Does energy always have mass?

Germano D'Abramo

v1

Sep 07, 2023

https://doi.org/10.32388/Z2NILS

Does energy always have mass?

Germano D'Abramo
Ministero dell'Istruzione, dell'Università e della Ricerca,
00041, Albano Laziale, RM, Italy
E–mail: germano.dabramo@gmail.com
ORCID: 0000-0003-1277-7418

Abstract

One of the most widespread interpretations of the mass-energy equivalence establishes that not only can mass be transformed into energy but that every type of energy also has mass (via the mass-energy equivalence formula $m = \mathcal{E}/c^2$). Here, we show that this is not always the case. By using two simple thought experiments, we show that, for instance, the electric potential energy of a charged capacitor should not contribute to the capacitor rest mass.

Keywords: special relativity; general relativity; mass-energy equivalence; gravitational frequency shift; conservation of energy; thought experiments

1 Introduction

In a recently published paper [1], we reexamined Einstein's 1905 derivation of mass-energy equivalence [2]. Einstein's original approach consisted in studying, in different reference frames, the energy balance of a body emitting electromagnetic radiation. In our paper, we showed that an unsupported assumption stands behind the validity of Einstein's celebrated result, namely that the motion of the body, in the form of its kinetic energy K relative to a stationary observer O, does contribute to the increase in the 'internal reservoir' of energy from which the electromagnetic emission originates with respect to O. We pointed out that with electromagnetic emissions or with

Figure 1: Does a gasoline tank in motion have more internal (chemical) energy than a stationary one? That appears to be a necessary consequence of the crucial assumption made by Einstein in his 1905 derivation of mass-energy equivalence [1].

any non-mechanical process, the consequences implied by that assumption are not unproblematic. As a matter of fact, in cases like those, it is much like taking for granted that, for instance, the kinetic energy of an electric battery in motion relative to us can contribute, for us, to the increase in the electrical energy content of that battery. Or that the kinetic energy of a car in motion relative to us can contribute, for us, to the increase in the energy content of the gasoline and, ultimately, to the increase in the gasoline mass (see Fig. 1).

Moreover, in the same paper, we gave strong evidence that the mentioned Einstein's assumption is logically equivalent, although not in a trivial way, to assuming mass-energy equivalence from the outset. We concluded that Einstein's original result was not *proving* that mass and energy are equivalent but, more properly, that *if* mass transforms into energy, it does it according to the relation $\mathcal{E} = mc^2$.

Furthermore, inspired by the above-mentioned results, we ended up asking whether energy always has mass. To be precise, if and when mass transforms into energy, like, for instance, in nuclear reactions (fission, fusion, annihilation, etc.), mass and energy are indeed related according to the equation $\mathcal{E} = mc^2$. However, the question is whether every form of energy (heat, electrical or gravitational potential energy, etc.) does always have an inertial/gravitational mass.

At the end of [1], we questioned that indiscriminate energy-to-mass conversion belief by analyzing and revising the following thought experiment by Misner, Thorne, and Wheeler [3] on the gravitational frequency shift derived from the conservation of energy:

> That a photon must be affected by a gravitational field Einstein (1911) showed from the law of conservation of energy, applied in the context of Newtonian gravitation theory. Let a particle of rest mass m start from rest in a gravitational field g at point $\mathcal{A}$ and fall freely for a distance h to point $\mathcal{B}$. It gains kinetic energy mgh. Its total energy, including rest mass, becomes
>
> $$m + mgh.$$
>
> Now, let the particle undergo an annihilation at $\mathcal{B}$, converting its total rest mass plus kinetic energy into a photon of the same energy. Let this photon travel upward in the gravitational field to $\mathcal{A}$. If it does not interact with gravity, it will have its original energy on arrival at $\mathcal{A}$. At this point it could be converted by a suitable apparatus into another particle of rest mass m (which could then repeat the whole process) plus an excess energy mgh that costs nothing to produce. To avoid this contradiction of the principal [*sic*] of conservation of energy, which can also be stated in purely classical terms, Einstein saw that the photon must suffer a red shift. [*The speed of light is set as* $c = 1$]

