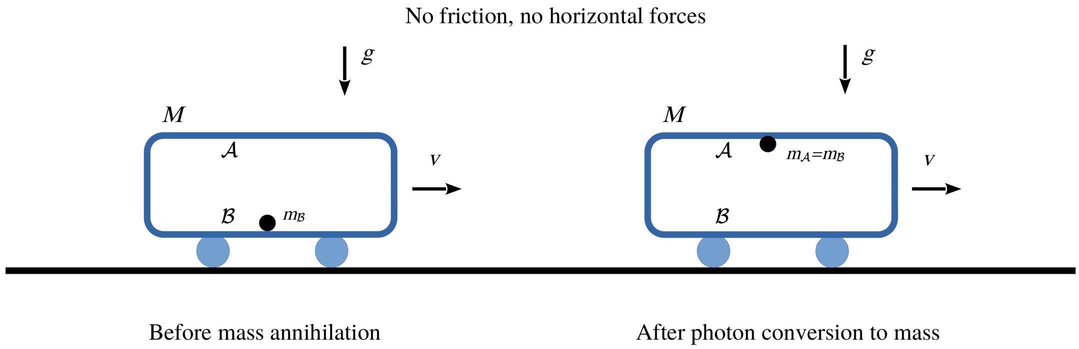

Figure 2: Pictorial representation of the thought experiment described in Section 2.

2 Gravitational frequency shift and linear momentum conservation

Here, we show that the gravitational potential energy of a body contributes to the total mass of the body, as assumed in our analysis of Misner, Thorne, and Wheeler's derivation and footnote 1 of the present paper. Consider the following ideal experiment. A closed wagon of mass M moves horizontally without friction in a vertical uniform gravitational field g at a constant velocity v (see Fig. 2). Inside the wagon, attached to floor $\mathcal{B}$, there is a particle of mass $m_{\mathcal{B}}$. At a certain point, mass $m_{\mathcal{B}}$ annihilates into a photon of energy $h\nu_{\mathcal{B}} = m_{\mathcal{B}}c^2$. Then, the photon travels upward toward ceiling $\mathcal{A}$ and is absorbed and converted by a suitable apparatus into another particle of mass $m_{\mathcal{A}}$. This particle also ends up stuck to the wagon frame. The whole process happens exclusively inside the closed wagon. Owing to the conservation of energy, we must have that $h\nu_{\mathcal{B}} = m_{\mathcal{A}}c^2 + m_{\mathcal{A}}gh$, but, as is widely believed, the mass of the generated particle at point A does not include the equivalent mass of its gravitational potential energy $m_{\mathcal{A}}gh/c^2$.

In reality, the total mass of the particle generated at point A must be $m_{\mathcal{A}} + m_{\mathcal{A}}gh/c^2 = h\nu_{\mathcal{B}}/c^2 = m_{\mathcal{B}}$, and therefore, it must include the equivalent mass of its own gravitational potential energy. Any different scenario seems to violate the conservation of (the horizontal) linear momentum of the closed system wagon+particle. No horizontal external forces act upon the system,

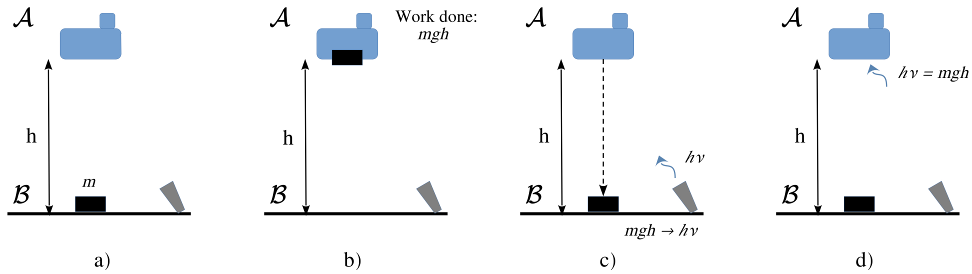

Figure 3: Pictorial representation of the thought experiment described in Section 3.

and no mass is ejected. Therefore, the total velocity v must be the same before and after the whole process. However, before the annihilation, the total horizontal linear momentum is $P_i = (M + m_{\mathcal{B}})v$ while, after the conversion of the photon energy into mass, the total horizontal linear momentum becomes $P_f = (M + m_{\mathcal{A}})v < P_i$. That is quite bizarre. On the other hand, by imposing the conservation of the horizontal linear momentum, we would have an equally strange consequence. Without any horizontal external force acting upon the wagon and without any mass ejection, we would see the wagon increase its velocity by itself at the end of the whole process.