Unfortunately, Misner, Thorne, and Wheeler's argument is problematic. If a particle of rest mass m starts from rest in a gravitational field g at point $\mathcal{A}$ and falls freely for a distance h to point $\mathcal{B}$, that particle possesses also an energy equal to mgh already at point $\mathcal{A}$. It is called gravitational potential energy. Therefore, *owing to the complete mass-energy equivalence*, at point $\mathcal{A}$, that particle already has a total mass/energy equal[1] to $m+mgh$. Now, if the energy of the photon generated in the particle annihilation and traveling upward does not have its original value on arrival at $\mathcal{A}$ (i.e., $m + mgh$), the mass of the particle created by the suitable apparatus at the end of the process would not have the same mass as the original particle (again, $m+mgh$), and the total energy/mass would not be conserved. When Misner, Thorne, and Wheeler say that the particle "gains kinetic energy mgh" on arrival

[1] It can be shown that, in a uniform gravitational field g, the mass m_h of a particle at height h is $m_h = me^{\frac{gh}{c^2}}$, where m is the proper mass at height taken as zero. The total energy E_{tot}, proper mass plus gravitational potential energy, at height h is given by $E_{tot} = mc^2 e^{\frac{gh}{c^2}}$. For small distances h, we have $m_h \simeq m + \frac{mgh}{c^2}$ and $E_{tot} \simeq mc^2 + mgh$. By assuming $c = 1$, like in [3], we have that the mass and the total energy of the particle at height h (point $\mathcal{A}$ in [3]) are $m + mgh$.

at point $\mathcal{B}$, and "its total energy, including rest mass, becomes $m + mgh$", they seem to forget that the particle already has a gravitational potential energy mgh, and total energy $m + mgh$, just before starting to fall. That is demanded by the principle of conservation of energy.

Therefore, the widely-held assumption that every energy always has mass is at odds with the conservation of energy and the existence of the gravitational frequency shift taken together. The thought experiment by Misner, Thorne, and Wheeler pits the above three assumptions one against the other. They cannot be simultaneously true. However, we concluded our paper [1] by saying that it is still not clear which one, among the three, is actually at fault. The only exception we felt like making was for the conservation of energy.

The aim of the present paper is to clarify that issue. By assuming the conservation of total energy as certain, we will show that the existence of the gravitational frequency shift, taken alone, is not compatible with energy conservation (section 2, see also [4]) and, consequently, that energy does not always have mass (section 3).

2 Gravitational frequency shift and the conservation of energy

Here, we show that photon (radiation) energy is not affected by a gravitational field. In the following thought experiment, the assumption of complete mass-energy equivalence is not used. The incompatibility of gravitational frequency shift and conservation of energy has been extensively treated in another paper [4], and what follows is an excerpt from that work.

Consider a body of mass m stationary at point $\mathcal{B}$ and a macroscopic apparatus stationary at point $\mathcal{A}$, at a height h above point $\mathcal{B}$ in a uniform gravitational field g (Fig. 2). Let the apparatus perform mechanical work on body m, raising it to point $\mathcal{A}$. The work done by the apparatus is equal to mgh, which is also equal to the gravitational potential energy of the body m relative to point $\mathcal{B}$. Now, if the mass is lowered back to point $\mathcal{B}$ and its potential energy conventionally (and entirely) converted into electrical energy and then into a single photon of energy mgh, the energy of the photon must always be the same while climbing up the gravitational field back to point $\mathcal{A}$. The photon energy at point $\mathcal{A}$ must still be equal to mgh. That is demanded

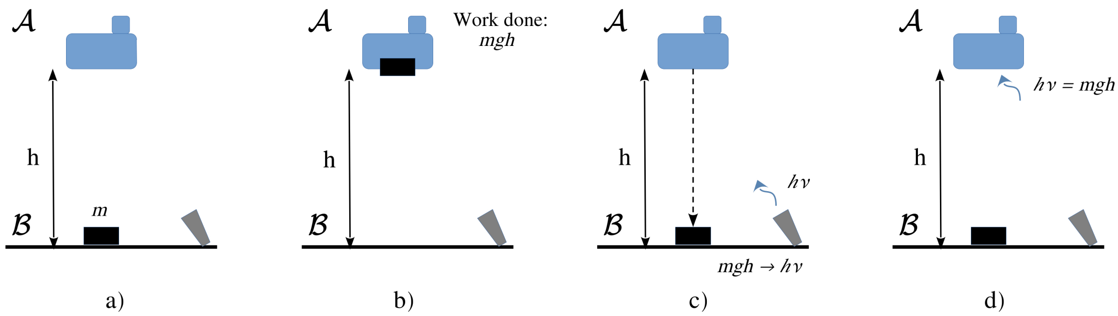

Figure 2: Pictorial representation of the thought experiment described in section 2.

by the conservation of energy. Through photon absorption, the apparatus must regain the same energy expended at the beginning of the cycle on m. Therefore, owing to the Planck-Einstein formula $E = h\nu$ (where h is the Planck constant), the photon frequency ν must be the same at points $\mathcal{A}$ and $\mathcal{B}$.

3 No, energy does not always have mass!

Now, we have all the tools to show that energy does not always have mass. With the following thought experiment, we prove that, for instance, the electrical potential energy of a capacitor does not contribute to the capacitor mass.

As in section 2, consider an apparatus of mass m initially standing at point $\mathcal{B}$ in a uniform gravitational field g (see Fig. 3). This time, the apparatus has the ability to convert the incoming radiation energy into electrical potential energy inside a capacitor. The first step of the cyclic process to be shown consists in rising the apparatus from point $\mathcal{B}$ to point $\mathcal{A}$ at a height h above $\mathcal{A}$. The work done on m is equal to mgh, which also corresponds to the gravitational potential energy of the apparatus at point $\mathcal{A}$. Then, a photon of energy $h\nu$ is emitted from a device at point $\mathcal{B}$ towards the apparatus at point $\mathcal{A}$. As established in section 2, that energy must not change in climbing up the gravitational field and, upon absorption by the apparatus, it is stored in a capacitor as electrical potential energy of the same value $h\nu$.