Incidentally, the above argument confirms that there is a problem with the gravitational redshift: if the total mass of the particle generated at point $\mathcal{A}$ is still $m_{\mathcal{B}}$, the energy of the photon from which it derives is $m_{\mathcal{B}}c^2 = h\nu_{\mathcal{B}}$, namely, the frequency of the photon at point $\mathcal{A}$ must be the same as that at point $\mathcal{B}$, $\nu_A = \nu_B$.

3 Gravitational frequency shift and the conservation of energy

Here, we give a different proof that photon (radiation) energy is not affected by a gravitational field. In the following thought experiment, the assumption of complete mass-energy equivalence is not used. In particular, we do not even need to assume that the gravitational potential energy of a body contributes to the total mass of the body as we have done in our revision of Misner, Thorne, and Wheeler's derivation.

Consider a body of mass m stationary at point $\mathcal{B}$ and a macroscopic apparatus stationary at point $\mathcal{A}$, at a height h above point $\mathcal{B}$ in a uniform gravitational field g (Fig. 3). Let the apparatus perform mechanical work on body m, raising it to point $\mathcal{A}$. The work done by the apparatus is equal to mgh, which is also equal to the gravitational potential energy of the body m relative to point $\mathcal{B}$. Now, if the mass is lowered back to point $\mathcal{B}$ and its potential energy conventionally (and entirely) converted into electrical energy and then into a single photon of energy mgh (ultimately emitted by a beacon), the energy of the photon must always be the same while climbing up the gravitational field back to point $\mathcal{A}$. The photon energy at point $\mathcal{A}$ must still be equal to mgh. That is demanded by the conservation of energy. Through photon absorption, the apparatus must regain the same energy expended at the beginning of the cycle on m. Therefore, owing to the Planck-Einstein formula $E = h\nu$ (where h is the Planck constant), the photon frequency ν must be the same at points $\mathcal{A}$ and $\mathcal{B}$.

To emphasize the above conclusion, consider the cycle in reverse. The first step now consists of the crane emitting a photon of energy E' (frequency ν') suitably lower than mgh. The original energy E' is such that when the photon arrives at the beacon, it becomes equal to $E_b = mgh$ ($> E'$) owing to the standard gravitational redshift (blueshift in this case). In this way, E_b is what is exactly needed to raise the mass m to the crane at the height h. Then, the mass is released back to the initial position, and the energy coming from that release (mgh) goes into the crane reservoir. At the end of the cycle, the crane will gain positive energy ($mgh - E' > 0$) out of nowhere.

4 No, energy does not always have mass!

Now, we have all the tools to show that energy does not always have mass. With the following thought experiment, we prove that, for instance, the electrical potential energy of a capacitor does not contribute to the capacitor mass.

As in Section 3, consider an apparatus of mass m initially standing at point $\mathcal{B}$ in a uniform gravitational field g (see Fig. 4). This time, the apparatus can convert the incoming radiation energy into electrical potential energy inside a capacitor. The first step of the cyclic process to be shown consists in raising the apparatus from point $\mathcal{B}$ to point $\mathcal{A}$ at a height h above $\mathcal{A}$. The work done on m is equal to mgh, which also corresponds to the gravitational

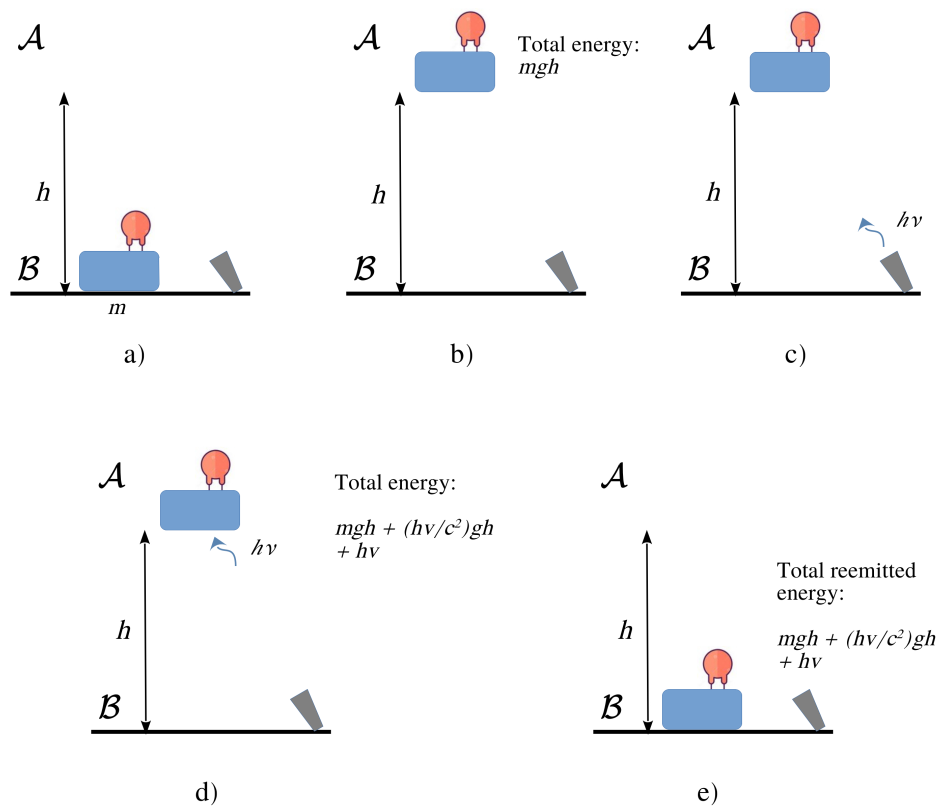

Figure 4: Pictorial representation of the thought experiment described in section 4.

potential energy of the apparatus at point $\mathcal{A}$. Then, a photon of energy $h\nu$ is emitted from a beacon at point $\mathcal{B}$ towards the apparatus at point $\mathcal{A}$. As established in Section 2, that energy must not change in climbing up the gravitational field, and, upon absorption by the apparatus, it is stored in a capacitor as electrical potential energy of the same value $h\nu$.

Now, if the widely-held interpretation that every energy always has mass is correct, then, upon absorption, the apparatus gains a mass equal to $\frac{h\nu}{c^2}$. Therefore, the total energy of the apparatus becomes

$$E_{tot} = mgh + \frac{h\nu}{c^2} gh + h\nu, \tag{1}$$

where mgh is the gravitational potential energy of the apparatus, $\frac{h\nu}{c^2}gh$ is the gravitational potential energy of mass $h\nu/c^2$, and $h\nu$ is the energy of the charged capacitor.

As soon as the cycle is completed by lowering the apparatus and discharging the capacitor, the total re-emitted energy E_{out} needs to be equal to that given by equation (1). That is required by the conservation of total energy. The problem should now be evident. The input energy E_{in} throughout the whole cycle is $E_{in} = mgh + h\nu$ while the output energy is $E_{out} = mgh + \frac{h\nu}{c^2}gh + h\nu$: we have gained an extra-energy $\frac{h\nu}{c^2}gh$ out of nowhere.

The only possibility to resolve this paradox in compliance with the principle of conservation of energy is to accept that the energy $h\nu$ stored as electrical potential energy in the capacitor does not have mass.

There remains one thing to notice. If we do not want to contradict the conservation of linear momentum, the energy stored in the charged capacitor has no mass but must still have linear momentum. If, like in the thought experiment in Section 2, the apparatus and the capacitor move horizontally at a constant velocity v, the charged capacitor must have an additional linear momentum equal to $\frac{\mathcal{E}}{c^2}v$, where $\mathcal{E}$ is the electrical potential energy in the capacitor (see also [17]). That is not strange. There is another well-known electromagnetic phenomenon that has momentum but no (rest) mass: that is light.

5 Conclusions

The actual meaning and correct interpretation of the celebrated mass-energy equivalence $\mathcal{E} = mc^2$ is still a matter of discussion among scholars. For a far-from-complete collection of references to the existing literature on mass-energy equivalence derivation, discussion, and interpretation, see, for instance, [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16], and references therein. However, it is not the goal of the present paper to enter such a debate. The aim is instead to present a simple thought experiment that shows that energy does not always have mass. For instance, when (radiation) energy is stored in a reusable form, e.g., the electrical potential energy of a capacitor, that energy does not contribute to the mass of the device storing it while still contributing to its linear momentum. We acknowledge that such a result has fundamental consequences for physics as we know it (e.g., the validity of the equivalence principle), but the derivation is too straightforward to ignore. Moreover, to this author, our results seem to answer a puzzle relative to a sort of 'doubling of energy'. For example, if radiation energy is transformed into and stored under the form of (capacitor) electrical potential energy, why should it become mass too? Isn't mass a further way to store the same energy already stored (and ready to use) as electrical potential energy? To this author, this always appeared to be a 'doubling of energy'.

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