Now, if the widely-held interpretation that every energy always has mass

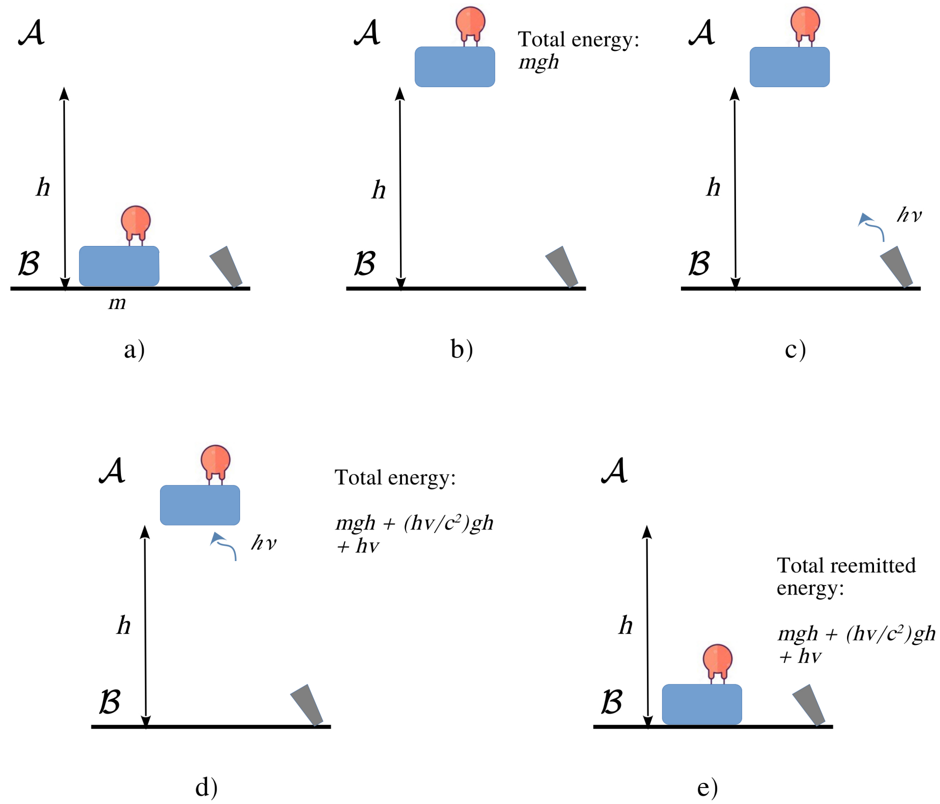

Figure 3: Pictorial representation of the thought experiment described in section 3.

is correct, then, upon absorption, the apparatus gains a mass equal to $\frac{h\nu}{c^2}$. Therefore, the total energy of the apparatus becomes

$$E_{tot} = mgh + \underbrace{\frac{h\nu}{c^2}gh}_{gravitational\ potential\ energy\ of\ mass\ h\nu/c^2} + \underbrace{h\nu}_{capacitor\ energy\ h\nu} . \tag{1}$$

As soon as the cycle is completed by lowering the apparatus and discharging the capacitor, the total re-emitted energy E_{out} needs to be equal to that given by equation (1). That is required by the conservation of total energy. The problem should now be evident. The input energy E_{in}

throughout the whole cycle is $E_{in} = mgh + h\nu$ while the output energy is $E_{out} = mgh + \frac{h\nu}{c^2}gh + h\nu$: we have gained an extra-energy $\frac{h\nu}{c^2}gh$ out of nowhere.

The only possibility to resolve this paradox in compliance with the principle of conservation of energy is to accept that the energy $h\nu$ stored as electrical potential energy in the capacitor does not have mass.

4 Discussion and conclusions

The actual meaning and correct interpretation of the celebrated mass-energy equivalence $\mathcal{E} = mc^2$ is still a matter of discussion among scholars (see, for instance, [5]). However, it is not the goal of the present paper to enter such a debate. The aim is instead to present two simple thought experiments that show that energy not always has mass. For instance, when (radiation) energy is stored in reusable form, e.g., the electrical potential energy of a capacitor, that energy does not contribute to the mass of the device storing it. We acknowledge that such a result has fundamental consequences for physics as we know it, but the derivation is too simple and straightforward to ignore. Moreover, to this author, our result seems to answer a puzzle relative to a sort of 'doubling of energy'. For example, if radiation energy is transformed into and stored under the form of (capacitor) electrical potential energy, why should it become mass too? Isn't mass a further way to store the same energy already stored (and ready to use) under the form of electrical potential energy? To this author, this always appeared to be a 'doubling of energy'.

References

[1] D'Abramo, G. Einstein's 1905 derivation of the mass-energy equivalence: is it valid? Is energy always equal to mass and vice versa? *Physics of Particles and Nuclei* **2023**, 54(5), pp. 966–971.

[2] Einstein, A. On the Electrodynamics of Moving Bodies. *Ann. Phys.* **1905**, *18*, 639–643. English translation at `https://einsteinpapers.press.princeton.edu/vol2-trans/154`

[3] Misner, C.W.; Thorne, K.S.; Wheeler, J.A. *Gravitation*; W.H. Freeman: San Francisco, USA, 1973; Sec. 7.2, p. 187.

[4] D'Abramo, G. Troubles With Gravitational Frequency Shift Derived From Energy Conservation. *Preprints* **2023**, 2023080531. https://doi.org/10.20944/preprints202308.0531.v1

[5] The Equivalence of Mass and Energy, Stanford Encyclopedia of Philosophy. https://plato.stanford.edu/entries/equivME/ (retrieved on Aug, 22, 2023).